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Mathematical modelling of the electrical discharge mechanical alloying process

S. Spadło^{a,*}, J. Kozak^b, P. Młynarczyk^a

^aKielce University of Technology, Al. Tysiąclecia P.P. 7, 25-314 Kielce, Poland

^bInstitute of Advanced Manufacturing Technology, ul. Wrocławska 37A, 30-011 Krakow, Poland

* Corresponding author. Tel.: +48-41 34-24-517; fax: +48 -41-24-48-698. E-mail address: sspadlo@tu.kielce.pl.

Abstract

In the paper, a comprehensive study of the electro-discharge mechanical alloying with using brush electrode is presented. This kind of a novel method is denoted as BEDMA (Brush Electro-Discharge Mechanical Alloying) and it combines features of electrical discharge machining with thermo-mechanical treatment. Electrode is being made of material which is to be alloyed on the part surface. A mathematical modelling of the thermal processes and mechanical action during the interaction of a single filament on the machined surface have been developed and used to estimation of the material transfer rate.

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1. Introduction

So far the machining process using filamentary metal brushes in the shape of disks has been used in surface machining to remove corroded layers, to prepare metal surfaces to be galvanized, and to produce surfaces of high adhesion to be coated with paint, glue, etc. Recently the process has been developed to include operations such as removing sharp edges and burrs, flashes and bosses from machine parts made of alloys of non-ferrous metals, as well as cleaning welds. To summarize, the typical uses of metal brush tools are limited to machining materials of a hardness lower than that of the material the filaments of the brush are made of.

On analysis of the advantages of using brush tools the authors suggests a new machining operation [1], [2] that combines mechanical, electrochemical, and electro-erosive processes acting on the machined item. Thanks to the synergetic effect, this type of hybrid machining - BEDMM (Brush Electrodischarge Mechanical Machining) makes the metal removal process more cost-effective [3], [4].

The use of brushing tools in an automation environment will necessitate a clear understanding of important brush performance characteristics. An understanding of such characteristics is important, as

surface preparation processes require a detailed knowledge of interrelationships between productivity of machining and brush operating conditions [5]. For example, it is recognized that electrical discharges generated during electroerosion-mechanical processes are closely related to the mechanical and thermal characteristics of the filament [6].

In BEDMM processing independently of their allocating (i.e. for removing the inequalities, smoothing etc.) or modify the properties of the superficial layer [7], [8] BEDMA (Brush Electrodischarge Mechanical Alloying) property it is possible to distinguish the following main sub-processes [9], [10]:

- electric: flow of the current through metal elements of the brush electrode and the joint-contact of machining surface (E-S), electrical discharge (sparking or in case of coarse machining arched), transient states in the working electric circuit,
- thermal: heating with Joule's heat of metal elements of the brush (wire, tape, etc.), melting in the zone of the joint, thermal phenomena in the plasma column while discharging,
- mechanical: the interaction between elements of hot electrode and machining surface (contact phenomena: pressing, deformation, friction, etc.)
- phase changes (thermal-chemical),
- transport of mass (mechanical, thermal, diffusion).

Take into account complexity of individual processes of components at mathematical modelling applying individual BEDMM processes of next approximations in the description is intentional.

From analysis of the BEDMM process [11] it results that considering the thermal phenomena associated with the flow of the current to begin with by the element of the brush and the E-S contact are the most rational presentation and with electric discharges. The propose of mathematical thermal BEDMM model supplemented with the mathematical model of electric phenomena will constitute the base to more distant of specifying as a result of taking into account mechanical phenomena and transformations in outer layer (phase changes, transport of mass).

In the first approximation a single element of the brush will be considered in the form of wire having an affect on flat area (fig. 1). In the period of the $t_p = mt_s$ (where $m \approx 1$ - rate taking appearing of the arc into account ($m > 1$), delay in straightening the single element of the brush out ($m < 1$, when the bow won't appear), t_s – theoretical time of the mechanical joint (or geometrical).

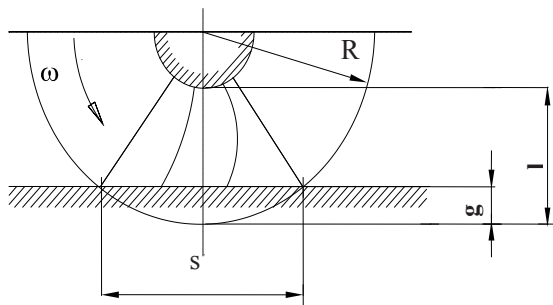


Fig. 1 Geometry of a particular filament deformation and surface contact with the worked area

The distance of the tip of the wire along the machining surface at the time of contact is equal to:

$$s = 2\sqrt{2Rg - g^2}$$

Time of "mechanical" contact without taking account of thermal and electrical phenomena, such as dynamic wire (i.e. vibration. "without power") will take out:

$$t_s = \frac{2}{\omega R} \sqrt{2Rg[1 - \frac{g}{2R}]} \quad (1)$$

where:

- t_s – theoretical time of contact
- ω – angular velocity of tool electrode,

- g – penetration depth,
- R – radius of hub,
- l – length of filament.

and with the assumption that $g \ll 2R$

$$t_s \cong \frac{2}{\omega} \sqrt{\frac{2g}{R}} \quad (2)$$

e.g.: for $R = 100$ mm, $g = 2$ mm and $n = 600$ rev/min (62.83 1/s), give $t_s = 6.6$ ms

2. Thermal model for BEDMA process

To estimate the time required for the filament temperature to reach melting point, the thermal model of a filament (length L under voltage U) has been developed (Figure 2).

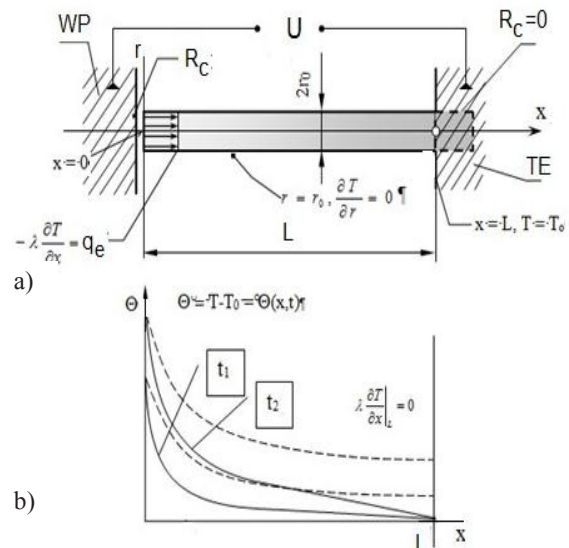


Fig. 2 (a) Scheme for modelling of heating before of melting; (b) example of distribution of temperature along the wire

Following assumption are made in developing the model for the heating of filament during mechanical contact with workpiece and electrical short circuit:

1. The cross-sectional area of the wire is constant throughout the length of the filament.
2. The changes in temperature, current density and electrical properties in the cross-section of the filament are negligible.
3. The changes in specific heat C and density ρ of the filament material with temperature are negligible.
4. The one-dimensional models i.e. neglecting the heat transfer through the filament surface.

5. The effects of friction between filament and surface on the heating are negligible in compare to high effect of the Joule heat and electrical discharges.
6. The change in the electric resistivity of the filament material is given by:

$$r = r_0(1 + \alpha\theta + \beta\theta^2)$$

Where: r_0 and α , β are the electrical resistivity and the temperature coefficient of resistivity at temperature T_0 , respectively, and $\theta = T - T_0$ is the increase in temperature of the filament.

Considering the Joule heat, the increase in temperature of the filament can be described as:

$$\frac{\partial\theta}{\partial t} = a \frac{\partial^2\theta}{\partial x^2} + r \frac{i^2}{\rho \cdot C} \tag{3}$$

Where: i – current density, and a – the thermal diffusivity.

The current density can be determined from Ohm’s law as follows:

$$i \cdot A \left(r \frac{L}{A} + R_C \right) = U \tag{4}$$

Where: A – the cross-sectional area of the filament and R_C – contact resistance.

At increasing temperature of the filament and the assumption that bond of wheel perform the role of a perfectly good conductor, the contact resistance is approximately described by expression [12]:

$$R_C = r_0 \left(1 + \frac{2}{3} \alpha \right) / d \tag{5}$$

Where: d – the characteristic dimension of contact spot. For circular contact, d is the diameter of spot.

Upon transformation of Eq. (4) and Eq. (5), the current density can be estimated by:

$$i = \frac{U}{r_0 \left[(1 + \alpha\theta + \beta\theta^2)L + (1 + 2\alpha\theta/3)A/d \right]} \tag{6}$$

Substituting Eq. (6) into Eq (3) and transforming

$$\frac{\partial\theta}{\partial t} = a \frac{\partial^2\theta}{\partial x^2} + \frac{U^2(1 + \alpha\theta)}{r_0 \left[(1 + \alpha\theta + \beta\theta^2)L + (1 + 2\alpha\theta/3)A/d \right]^2 \rho \cdot C} \tag{7}$$

The estimations of the magnitude of terms in the right side of Eq. (7) for typical condition of BEDMA

shown, that the order of mentioned magnitudes in K/s, are:

$$a \frac{\partial^2\theta}{\partial x^2} \sim a \frac{T_m - T_0}{L^2} \propto 10^3 - 10^5$$

and

$$\frac{U^2(1 + \alpha\theta)}{r_0 \left[(1 + \alpha\theta + \beta\theta^2)L + (1 + 2\alpha\theta/3)A/d \right]^2 \rho \cdot C} \propto 10^7 - 10^{10}$$

Therefore, the first term, which is connected with heat conduction along axis x , can be neglected, and Eq. (7) becomes:

$$\frac{\partial\theta}{\partial t} = \frac{U^2(1 + \alpha\theta)}{r_0 \left[(1 + \alpha\theta + \beta\theta^2)L + (1 + 2\alpha\theta/3)A/d \right]^2 \rho \cdot C} \tag{8}$$

With initial condition $\theta(t=0) = 0$.

For BEDMA process when voltage during short circuit is approximately stable, after integrating Eq. (8) and transforming and neglecting second order term, the time, t_m , for melting can estimated from:

$$t_m = \frac{r_0 \rho \cdot C L^2 \theta}{U^2} \left[1 + \frac{1}{2} \alpha \theta + \left(1 + \frac{1}{3} \alpha \theta \right) \frac{d}{L} \right] \tag{9}$$

For the case of BEDMA when pulse generator with constant amplitude of pulse current I is used, the heating can be approximately described by:

$$\frac{\partial\theta}{\partial t} = \frac{r_0(1 + \alpha\theta)i^2}{\rho \cdot C} \tag{10}$$

$$\theta(t = 0) = 0,$$

where:

$$i = \frac{I}{n \cdot A(x)},$$

I is the setting current and n is number of filaments contacted with the workpiece.

Solution of Eq. (10) for this case is

$$t = \frac{\rho \cdot C \cdot n^2 A^2}{\alpha \cdot r_0 \cdot I^2} \ln(1 + \alpha\theta) \tag{11}$$

The Figure 3 shows the change in time heating of the

temperature of the filament.

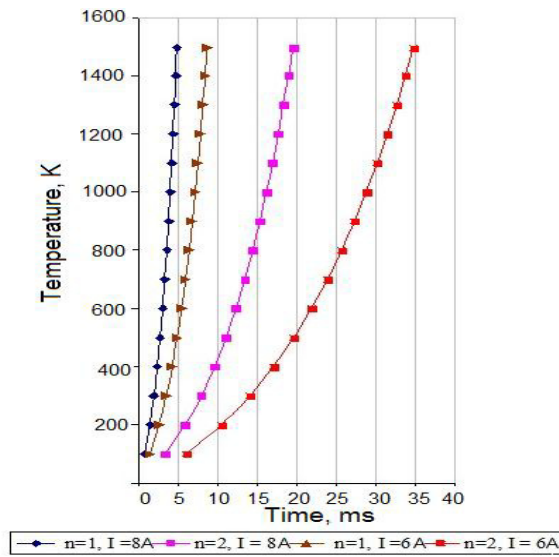


Fig. 3 Temperature versus time for tungsten filaments (cross sectional area of the filament $A = 10000 \mu\text{m}^2$)

After reaching of the melting point T_m by the filament end ($x=0$) ($\Theta_m = T_m - T_0$), the boundary conditions and properties of materials (melted) are changing. The new phenomena at the end of filament are appearing such as electrical discharges (Figure 4).

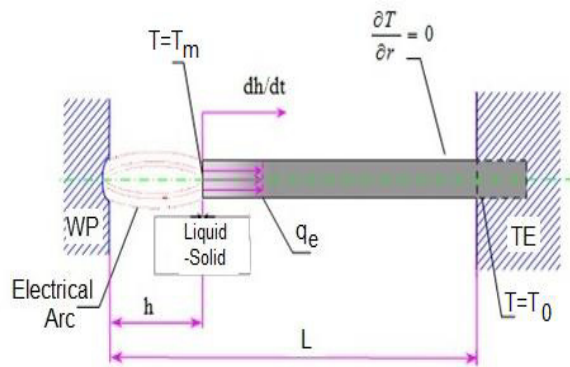


Fig. 4 Scheme for modelling of heating after reaching of the melting point T_m by the filament

For modelling this stage the additional assumptions is introducing as following:

- the effect of melted part of filament on total resistance are neglecting,
- the effect of melted part and electrical arc on current is accounted for by introducing the drop voltage ΔU_d into the boundary condition.
- the temperature of melted part is constant and equal T_m ,

The change phase of material solid – liquid (S-L) led to Stefan boundary conditions as following:

$$\frac{dh}{dt} = \frac{I}{\rho_m L} [q_e + \lambda \frac{\partial T}{\partial x}] \text{ at } x=h \quad (12)$$

Where

$$q_e = \eta_e i \Delta U_d$$

is the power density transferred into the filament from electrical arc, η_e is energy distribution ratio to filament, ρ_m , L – density and heat fusion of filament material, respectively.

The Eq. (12) described the rate of melting and transferring material from the filament into the workpiece surface.

Theoretical maximal rate with neglecting heat conduction is equal

$$V_m = \frac{dh}{dt} |_{\max} = \frac{q_e}{\rho_m L} = \eta_e \frac{i \Delta U_d}{\rho_m L} \quad (13)$$

or

$$V_m = \eta_e \frac{\Delta U_d I}{\pi r_0^2 \rho_m L} \quad (14)$$

The mathematical model described of melting i.e. alloying phase of BEDMA is as following:

$$\left. \begin{aligned} \frac{\partial \Theta}{\partial t} &= a \frac{\partial^2 \Theta}{\partial x^2} + \rho_0 (1 + \alpha \Theta + \beta \Theta^2) \frac{i^2}{\rho_m C} \\ i &= \frac{U - \Delta U_d}{\rho_0 \int_0^{L-h} (1 + \alpha \Theta + \beta \Theta^2) dx} \end{aligned} \right\} \quad (15)$$

with initial condition:

$$t = t_m \quad \theta(x, t_m) = \theta(x, t_m) \text{ from Eq. 7}$$

and boundary conditions as follows:

$$\left. \begin{aligned} x = h \quad \theta = \theta_m \\ \frac{dh}{dt} = \frac{1}{\rho_m L} \left[q_e + \lambda \frac{\partial \Theta}{\partial x} \Big|_h \right] \\ x = L \quad \frac{\partial \Theta}{\partial x} = 0 \end{aligned} \right\}$$

Solution of the system equations (7) and (15) has been obtained by using Finite Difference Methods for Stefan Problems.

Based on developing of thermal model BEDMA, the value of decrement of filament length during single action has been estimated as following:

$$\Delta h = \eta_e \frac{\Delta U_d}{\rho_m L \pi r_0^2} I(t_p - t_m) \quad (16)$$

With using Eq. (16) the amount of transferred material from the filament into the workpiece can be estimated and effects of parameters BEDMA on productivity analyzed.

Conclusion

The thermal analysis of BEDMA process has been applied to develop mathematical models of the material transfer rate. These models may be helpful to the manufacturers for selecting the machining parameters during electrical discharge alloying.

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