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Chemical reaction effect on MHD viscoelastic fluid flow over a vertical stretching sheet with heat source/sink

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Abstract The present paper intended to analyze the effect of thermal diffusion (Soret) and diffusion-thermo (Dufour) effect on MHD viscoelastic fluid flow over a porous vertical stretching sheet subject to variable magnetic field embedded in a porous medium in the presence of chemical reaction and heat source/sink. The method of solution involves similarity transformation. The coupled nonlinear partial differential equations governing flow, heat and mass transfer phenomena are reduced into set of nonlinear ordinary differential equations. The transformed equations are solved numerically by using Runge-Kutta fourth order method followed by shooting technique. The effects of various parameters on the velocity, temperature and concentration fields are analyzed with the help of graphs. The numerical computation of skin friction, Nusselt number and Sherwood number is presented in a table. For validity of the numerical method applied here the work of previous authors is compared with the present one as a particular case by omitting the porosity, heat source/sink and chemical reaction parameters.

1. Introduction

In recent years, a great deal of interest has been created on heat and mass transfer of the boundary layer flow over a stretching sheet, in view of its numerous applications in various fields such as polymer processing industry in manufacturing processes. Crane [1] computed an exact similarity solution for the boundary layer flow of a Newtonian fluid toward an elastic sheet which is stretched with the velocity proportional to the distance from the origin. Sakiadis [2,3] first studied the boundary layer problem assuming velocity of a bounding surface as constant. The convection problem in porous medium has also important applications in geothermal reservoirs and geothermal energy extractions. A comprehensive review of the studies of convective heat transfer mechanism through porous media has been made by Nield and Bejan [4]. Hiremath and Patil [5] studied the effect on free convection currents on

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Nomenclature

a, b, c	constant values
u, v	velocity components (m s^{-1})
C	concentration (kg m^{-3})
D_e	coefficient of mass diffusivity (m^2 s^{-1})
Le	Lewis number
Re_x	local Reynolds number
Sr	Soret number
K_p	porosity parameter (m^2)
S	heat source\sink
Rc	viscoelasticity parameter
g	acceleration due to gravity (m s^{-2})
c_s	concentration susceptibility (kg m^{-3})
$B(x)$	magnetic field
c_p	specific heat (kj K g^{-1} K^{-1})
Du	Dufour number
M	magnetic field parameter
N	concentration buoyancy parameter
T_m	mean fluid temperature (K)
K_c	chemical reaction parameter
p_r	Prandtl number
Gr_x	Grashof number
k_T	thermal diffusion ratio
Sc	Schmidt number

Greek symbols

η	similarity variable
θ	dimensionless temperature
σ	electrical conductivity of the fluid (S m^{-1})
υ	kinematic fluid viscosity (m^2 s^{-1})
ψ	stream function
φ	dimensionless fluid concentration
β_T	thermal expansion coefficient (K^{-1})
γ	second auxiliary parameter
ψ	stream function
T	fluid temperature (K)
ρ	fluid density (kg m^{-3})
λ	buoyancy parameter
α	thermal diffusivity
β_C	solutal expansion coefficient (K^{-1})

the oscillatory flow through a porous medium, which is bounded by vertical plane surface of constant temperature. Fluctuating heat and mass transfer on unsteady free convective MHD flow through porous media in a rotating system has been discussed by Dash et al. [6].

Subhashini et al. [7] studied the effect of mass transfer on the flow past a vertical porous plate. Unsteady free convective flow with mass transfer phenomenon past an infinite vertical porous plate with constant suction was studied by Soundalgekar and Wavre [8]. Soundalgekar [9] studied the effects of mass transfer and free convection currents on the flow past an impulsively started vertical plate. In these studies the magnetohydrodynamic effect has been ignored. Lai and Kulacki [10] studied the coupled heat and mass transfer with natural convection from vertical surface in a porous medium. Benazir et al. [11] have studied unsteady MHD Casson fluid flow over a vertical cone and flat plate with non-uniform heat source/sink. Kumar et al. [12] studied chemically reacting MHD free convective flow over a vertical cone with a variable electric conductivity. Further, Prakash et al. [13] contributed through their publication entitled radiation and Dufour effects on unsteady MHD mixed convective flow in an accelerated vertical wavy plate with varying temperature and mass diffusion. They have considered an unsteady mixed convective flow past a vertically wavy plate with varying temperature and mass diffusion. Sivaraj and Kumar [14] have analyzed unsteady MHD dusty viscoelastic fluid Couette flow in an irregular channelwith varying mass diffusion. They have pointed out the effect of dusty viscoelastic fluid on flow, heat and mass transfer in an irregular channel.

Investigating the references regarding the existence of multiple solutions of the governing equation the following are of great works. Turkyilmazoglu [15] studied multiple solutions of heat and mass transfer of MHD slip flow for the viscoelastic fluid over a stretching sheet. Turkyilmazoglu [16] also discussed Dual and triple solutions for MHD slip flow of non-Newtonianfluid over a shrinking surface. Multiple analytic solutions of heat and mass transfer of MHD slip flow for two types of viscoelastic fluids over a stretching surface have been investigated by Turkyilmazoglu [17]. Moreover, Elbashbeshy and Ibrahim [18] investigated the effect of steady free convection flow with variable viscosity and thermal diffusivity along a vertical plate. Kafoussias and Williams [19] studied the thermal-diffusion and diffusion-thermo effects on the mixed free-forced convective and mass transfer steady laminar boundary layer flow over a vertical plate, with temperature-dependent viscosity. Sajid and Hayat [20] investigated the radiation effects on the mixed convection flow over an exponentially stretching sheet and solved the problem analytically using homotopy analysis method. The numerical solution for the same problem was then given by Bidin and Nazar [21]. Recently, Poornima and Bhaskar Reddy [22] presented an analysis of the radiation effects on MHD free convective boundary layer flow of nanofluids over a nonlinear stretching sheet. However, the interaction of radiation with mass transfer due to a stretching sheet has received little attention. Abolbashari et al. [23] studied entropy analysis for an unsteady MHD flow past a stretching permeable surface in nanofluid. Rashidi and Erfani [24] applied an analytical method for solving steady MHD convective and slip flow due to a rotating disk with viscous dissipation and Ohmic heating. Mixed convective heat transfer for MHD viscoelastic fluid flow over a porous wedge with thermal radiation is studied by Rashidi et al. [25]. Further, Rashidi et al. [26] studied an analytic approximate solution for MHD boundary layer viscoelastic fluid flow over continuously moving stretching surface by HAM with two auxiliary parameters.

It has been experimentally verified that the diffusion of energy is caused by a composition gradient. This fact is known as the Dufour effect or the diffusion-thermo effect. The diffusion of species by a temperature gradient is termed as the Soret effect or the thermal diffusion effect. In most of the studies dealing with heat and mass transfer, these effects are neglected under the assumption that these are of smaller orders of magnitude described by Fourier's and Fick's laws.

Anwar Bég et al. [27] obtained the numerical solutions for the free convective flow induced by a stretching surface in the presence of Dufour and Soret effects. Tsai and Huang [28] analyzed Dufour and Soret effects on the Hiemenz flow over a stretching surface immersed in a porous medium. Afify [29] studied Dufour and Soret effects in heat and mass transfer in the flow induced by a stretching surface. Mishra [30] analyzed the effect of free convection and mass transfer on the flow of an elastico-viscous fluid (Walters B' model) in a vertical channel. The diffusion thermo effect has been considered on the fully developed laminar flow with uniform plate temperature and concentration. Tripathy et al. [31] studied Chemical reaction effect on MHD free convective surface over a moving vertical plane through porous medium. Rashidi et al. [32] examined the heat and mass transfer for MHD viscoelastic fluid flow over a vertical stretching sheet with considering Sore and Dufour effects. Further, Baag et al. [33] discussed the numerical investigation on MHD micropolar fluid flow toward a stagnation point on a vertical surface with heat source and chemical reaction.

Problems on fluid flow and mass transfer through porous media are the interest of not only mathematicians but also chemical engineers who have generally concerned with reacting and absorbing species, petroleum engineers, who have concerned with the miscible displacement process and civil engineers who are confronted with the problem of salt water encroachment of careful costal equiforce. Also, porous media are very widely used for heated body to maintain its temperature. To make the heat insulation of the surface more effective it is necessary to study the free convective effects under flow through porous media and to estimate effects on the heat transfer. Further, Gebhart and Pera [34] studied the laminar flows which arise in fluid due to the interaction of the gravity forces and density differences caused by the simultaneous diffusion of thermal energy and of chemical species neglecting the thermal diffusion and diffusion-thermo (Soret – Dufour) effects because the level of species concentration is very low. Sparrow et al. [35] analyzed the free convection flow with Soret–Dufour effects.

In the present study we have considered the level of species concentration is moderately high. So that the thermal diffusion (Soret) and diffusion-thermo (Dufour) effects cannot be neglected. So in view of above discussion the necessity of considering the flow through porous media and the inclusion of additional heat source, Soret and Dufour effect and first order chemical reaction not only enrich the present analysis but also complement the earlier studies. The inclusion of these phenomena gives rise to additional parameters such as permeability parameter, heat source parameter and chemical reaction parameter.

Thus, inclusion of additional parameters contributes to the complexity of the mathematical model representing the flow, heat and mass transfer phenomena. To the best of authors' knowledge no work has been so far reported considering above phenomenal together.

2. Mathematical analysis

Consider a steady two-dimensional flow of an incompressible and electrically conducting viscoelastic fluid of Walters' B' model over a stretching vertical surface with a variable magnetic field of strength $B(x) = B_0 x^{n-1/2}$ applied to the surface. Rashidi et al. [32] have considered the variable magnetic field $B(x)$ is proportional to the power of x-coordinate $B(x) = B_0 x^{(n-1)/2}$ and the significance of this form is that when $n = 1$, this represents a constant magnetic field strength. When $n \neq 1$, magnetic field varies with x-coordinate that means non uniform field strength. Two equal and opposite forces are applied along x-axis such that the position of the origin remains unaffected. The stretching velocity is assumed to be of the form $U_w(x) = ax^n$ where a and n are constants. The strength of the applied magnetic field is considered to be low so that induced magnetic field and Hall current are neglected. Under the usual boundary layer assumptions with Boussinesq approximations, the governing equations for the double diffusive natural convection flow of viscoelastic fluid following Rashidi et al. [32] and Hayat et al. [36] can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\begin{aligned} \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = {} & \mu\frac{\partial^2 u}{\partial y^2} + k_0\left(u\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right) \\ & + \rho g(\beta_T(T - T_\infty) + \beta_C(C - C_\infty)) \\ & - \left(\sigma B^2(x) + \frac{\mu}{k_p'}\right)u \end{aligned} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{D_e k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} + S'(T - T_\infty) \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_e\frac{\partial^2 C}{\partial y^2} + \frac{D_e k_T}{T_m}\frac{\partial^2 T}{\partial y^2} - k_c'(C - C_\infty) \tag{4}$$

The appropriate boundary conditions are

$$\begin{aligned} & u = U_w(x),\ v = v_w,\ T = T_w(x),\ C = C_w(x) \quad \text{at } y = 0, \\ & nu \to 0,\ \frac{\partial u}{\partial y} \to 0,\ T \to T_\infty,\ C \to C_\infty \quad \text{as } y \to \infty, \end{aligned} \tag{5}$$

3. Solution of the flow field

Assume that $T_w(x) = T_\infty + bx$ and $C_w(x) = C_\infty + cx$ where b and c are constants. Yang et al. [37] have shown that a similarity relation is not possible for flows in stably stratified media. Similarity transformation is possible only for the case of temperature decreasing with height, which is physically unstable. Such is the present case and hence there exists similarity transformations.

Now let us define the following dimensionless functions $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ and the similarity variable η as

$$\begin{aligned} & \eta = \sqrt{\frac{u_w}{\upsilon x}}y,\ \psi(x,y) = \sqrt{u_w \upsilon x}f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \\ & \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \end{aligned} \tag{6}$$

where $\psi(x,y)$ is the stream function defined as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

Using Eq. (6), the governing Eqs. (1)–(4) are transformed to the ordinary differential equations as follows:

$$nf'^2 - \frac{n+1}{2}ff'' - f''' - Rc\left\{(3n-1)f'f''' - \frac{3n-1}{2}f''^2 - \frac{n+1}{2}ff^{(4)}\right\} + \left(M + \frac{1}{K_p}\right)f' - \lambda(\theta + N\phi) = 0 \quad (8)$$

$$\theta'' + p_r\left(\frac{n+1}{2}f\theta' - f'\theta + Du\phi'' + S\theta\right) = 0 \quad (9)$$

$$\phi'' + Le\left\{p_r\left(\frac{n+1}{2}f\phi' - f'\phi\right) + Sr\theta''\right\} - ScK_c = 0 \quad (10)$$

with boundary conditions

$$\begin{array}{ll} f(\eta) = f_w,\ f'(\eta) = 1,\ \theta(\eta) = 1,\ \phi(\eta) = 1 & \text{at } \eta = 0, \\ f'(\eta) = 0,\ f''(\eta) = 0,\ \theta(\eta) = 0,\ \varphi(\eta) = 0 & \text{as } \eta \to \infty, \end{array} \quad (11)$$

where $p_r = \upsilon/\alpha$ is the Prandtl number, $M = \sigma B_0^2/(\rho a)$ is the magnetic parameter, K_p is the porous matrix and $S = S'/a$ is the heat source/sink, $Sc = \upsilon/D_e$ is the Schmidt number, $K_c = K_c'/a$ is the chemical reaction parameter, $Rc = k_0 a x^{n-1}/\upsilon$ is the viscoelastic parameter, $\lambda = g\beta_T(T_w - T_\infty)x/a^2x^{(2n-1)}x = Gr_x/Re_x^2$ is the buoyancy parameter, $Gr_x = g\beta_T(T_w - T_\infty)x^3/\upsilon^2$ is the Grashof number, $Re_x = u_w x/\upsilon$ is the Reynolds number, $N = \beta_c(C_w - C_\infty)/\beta_T(T_w - T_\infty)$ is the constant dimensionless concentration buoyancy parameter, $Le = \alpha/D_e$ is the Lewis number, $Sr = D_e k_T(T_w - T_\infty)/T_m\alpha(C_w - C_\infty)$ is the Soret number, and $Du = D_e k_T(C_w - C_\infty)/c_s c_p \upsilon(T_w - T_\infty)$ is the Dufour number. The primes denote differentiation with respect to η.

Now, let us discuss the criteria of dual solutions. If we substitute $n = 1, \lambda = 0, Kp \to \infty$ then Eq. (8) reduces to

$$f''' + ff'' - f'^2 + Rc[(2f'f''' - f''^2 - ff^{(4)}] - Mf' = 0$$

The boundary conditions prescribed by Eq. (11) are sufficient to solve Eq. (8) uniquely. In fact $f''(\infty) = 0$ is the property of the boundary layer in the asymptotic region. In the absence of the viscoelastic parameter $Rc = 0$ Eq. (8) reduces to third order ODE for which these four conditions are also applicable. The adequacy of the boundary condition was also discussed in the work of Rajgopal et al. [38]. For $-1 < Rc < 0$ Troy et al. [39] found a solution in the form of

$$f(\eta) = \sqrt{(1 + Rc)}\left(1 - e^{-\frac{1}{\sqrt{1+Rc}}\eta}\right).$$

Afterward, Chang [40] showed that solution of Eq. (8) with boundary condition (11) for $M = f_w = 0$ is not unique and presented another solution by considering $Rc = -0.5$ in the form of

$$f(\eta) = \sqrt{2}\left(1 - e^{-\frac{1}{\sqrt{2}}\eta}\cos\left(\frac{\sqrt{3}}{2}\eta\right)\right)$$

The occurrence of this kind of solution was dealt with Rao [41] in an extension to the interval $-1 < Rc < 0$ and the following closed form solution was derived:

$$f(\eta) = A\left(1 - e^{-\frac{A}{2}\eta}\cos\left(\sqrt{3}A\eta\right) + \frac{1 + 2Rc}{\sqrt{3}}\sin\left(\frac{\sqrt{3}A\eta}{2}\right)\right)$$

where $A = \sqrt{-1/Rc}$

Further, Lawrence and Rao [42] presented a general method and obtained all the non-unique solutions with a transverse magnetic field M. Pop and Na [43] suggested an analytical solution in the absence of viscoelasticity which is given by

$$f(\eta) = f_w + \left(\frac{1 - e^{-\beta\eta}}{\beta}\right) \quad (12)$$

with $\beta = \frac{1}{2}\left(f_w + \sqrt{4 + 4M + f_w^2}\right)$

Among all the suggested solutions, solutions of the form proposed by Troy et al. [39], Pop and Na [43] are the realistic ones as we can recover the boundary layer approximation of Navier-Stokes solution (12) only in its limiting case $Rc = 0$ and for slightly viscoelastic fluid. Thus, sinusoidal solution given above was denied to be physical.

For the present problem the suggested analytical solution in the absence of additional buoyancy forces but in the presence of magnetic field and porous matrix is as follows:

$$f(\eta) = f_w + \left(\frac{1 - e^{-\beta\eta}}{\beta}\right)$$

with $\beta = \frac{1}{2}\left(f_w + \sqrt{4 + 4\left(M + \frac{1}{Kp}\right) + f_w^2}\right)$

4. Numerical method

From the basic theory of ordinary differential equations (ODEs), there are two ways of solving nonlinear system of ODEs either as an IVP or as an BVP. One of the most popular methods for solving general BVP with shooting technique (Roberts and Shipman, 1972) with the simple shooting method one turns the BVP into a first order IVP, tries to obtain the solution based on a set of un specified initial conditions which are then converted through an iterative procedure (Newton's method) to satisfy the boundary conditions. The procedure requires the transformation into the state-space form and also requires evaluation of the Jacobian. The success of the procedure depends on using an appropriate initial guess. The solutions of the governing Eqs. (8)–(10) have been solved with boundary conditions (11) using MATLAB code to implement Runge-Kutta fourth order method with shooting technique.

5. Results and discussion

The effects of physical parameters such as magnetic parameter (M), Porous matrix (K_p), viscoelastic parameter (Rc), Radiation parameter (N), Buoyancy parameter (λ), source/sink parameter (S), Chemical reaction parameter (K_c) and Soret and Dufour numbers (Sr and Du) are discussed in the following lines. The effect of these pertinent parameters is shown in the graphs. We assign $n = 0.5$ throughout. In all the figures, the continuous and dotted lines represent the cases of presence and absence of porous matrix respectively (see Fig. 1).

Fig. 2 exhibits the effect of magnetic parameter M on the velocity profile in both presence of porous matrix ($K_p = 0.5$) and absence of porous matrix ($K_p = 100$). It is observed that due to increase in magnetic parameter the velocity profile decreases. To be specific, magnetic field generates a force of electromagnetic origin, which is a resistive force, as a result of which the velocity decreases. In the absence of porous matrix, ($K_p = 100$), heat source, ($S = 0$) and chemical reaction($K_c = 0$), the present result is in good agreement with

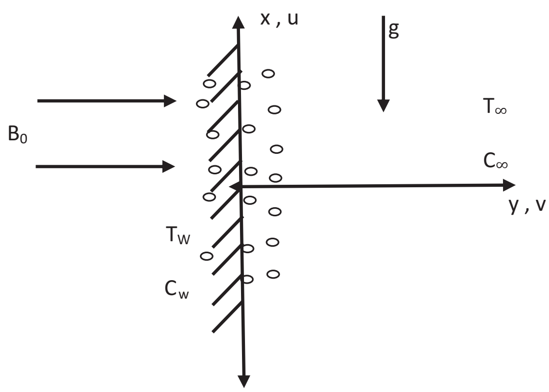

Figure 1 Flow geometry.

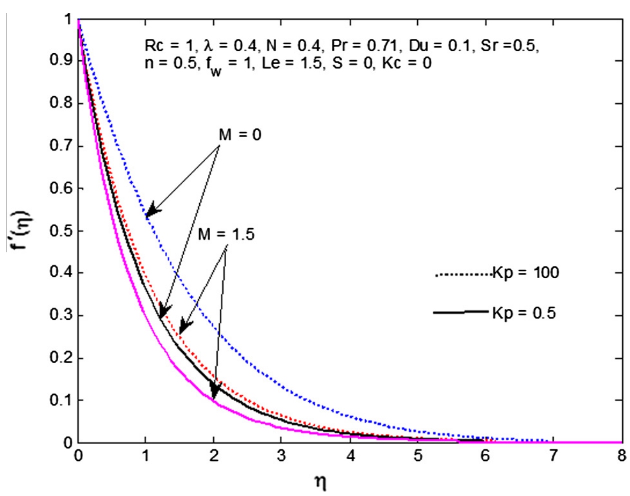

Figure 2 Variation of M and Kp on velocity profile.

earlier published result of Rashidi et al. [32]. The effects of the parameters agree with the result reported in the present study.

Fig. 3 exhibits the effect of chemical reaction on velocity distribution. It is seen that in case of constructive reaction ($K_c < 0$) velocity decreases for small values of K_c i.e. $K_c = -0.5$. However, for destructive reaction velocity increases i.e. $K_c = 0.5$. But the striking feature of the effect of the chemical reaction is that for higher value of K_c i.e. for. $K_c = -4$ and $K_c = 4$ the order of effect of chemical reaction on velocity is reversed. Thus, it is concluded that the magnitude of coefficient of chemical reaction K_c plays a vital role on velocity distribution.

Fig. 4 shows the effect of heat source/sink. The conclusion is very straightforward because the presence of sink absorbs the thermal energy which reduces the velocity whereas presence of source increases the velocity.

Fig. 5 presents the effect of magnetic field on temperature distribution. An interesting point is to note that effect of magnetic field is to increase the temperature at all points which is a natural consequence of resistive force offered by magnetic field opposing the motion of the fluid, thereby, enhancing the thermal energy as a result of which temperature increases in the thermal boundary layer. Further, on careful observation Fig. 5 reveals that the presence of porous matrix increases the temperature in the flow domain; this is due to the resistance

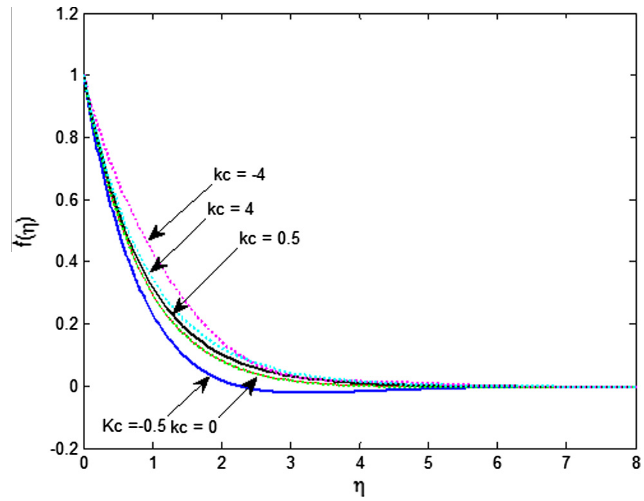

Figure 3 Variation of Kp and Kc on velocity profile.

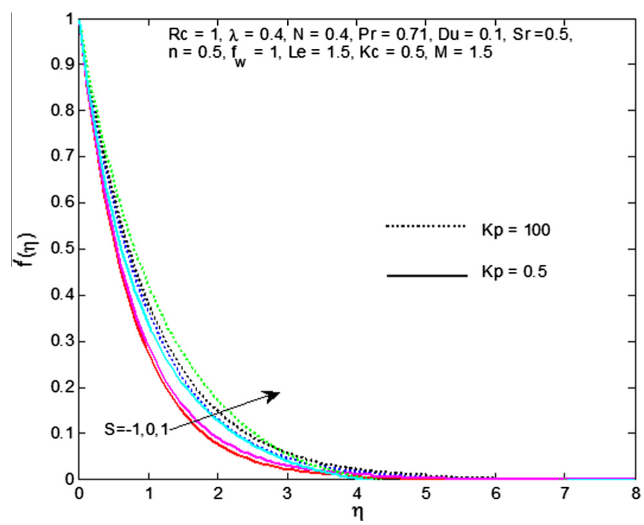

Figure 4 Variation of S and Kp on velocity profile.

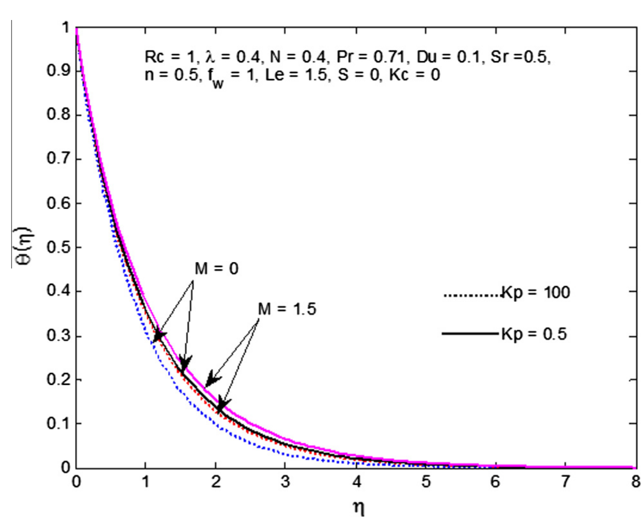

Figure 5 Variation of M and Kp on temperature profile.

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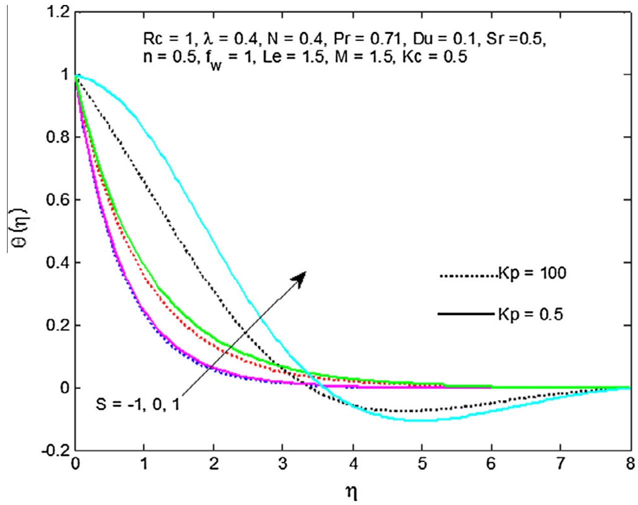

Figure 6 Variation of S and Kp on temperature profile.

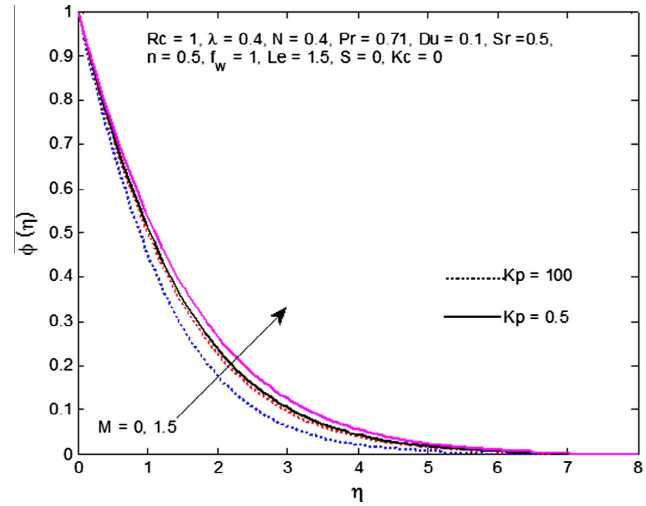

Figure 9 Variation of M and Kp on concentration profile.

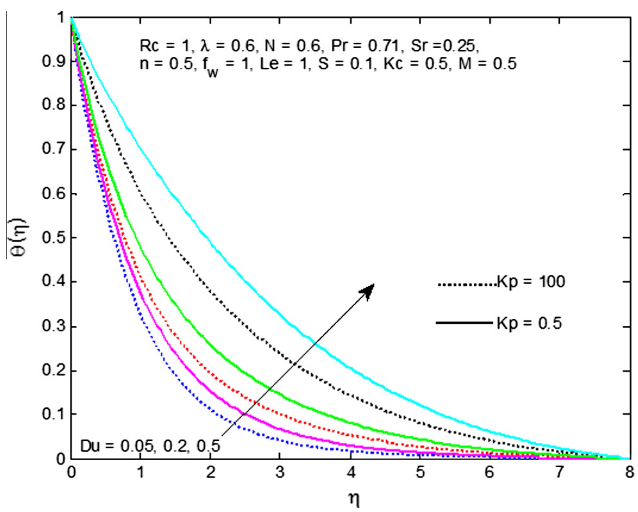

Figure 7 Variation of Du and Kp on temperature profile.

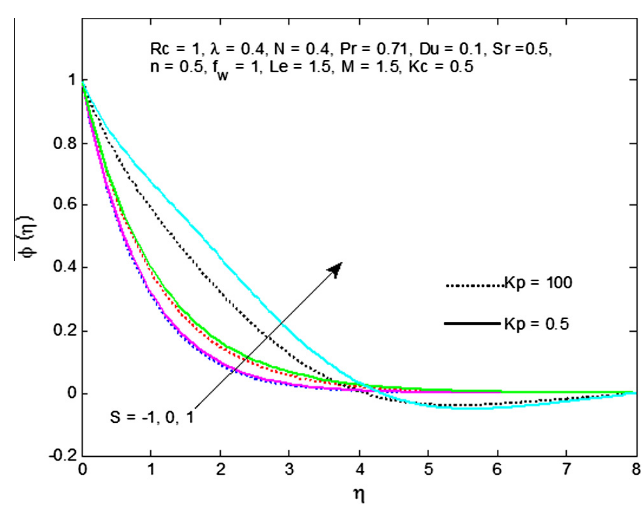

Figure 10 Variation of S and Kp on concentration profile.

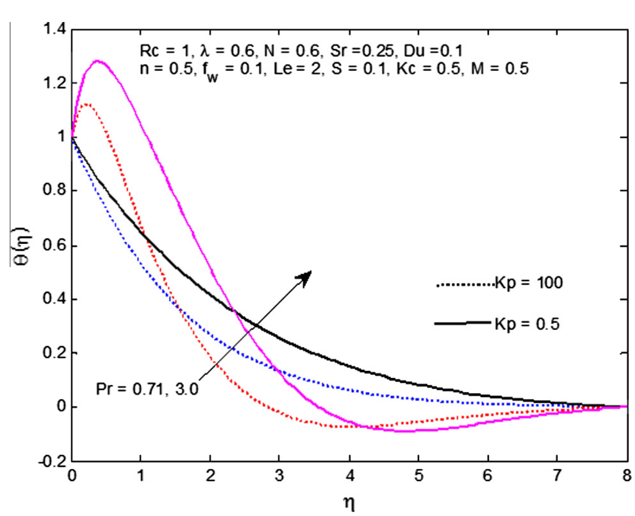

Figure 8 Variation of Pr and Kp on temperature profile.

offered by the porous matrix to the flow of fluid under consideration.

Fig. 6 depicts variation of temperature distribution for various values of source/sink through porous and non-porous medium. It is quite interesting to record the variation of temperature near $\eta = 3.5$, where profiles intersect each other. Another point is to note that presence of porous medium lowers down the temperature up to $\eta < 3.5$ and then reverse effect is observed. The striking feature of the temperature distribution is that in the presence of heat source negative temperature is recorded when $\eta > 4.0$ i.e. heat absorption occurs for the layer away from the plate surface.

Fig. 7 shows the effect of Dufour number *Du* i.e. mass diffusion of chemical species in the flow domain contributing to thermal energy as we have considered the level of species concentration is not very low in the present study.

Fig. 8 also shows the same type of effect in respect of Prandtl number. On careful study the graph depicts that for Non-Newtonian fluid ($p_r = 3.0$; for saturated liquid Freon at 273.3 k) the sharp rise of temperature is marked near the plate in both presence and absence of porous medium but this effect

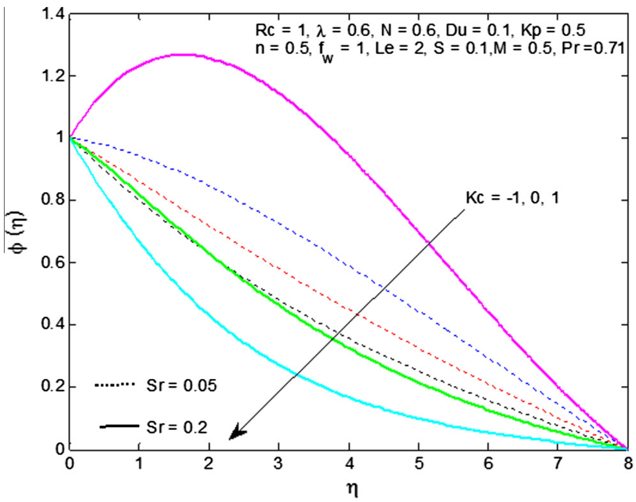

Figure 11 Variation of Sr and Kc on concentration profile.

is absent in case of Newtonian fluid ($p_r = 0.71$). The sudden hike is attributed to the non-Newtonian property i.e. viscoelastic property of the fluid. Moreover, negative temperature is recorded beyond the layers $\eta > 3.0$.

Fig. 9 presents the growth of solutal boundary layer in the presence of magnetic field as well as porous matrix.

Fig. 10 exhibits the solutal distribution for various values of source/sink parameters. The similar discussions have already been carried out in case of temperature distribution (Fig. 6).

Fig. 11 shows the Soret effect i.e. diffusion of thermal energy contributing to solutal boundary layer i.e. species concentration. It is evident from the figure that there is an increase in Soret number in the presence of destructive chemical reaction, and concentration level decreases whereas for constructive chemical reaction reverse effect is encountered. One striking feature i.e. a hike in concentration level is marked in case of combined effect of constructive chemical reaction ($K_c < 0$) with moderately high value of Sr ($Sr = 0.2$) near the plate. Thus, it is inferred that constructive reaction combined with thermal diffusion contributes to the growth of solutal boundary layer.

The numerical computations of local skin friction coefficient $f''(0)$, Nusselt number $-\theta'(0)$ and Sherwood number $-\phi'(0)$ are presented in Table 1 using the pertinent parameters that arise in the flow problem. It is observed that the skin friction coefficient increases with the increasing value of the Dufour number Du, Chemical reaction parameter K_c and heat source/sink S in both the absence/presence of porous matrix whereas, it decreases with the increasing value of Magnetic field parameter M, Prandtl number p_r and Soret number Sr in both presence/absence of K_p. Further, it is noticed that Nusselt number increases with an increasing value of the parameters K_c, p_r and Sr whereas the reverse effect is observed for the increasing value of M, Du and S. Again the Sherwood number increases with increasing values of Du, p_r and Sr but the reverse effect is marked with the increasing values of M, K_c and S.

Thus, it is concluded that interaction of magnetic field and thermal diffusion in the solutal boundary layer reduces the skin friction irrespective of presence/absence of porous matrix. Further, it is concluded that rate of heat transfer from the bounding surface increases with chemical reaction and Soret

Table 1 Skin friction coefficient, Nusselt number and Sherwood number for the various thermo-physical parameters.

$Rc = 1, \lambda = 0.4, N = 0.4, Pr = 0.71, Du = 0.1, Sr = 0.5, n = 0.5, f_w = 1, Le = 1.5, S = 0, Kc = 0$

M	Kp	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0	100	−0.56758	1.146877	0.687201
0	0.5	−1.03575	1.064889	0.605688
1.5	100	−0.94517	1.080288	0.620725
1.5	0.5	−1.26437	1.027477	0.569875
2	100	−1.0375	1.064595	0.605402
2	0.5	−1.32922	1.017289	0.560319

$Rc = 1, n = 0.5, N = 0.5, M = 1.5, Le = 1.5, Sr = 0.5, Pr = 0.71, f_w = 1, \lambda = 0.4, S = 0.1, Kc = 0.5$

Du	Kp	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.05	100	−0.94113	1.041014	0.897083
0.05	0.5	−1.26022	0.977918	0.876705
0.2	100	−0.92952	0.8991976	0.940877
0.2	0.5	−1.25203	0.828161	0.923546
1	100	−0.78394	−0.724711	1.393018
1	0.5	−1.14316	−1.010489	1.444838

$Rc = 1, n = 0.5, N = 0.5, M = 1.5, Le = 1.5, Sr = 0.5, Pr = 0.71, f_w = 1, \lambda = 0.4, S = 0.1, Du = 0.1$

Kc	Kp	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
−0.5	100	−1.08657	0.847862	1.177963
−0.5	0.5	−1.35964	0.780913	1.408813
0	100	−0.91277	1.030435	0.5923436
0	0.5	−1.23617	0.968812	0.5363016
0.5	100	−0.90743	1.096157	0.0910993
0.5	0.5	−1.21761	1.0930653	0.08915414

$Rc = 1, n = 0.5, N = 0.5, M = 1.5, Le = 1.5, Sr = 0.5, Du = 0.1, f_w = 1, \lambda = 0.4, S = 0.1, Kc = 0.5$

Pr	Kp	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.71	100	−0.93743	0.996157	0.910993
0.71	0.5	−1.25761	0.930653	0.8915414
1	100	−0.97374	1.251544	1.123225
1	0.5	−1.28417	1.18556	1.100486
3	100	−2.46037	11.83234	8.125968
3	0.5	−1.38287	2.994609	4.851173

$Rc = 1, n = 0.5, N = 0.5, M = 1.5, Le = 1.5, Sr = 0.5, Pr = 0.71, f_w = 1, \lambda = 0.4, Du = 0.1, Kc = 0.5$

S	Kp	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
−1	100	−0.98494	1.458862	1.1270457
−1	0.5	−1.29636	1.433126	1.111697
0	100	−0.94494	1.052590	0.9356789
0	0.5	−1.26397	0.996524	0.9172989
1	100	−0.85955	0.3490839	0.586000
1	0.5	−1.16827	0.095030	0.5005635

$Rc = 1, n = 0.5, N = 0.5, M = 1.5, Le = 1.5, Du = 0.1, Pr = 0.71, f_w = 1, \lambda = 0.4, S = 0.1, Kc = 0.5$

Sr	Kp	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.05	100	−0.82043	0.944131	0.262815
0.05	0.5	−1.18664	0.863213	0.250418
0.2	100	−0.88770	0.963889	0.532694
0.2	0.5	−1.22416	0.893075	0.512759
1	100	−0.96916	1.0483634	1.432416
1	0.5	−1.28216	0.984801	1.419563

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number and on the other hand magnetic field, Dufour effect and heat source reduce the surface heat transfer. Moreover, rise of rate of species concentration at the plate is favored by Dufour effect and heat source.

6. Conclusions

- In the absence of porous matrix, heat source and chemical reaction observation of the present study coincides with earlier published work of Rashidi et al. [32] (Fig. 2).
- For low value of chemical reaction parameter $K_c = 0.5$ the velocity increases but for $K_c < 0$ i.e. $K_c = -0.5$ the velocity decreases whereas, for higher value of chemical reaction parameter $K_c = 4.0$ and $K_c = -4.0$ the anomalous behavior is marked.
- The reaction rate of chemical species plays an important role on velocity boundary layer.
- Presence of heat source and higher value of Prandtl number induce absorption of temperature for a few layers away from the plate.
- The constructive reaction combined with Soret effect favors the growth of solutal boundary layer.
- Dufour number, chemical reaction parameter and heat source enhance the skin friction whereas Soret number and magnetic field reduce the skin friction at the plate.
- Chemical reaction, Prandtl number and Soret number enhance the rate of heat transfer at the plate.

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