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Ramification of Some Automorphisms of Local Fields

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Let k be a perfect field of characteristic p and let $\gamma \in \mathrm{Aut}(k((t)))$. Define the ramification numbers of γ by $i_m = v_t(\gamma^{p^m}(t) - t) - 1$. We give a characterization of the sequences (i_m) which are the sequences of ramification numbers of infinite order automorphisms of formal power series fields over finite fields. Then, given a perfect field k, we give sufficient conditions on (i_m) to be the sequence of ramification numbers of an autormorphism $\gamma \in \mathrm{Aut}_k(k((t)))$ and we investigate these sequences (i_m) in the case where there exists $\sigma \in \mathrm{Aut}_k(k((t)))$ such that $\sigma\gamma = \gamma\sigma$ with $\sigma \neq \gamma^\nu$ for all $\nu \in \mathbb{Z}_p$. © 1998 Academic Press

1. INTRODUCTION

Let k be a perfect field of characteristic $p > 0$ and set $K = k((t))$. For $\gamma \in \mathrm{Aut}_k(K)$ define $i(\gamma) = v_t((\gamma(t) - t)/t)$ and for $m \geqslant 0$ define $i_m = i(\gamma^{p^m})$.

In [2] K. Keating determines upper bounds for the i_m in some cases where γ has infinite order. He uses only elementary methods. In [6] the authors improve his results using Wintenberger's theory of fields of norms [13, 14]. In this paper we answer the question: Which sequences (i_m) are sequences of ramification numbers of automorphisms of finite extensions of K?

This question is closely related to the characterization of ramification numbers of the totally ramified $\mathbb{Z}_p$-extensions of local fields which was investigated in [1, 7, 9] and completely solved in [10]. The connection between the automorphisms of K and the $\mathbb{Z}_p$-extensions of local fields is given by the fields of norms theory [13, 14].

We use the same methods to give some indications on the ramification of two automorphisms σ and τ of K such that $\sigma\tau = \tau\sigma$.

Finally we give an erratum relating to Theorem 1 of [6].

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2. MAIN THEOREMS

Let K be as before and let $\gamma \in \mathrm{Aut}_k(K)$ be of infinite order. For $m \geqslant 0$ we set $i_m = i(\gamma^{p^m})$ as before and $u_m = i_0 + (i_1 - i_0)/p + \cdots + (i_m - i_{m-1})/p^m$. The u_m are called the ramification numbers of γ in the upper numbering or more simply the upper ramification numbers of γ. In [11] S. Sen proved that the sequence (i_m/p^m) is strictly increasing and that, for every $m \geqslant 0$, $i_{m+1} \equiv i_m \bmod p^{m+1}$; thus the u_m are integers. Moreover it can be easily deduced that $\lim_{m \to +\infty} i_m/p^m = pe/(p-1)$ where e is an integer $\geqslant 1$ or $+\infty$ and if e is finite then $u_{m+1} = u_m + e$ for any large integer m.

THEOREM 1. *Let (u_m) be an infinite increasing sequence of integers such that $u_0 \geqslant 1$ and set*

$$\mu = \min\{m \in \mathbb{N};\, u_{m+1} < pu_m\} \in \mathbb{N} \cup \{+\infty\}.$$

The following assertions are equivalent:

(i) *there exists* $e \in \mathbb{N} \cup \{+\infty\}$ *with* $(p-1)\,u_\mu/p \leqslant e < (p-1)\,u_\mu$ *(therefore:* $e = +\infty \Leftrightarrow \mu = +\infty$*) such that*

$$p \mid u_m \Rightarrow \text{either} \quad m \geqslant 1 \quad \text{and} \quad u_m \leqslant pu_{m-1},$$

$$\text{or} \quad m = \mu \quad \text{and} \quad e = \frac{(p-1)\,u_\mu}{p},$$

$$m \geqslant \mu \Rightarrow u_{m+1} = u_m + e.$$

(ii) *There exists a finite extension k' of k and $\gamma \in \mathrm{Aut}_{k'}(k'((t)))$ such that (u_m) is the sequence of upper ramification numbers of γ.*

(iii) *There exists a finite field $\mathbb{F}_q$ of characteristic p and $\gamma \in \mathrm{Aut}_{\mathbb{F}_q}(\mathbb{F}_q((t)))$ such that (u_m) is the sequence of upper ramification numbers of γ.*

Of course a translation of Theorem 1 in terms of sequences (i_m) instead of (u_m) is possible. It is useless but for the following corollary and Corollary 2 below which improve Theorem 7 of [2] and Theorem 2 of [6].

COROLLARY 1. *Let i_0 and i_1 be two integers such that $0 < i_0 < i_1$. Then there exists a finite field $\mathbb{F}_q$ and $\gamma \in \mathrm{Aut}_{\mathbb{F}_q}(\mathbb{F}_q((t)))$ such that $i(\gamma) = i_0$ and $i(\gamma^p) = i_1$ if and only if $p \mid i_1 - i_0$ and either $i_1 = pi_0$ (with $p \mid i_0$) or $i_1 > pi_0$ with $p \nmid i_0$. Moreover if $p \mid i_0$ then $i(\gamma^{p^n}) = p^n i_0$ for all $n \in \mathbb{N}$; if $p \nmid i_0$ and $i_1 < (p^2 - p + 1)\, i_0$ then $i(\gamma^{p^n}) = i_0 + ((p^n - 1)/(p-1))(i_1 - i_0)$ for all $n \in \mathbb{N}$.*

In the case where the field $K=k((t))$ is given, we obtain only sufficient conditions on the sequences (u_n) to be the sequence of upper ramification numbers of some infinite order automorphism γ of K:

THEOREM 2. *Let (u_m) be an increasing sequence of integers such that $u_0 \geqslant 1$ and let μ be as in Theorem* 1.

Let us suppose that there exists $e \in \mathbb{N} \cup \{+\infty\}$ with $(p-1)\, u_\mu/p \leqslant e < (p-1)\, u_\mu$ such that

$$p \mid u_m \Rightarrow m \geqslant 1 \qquad \text{and} \qquad u_m \leqslant p u_{m-1},$$

$$m \geqslant \mu \Rightarrow u_{m+1} = u_m + e.$$

Then there exists $\gamma \in \mathrm{Aut}_k(K)$ such that (u_m) is the sequence of upper ramification numbers of γ.

COROLLARY 2. *Let i_0 and i_1 be two integers such that $0 < i_0 < i_1$ and $p \mid i_1 - i_0$. Let us suppose that $p \nmid i_0$ and either $p i_0 \leqslant i_1 \leqslant (p^2 - p + 1)\, i_0$, or $i_1 > (p^2 - p + 1)\, i_0$ with $p \nmid i_0 + (i_1 - i_0)/p$. Then there exists $\gamma \in \mathrm{Aut}_k(K)$ such that $i(\gamma) = i_0$ and $i(\gamma^p) = i_1$.*

The proof of Theorem 1 is based on the following straightforward consequence of Theorems 7 and 8 of [7] (see also Remark 1 of [7] and for further details, the introduction of [10]).

THEOREM 3. *Let E be either a local field of equal characteristic p or a local field of inequal characteristics which contains a primitive pth root of unity. Let us suppose that the Galois group of the maximal Abelian pro-p-extension of E is a free Abelian pro-p-group. Let $e \in \mathbb{N}^* \cup \{+\infty\}$ be the absolute ramification index of E and let (u_m) be an increasing sequence of integers. Then the following assertions are equivalent.*

(iv) *There exists a totally ramified $\mathbb{Z}_p$-extension of E whose sequence of the upper ramification numbers is (u_m).*

(v) (a) *either $1 \leqslant u_0 < ep/(p-1)$ and $p \nmid u_0$ or $u_0 = ep/(p-1)$;*

(b) *if $u_m < e/(p-1)$ then $u_{m+1} = p u_m$; or $p u_m < u_{m+1} < ep/(p-1)$ and $p \nmid u_{m+1}$; or $u_{m+1} = ep/(p-1)$;*

(c) *if $u_m \geqslant e/(p-1)$ then $u_{m+1} = u_m + e$.*

Moreover if the local field E doesn't contain a primitive pth root of unity then the cases where $u_m = ep/(p-1)$ with $m = 0$ or $u_m > p u_{m-1}$ don't happen.

Remark 1. In fact the assertion (i) of Theorem 1 is equivalent to: there exists $e \in \mathbb{N}^* \cup \{+\infty\}$ such that the conditions (a), (b), and (c) of the assertion (v) in Theorem 3 are satisfied. But the formulation (i) is more adapted to the context of Theorem 1 because the index e is not given in the hypothesis.

Proof of the Remark. Let us prove that (v) $\Rightarrow$ (i). We have

$$u_{m+1} < pu_m \Rightarrow u_m \geqslant \frac{e}{p-1} \qquad \text{by (b)},$$

$$\Rightarrow u_{m+1} = u_m + e \qquad \text{by (c)},$$

$$\Rightarrow u_m > \frac{e}{p-1} \qquad \text{because} \quad \frac{e}{p-1} + e = p\,\frac{e}{p-1},$$

$$u_m > \frac{ep}{p-1} \Rightarrow u_{m-1} \geqslant \frac{e}{p-1} \qquad \text{by (b)}$$

$$\Rightarrow \mu \leqslant m-1.$$

Therefore, for $m = \mu$, we get $(p-1)\,u_\mu/p \leqslant e < (p-1)\,u_\mu$ and, for all $m \geqslant \mu$, $u_{m+1} = u_m + e$ by (c).

Now, if $p \mid u_{m+1}$ then $u_{m+1} = pu_m < u_m + e$ or $u_{m+1} = u_m + e \leqslant pu_m$; or $u_{m+1} = ep/(p-1)$ with either $u_m = e/(p-1)$ or $u_m < e/(p-1)$ and $\mu = m+1$.

Finally if $p \mid u_0$ then $u_0 = ep/(p-1)$ and $\mu = 0$.

Now let us prove that (i) $\Rightarrow$ (v).

(a) We have $u_0 \leqslant u_\mu \leqslant ep/(p-1)$ and if $p \mid u_0$ then $\mu = 0$ and $e = (p-1)\,u_0/p$.

(b) Let m be an integer such that $u_m < e/(p-1)$. Then $u_m < u_\mu$, $m < \mu$ and $u_{m+1} \leqslant u_\mu \leqslant ep/(p-1)$; moreover, since $u_{m+1} \geqslant pu_m$, in the case where $p \mid u_{m+1}$, we have either $u_{m+1} = pu_m$ or $u_{m+1} = ep/(p-1)$ and $\mu = m+1$.

(c) Let m be an integer such that $u_m \geqslant e/(p-1)$. Then we have $pu_{\mu-1} \leqslant u_\mu \leqslant pu_m$, $\mu - 1 \leqslant m$ and therefore either $m \geqslant \mu$ and $u_{m+1} = u_m + e$; or $m = \mu - 1$ and $u_\mu = pu_m = ep/(p-1)$; in this last case $u_m = e/(p-1)$ and $u_{m+1} = u_\mu = ep/(p-1) = u_m + e$.

3. PROOF OF THE MAIN RESULTS

Proof of Theorem 1. Let F be a local field with an algebrically closed residue field of characteristic p. We consider the two following cases: either

F is of equal characteristic p, or F contains a primitive pth root of unity. Let e be the absolute ramification index of F (thus $e = +\infty$ in the first case). From the local class field theory, the Galois group of the maximal Abelian pro-p-extension of F is a free Abelian pro-p-group (see also [7, Theorem 8 and Remark 5]).

By Theorem 3, there exists a totally ramified $\mathbb{Z}_p$-extension of F whose upper ramification numbers are (u_m). Now there exists a local field E whose residue field is finite and whose absolute ramification index is e and there exists a totally ramified $\mathbb{Z}_p$-extension M of E whose upper ramification numbers are the u_m (see [3, Theorem 3] if $p \neq 2$ and [4, Theorem 2] if $p = 2$).

Let N be the field of norms of M/E. There is a canonical isomorphism from the Galois group of M/E onto an automorphism group Γ of N which preserves the ramification filtrations [13]. Therefore if γ is a topological generator of Γ then the sequence of upper ramification numbers of γ is (u_m). Thus the implication (i) $\Rightarrow$ (iii) is proved.

Now let γ be an infinite order automorphism of $k((t))$ with k perfect field and let (u_m) be its sequence of upper ramification numbers. We consider the $\mathbb{Z}_p$-extension F/E corresponding to $\gamma \in \mathrm{Aut}_k(k((t)))$ by the equivalence of categories given by the field of norms functor of Fontaine and Wintenberger [14]. Then E is a local field with residue field k and the Galois group of F/E has the same upper ramification numbers as γ. Now it suffices to apply Theorem 3 to get the implications (iii) $\Rightarrow$ (i) and (ii) $\Rightarrow$ (i). Finally the last implication (iii) $\Rightarrow$ (ii) is clear: given a perfect field k of characteristic p and a finite extension $\mathbb{F}_q/\mathbb{F}_p$, every automorphism of $\mathbb{F}_q((t))$ can be considered as an automorphism of $\mathbb{F}_q((t)) \otimes_{\mathbb{F}_q \cap k} k$ with the same ramification.

Proof of Corollary 1. Let i_0 and i_1 be two integers such that $0 < i_0 < i_1$ and $p \mid i_1 - i_0$; we recall that $u_0 = i_0$ and $u_1 = i_0 + (i_1 - i_0)/p$. We know by Theorem 1 that u_0 and u_1 are the first numbers of an infinite ramification sequence (u_m) of an automorphism of $\mathbb{F}_q((t))$ if and only if

$$p \mid u_0 \Rightarrow u_1 = u_0 + \frac{(p-1)\,u_0}{p},$$

$$p \nmid u_0 \Rightarrow \text{either} \quad u_1 \geqslant p u_0 \qquad \text{or}$$

$$p u_0 < u_1 = u_0 + e \qquad \text{with} \quad e > \frac{(p-1)\,u_0}{p},$$

that is to say, $u_1 > (2p-1)\,u_0/p$ for this last case.

We translate these conditions in terms of sequences (i_m):

$$p \mid i_0 \Rightarrow i_1 = pi_0,$$
$$p \nmid i_0 \Rightarrow \text{either} \quad i_1 \geqslant (p^2 - p + 1)\, i_0, \qquad \text{or}$$
$$(p^2 - p + 1)\, i_0 > i_1 > pi_0.$$

Moreover if $p \mid i_0$ then for every $m \geqslant 0$, $u_{m+1} = u_m + (p-1)\, u_0/p$ therefore $i_{m+1} = pi_m$; if $p \nmid i_0$ and if $i_1 < (p^2 - p + 1)\, i_0$ then $u_1 < pu_0$ and for every $m \geqslant 0$, $u_{m+1} = u_m + e$ with $e = (i_1 - i_0)/p = (i_{m+1} - i_m)/p^{m+1}$ therefore $i_m = i_0 + ((p^m - 1)/(p-1))(i_1 - i_0)$.

Proof of Theorem 2. The conditions on (u_m) in Theorem 2 are equivalent to the assertion (v) of Theorem 3 where we exclude (a′) $u_0 = ep/(p-1)$ and (b′) $u_{m+1} = ep/(p-1)$ with $u_m < e/(p-1)$. We already noted (Remark 1) that if (u_m) is the sequence of upper ramification numbers of a $\mathbb{Z}_p$-extension of a local field E then the conditions (a′) or (b′) can happen only if E contains a pth root of unity ζ. Moreover if the residue field of E is finite then the Galois group of the maximal Abelian pro-p-extension of E is a free Abelian pro-p-group if and only if $\zeta \notin E$ (see [7, Theorem 8; 8] for more details).

Therefore, given a local field E with a perfect residue field k such that $\zeta \notin E$ and a sequence of integers (u_m) as in Theorem 2, there exists a $\mathbb{Z}_p$-extension F/E whose upper ramification numbers are the u_m (Theorem 3). Since $k((t))$ is isomorphic to the field of norms of F/E there exists an automorphism $\gamma \in \mathrm{Aut}_k(k((t)))$ whose sequence of upper ramification numbers is (u_m).

Proof of Corollary 2. Let $i_0, i_1, u_0 = i_0$ and $u_1 = i_0 + (i_1 - i_0)/p$ be integers as before. If we exclude the cases where $p \mid i_0$ and $p \mid i_0 + (i_1 - i_0)/p$ with $i_1 > (p^2 - p + 1)\, i_0$ (that is to say, $p \mid u_0$ and $p \mid u_1$ with $u_1 > pu_0$) from the conditions on (i_m) written in Corollary 1, then we obtain exactly the conditions of Corollary 2. Therefore Corollary 2 follows from Theorem 2.

4. RAMIFICATION OF SOME COMMUTING AUTOMORPHISMS

Let k and K be as in the Introduction and let σ and $\tau \in \mathrm{Aut}_k(K)$ be such that the closed subgroup G of $\mathrm{Aut}_k(K)$ generated by σ and τ is isomorphic to an extension of $\mathbb{Z}_p$ by $\mathbb{Z}_p$. This implies that σ and τ have infinite order and $\tau \neq \sigma^\nu$ for all $\nu \in \mathbb{Z}_p$ because the closed subgroup of $\mathrm{Aut}_k(K)$ generated by σ is constituted by the σ^ν, $\nu \in \mathbb{Z}_p$. For example, if there exists a p-adic unit $\nu \equiv 1 \bmod p$ such that $\tau\sigma\tau^{-1} = \sigma^\nu$ then the group G is such an extension of $\mathbb{Z}_p$ by $\mathbb{Z}_p$.

As usual, the groups of ramification of G are defined by $G_i = \{g \in G;\ i(g) \geqslant i\}$.

PROPOSITION 1. *With the hypothesis above, the sequences* $(i(\sigma^{p^n})/p^n)$ *and* $(i(\tau^{p^n})/p^n)$ *are not bounded.*

Proof. By Theorem 1.5.2 of [5], the automorphism group G of K lies in the image of Wintenberger's functor "field of norms." This means that there exists a local field E and a Galois pro-p-extension F of E whose field of norms is isomorphic to K and there is a canonical isomorphism of G onto $\mathrm{Gal}(F/E)$ preserving the filtrations of ramification. (The lower numbering of the ramification of $\mathrm{Gal}(F/E)$ exists because F/E is arithmetically profinite (A P F) in the sense of [13].) Let H be the closed subgroup of G generated by σ and let $E' = F^H$. The extension E'/E is APF [13, Proposition 3.4.1]. Let N be the field of norms of E'/E. The $\mathbb{Z}_p$-extension F/E' can be identified with a $\mathbb{Z}_p$-extension M of N and the field of norms of M/N can be identified with K [13, Proposition 3.4.1]. Therefore there is a canonical isomorphism h from H onto $\mathrm{Gal}(M/N)$ such that for every $i \in \mathbb{N}$, $h(H_i) = \mathrm{Gal}(M/N)_i$. Thus the jumps of the filtration $\mathrm{Gal}(M/N)_i$ are the $i(\sigma^{p^n})$. Since N is a local field of equal characteristic, the sequence $((i(\sigma^{p^n})/p^n)$ is not bounded. Clearly we can prove analogously that $((i(\tau^{p^n})/p^n)$ is not bounded.

PROPOSITION 2. *Let* σ *and* τ *be two automorphisms of* K *such that* $\sigma\tau = \tau\sigma$, $\tau \neq \sigma^\nu$ *for all* $\nu \in \mathbb{Z}_p$ *and* $1 \leqslant i(\sigma) < i(\tau) < i(\sigma^p) < p(p^2-p+1)\, i(\sigma) - (p-1)\, i(\tau)$. *Then for all* $n \geqslant 1$,

$$i(\sigma^{p^n}) = i(\sigma) + (i(\sigma^p) - i(\sigma))\,\frac{p^{2n}-1}{p^2-1}$$

$$i(\tau^{p^n}) = i(\sigma) + (i(\tau) - i(\sigma))\, p^{2n} + (i(\sigma^p) - i(\sigma))\,\frac{p^{2n}-1}{p^2-1}.$$

Proof. Let G be the closed subgroup of $\mathrm{Aut}_k(K)$ generated by σ and τ and let F be a totally ramified extension of a local field E corresponding to G by the equivalence of categories given by the norms field theory [14]. We identify $\mathrm{Gal}(F/E)$ with G. Let ψ and φ the Hasse–Herbrand functions of F/E: $\psi(x) = \int_0^x (G\colon G^t)\, dt$ for $x \geqslant 0$ and $\varphi = \psi^{-1}$. The first three jumps of the filtration of ramification (G^u) of F/E are $\alpha_0 = \varphi(i(\sigma))$, $\beta_0 = \varphi(i(\tau))$, and $\alpha_1 = \varphi(i(\sigma^p))$.

Therefore

$$i(\sigma) = \psi(\alpha_0) = \alpha_0$$
$$i(\tau) = \psi(\beta_0) = \alpha_0 + p(\beta_0 - \alpha_0)$$
$$i(\sigma^p) = \psi(\alpha_1) = \alpha_0 + p(\beta_0 - \alpha_0) + p^2(\alpha_1 - \beta_0).$$

Thus $p^2\alpha_1 = p(p-1)\, i(\sigma) + (p-1)\, i(\tau) + i(\sigma^p)$, and the condition $i(\sigma^p) < p(p^2 - p + 1)\, i(\sigma) - (p-1)\, i(\tau)$ is equivalent to $\alpha_1 < p\alpha_0$.

Let F^τ be the fixed field of the subgroup of G generated by τ. By Herbrand's theorem [11, Chap. 4, Proposition 14] the first two jumps of the filtration of ramification of the $\mathbb{Z}_p$-extension F^τ/E are α_0 and α_1. Since $\alpha_1 < p\alpha_0$ the local field E is of characteristic 0, the absolute ramification index of E is $e = \alpha_1 - \alpha_0$, and $\alpha_0 \geqslant e/(p-1)$. Therefore for all $n \geqslant 1$

$$\begin{aligned} i(\sigma^{p^n}) = \psi(\alpha_0 + ne) &= \alpha_0 + p(\beta_0 - \alpha_0) + p^2(\alpha_0 + e - \beta_0) + p^3(\beta_0 - \alpha_0) \\ &\quad + \cdots + p^{2n}(\alpha_0 + e - \beta_0) \\ &= \alpha_0 + p(1 + p^2 + \cdots + p^{2n-2})(\beta_0 - \alpha_0) \\ &\quad + p^2(1 + p^2 + \cdots + p^{2n-2})(\alpha_1 - \beta_0) \\ &= i(\sigma) + (i(\sigma^p) - i(\sigma)) \cdot \frac{p^{2n} - 1}{p^2 - 1} \end{aligned}$$

because $i(\tau) - i(\sigma) = p(\beta_0 - \alpha_0)$ and $i(\sigma^p) - i(\tau) = p^2(\alpha_1 - \beta_0)$.

By the same proof,

$$\begin{aligned} i(\tau^{p^n}) = \psi(\beta_0 + ne) &= \alpha_0 + p(1 + p^2 + \cdots + p^{2n-2} + p^{2n})(\beta_0 - \alpha_0) \\ &\quad + p^2(1 + p^2 + \cdots + p^{2n-2})(\alpha_1 - \beta_0) \\ &= i(\sigma) + p^{2n}(i(\tau) - i(\sigma)) + \frac{p^{2n} - 1}{p^2 - 1}(i(\sigma^p)) - i(\sigma)). \end{aligned}$$

Remark. A local field K and two automorphisms σ and τ of K as in Proposition 2 do exist.

Let α_0, β_0, α_1, and e be positive integers such that $e/(p-1) < \alpha_0 < \beta_0 < \alpha_1 < p\alpha_0$ and let E be a local field of absolute ramification index e and with a finite residue field. By the local class field theory there exists a $\mathbb{Z}_p$-extension whose first two upper ramification numbers are α_0 and α_1 and another $\mathbb{Z}_p$-extension whose first ramification number is β_0; then taking the field of norms N of the composition of these two extensions we get an automorphism group of N whose generators satisfy our conditions.

Note. Theorem 1 of [6] must be modified as follows.

Assume k is the finite field $\mathbb{F}_{p^f}$, $p \nmid i_0$, and $i_1 < (p^2 - p + 1)\, i_0$. Then the p-rank of Gal$(K(\gamma)/K)$ is bf or bf$\,+\,1$.

The end of the proof must be modified as follows. $W(E^a)$ is the maximal pro-p-extension of K containing $K(\gamma)$. The p-rank of Gal(E^a/E) is $ef + 2$ if E contains a primitive pth root of unity or $ef + 1$ if not. So the p-rank of Gal$(K(\gamma)/K)$ is $ef + 1$ or ef. The rest of the proof need not be modified.

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