Reconciling Planck with the local value of H_0 in extended parameter space

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A R T I C L E   I N F O

A B S T R A C T

The recent determination of the local value of the Hubble constant by Riess et al., 2016 (hereafter R16) is now 3.3 sigma higher than the value derived from the most recent CMB anisotropy data provided by the Planck satellite in a ΛCDM model. Here we perform a combined analysis of the Planck and R16 results in an extended parameter space, varying simultaneously 12 cosmological parameters instead of the usual 6. We find that a phantom-like dark energy component, with effective equation of state $w = -1.29^{+0.15}_{-0.12}$ at 68% c.l. can solve the current tension between the Planck dataset and the R16 prior in an extended ΛCDM scenario. On the other hand, the neutrino effective number is fully compatible with standard expectations. This result is confirmed when including cosmic shear data from the CFHTLenS survey and CMB lensing constraints from Planck. However, when BAO measurements are included we find that some of the tension with R16 remains, as also is the case when we include the supernova type Ia luminosity distances from the JLA catalog.

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1. Introduction

Since the first data release of 2013 ([1]), the constraints on the Hubble constant coming from the Planck satellite have been in significant tension with the results of Riess et al., 2011 ([2], hereafter R11), based on direct measurements made with the Hubble Space Telescope. This tension was further confirmed in the 2015 Planck data release [3]. Assuming standard ΛCDM the Planck data gives $H_0 = 67.27 \pm 0.66$ km/s/Mpc that is about two standard deviations away from the Riess et al., 2011 value of $H_0 = 73.8 \pm 2.4$ km/s/Mpc ([2]).

Given that the Planck constraint is derived under the assumption of the "standard" ΛCDM model, a large number of authors (including the Planck collaboration itself, see [1] and [3]) have proposed several different mechanisms to explain this tension by considering, for example, an increased value in the effective number of relativistic particles N_{eff} ([4]), phantom dark energy (see e.g. [1]), interacting dark energy ([5]), or cosmic voids ([6]). Cos-

mic variance can affect the local measurement ([7]), but probably introduces too small uncertainty to explain the discrepancy ([8]).

On the other hand, Efstathiou ([9]) questioned the reliability of some fraction of the Riess et al. (2011) dataset. Using the revised geometric maser distance to NGC 4258 and neglecting the Large Magellanic Cloud and Milky Way distance anchors, Efstathiou derived a conservative constraint of 70.6 ± 3.3 km/s/Mpc at 68% c.l. (EST14, hereafter), consistent in between one σ with the Planck result. Therefore, he concluded in [9] that the discrepancies between the Planck results and the R11 measurements were not large enough to provide significant evidence for deviations from ΛCDM.

However, the recent analysis of [10] (R16, hereafter), confirmed and improved the constraint presented in [2] with $H_0 = 73.24 \pm 1.74$ km/s/Mpc at 68% c.l., finding no compelling argument to not combine the three distance anchors as in [9] and including a detailed discussion of possible systematics. At the same time, the new constraints on the reionization optical depth, obtained with Planck HFI data [11], bring the Planck constraint on H_0 to an even lower value, with $H_0 = 66.93 \pm 0.62$ km/s/Mpc at 68% c.l. (see Table 8 in [11]). The new R16 value, which we may refer to as the

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local value of H_0, is therefore more than 3.3 standard deviations above the global value, the Planck constraint obtained assuming ΛCDM.

In other words, after three years of improved analyses and data sets, the tension in the Hubble constant between the various cosmological datasets not only persists but is even more statistically significant.

Following previous analyses (see [3] and [10] and references therein), two possible extensions to the ΛCDM scenario have been suggested to solve the tension. It has been found that considering a neutrino effective number $N_{eff} \sim 3.5$, i.e. the possibility of a dark radiation component, or having a dark energy equation of state with $w \sim -1.1$ could bring the Planck constraint into better agreement with higher values of the Hubble constant.

In this paper, we further investigate these possible solutions to the Hubble constant tension by performing an analysis in an extended parameter space, varying simultaneously 12 parameters instead of the usual 6 assumed in ΛCDM. As we argued in [13], many of the assumptions made in ΛCDM are indeed not fully justified. For example, there is clearly no theoretical argument that requires us to restrict the dark energy component to a cosmological constant. Moreover, neutrinos are massive and there is no current laboratory measurement that could constrain their absolute mass scale to be less than, say, $\Sigma m_\nu < 1$ eV. Assuming the minimal value of $\Sigma m_\nu = 0.06$ eV as in ΛCDM could therefore introduce a strong bias in the analysis since it is equivalent to removing a large portion of the physically allowed parameter space. Hence especially in view of the new precise measurements made by Planck, it seems reasonable to consider a larger parameter space.

It is also important to stress that simply increasing the number of parameters would not necessarily bring the two datasets in agreement. The neutrino mass, for example, anti-correlates with the value of the Hubble constant when constrained from CMB data, and the Planck constraint would be even lower when variations in Σm_ν are considered.

Following the method presented in [13], we therefore consider as additional parameters the dark energy equation of state w, the neutrino effective number N_{eff}, the running of the spectral index $dn_s/dlnk$, the tensor to scalar ratio r, the neutrino mass Σm_ν and, finally, the amplitude of the gravitational lensing on the CMB angular spectra A_{lens} (see [14] for a definition). The inclusion of the last parameter comes from the Planck data itself that suggests an anomalous value of $A_{lens} = 1.15^{+0.13}_{-0.12}$ at 95% c.l. [11], but see also [12].

However, respect to [13], here we include the new R16 result, studying the compatibility not only with the Planck data, but also with several combination of datasets. Indeed the goal of this paper is to identify a new "concordance" model in an extended parameter space, where the new R16 result could be accommodated.

Moreover, another anomaly is present when the Planck dataset alone is considered: indeed, Planck is suggesting also a non flat universe, with positive curvature such that the curvature density parameter is constrained to be $\Omega_k = -0.052^{+0.049}_{-0.055}$ at 95% c.l. (see [3]). It is therefore interesting to consider also this possibility and in this paper we further extend the analysis presented in [13] by considering an extended parameter space where curvature, instead of A_{lens}, is varied.

Our brief paper is structured as follows: in the next Section we describe the data analysis method adopted, in Section 3 we present our results and in Section 4 we derive our conclusions.

2. Method

As in [13] we analyze current cosmological data by making use of publicly available code `cosmomc` [15,16].

As discussed in the introduction, following [13], we consider an extended ΛCDM scenario where we vary a total of 12 cosmological parameters simultaneously.

We indeed vary the "standard" six parameters of the ΛCDM model: the baryon ω_b and cold dark matter ω_c energy densities, the angular diameter distance to the sound horizon at last scattering θ, the amplitude A_s and tilt n_s of primordial scalar fluctuations and the reionization optical depth τ. In addiction to these parameters, we vary at the same time also 6 extra parameters: the absolute neutrino mass scale Σm_ν, the neutrino effective number N_{eff}, the tensor-to-scalar ratio r, the running of the scalar spectral index $dn_s/dlnk$, the dark energy equation of state w and the lensing amplitude in temperature and polarization angular spectra A_{lens}.

Moreover, as mentioned in the introduction, we also consider a slightly different extended parameter space by fixing the values of the neutrino effective number and of the lensing amplitude to their LCDM values of $N_{eff} = 3.046$ and $A_{lens} = 1$, but letting now the curvature density Ω_k to vary. In this way we could not only test the possibility of a curved universe, as suggested by Planck data alone, but also in someway quantify how much the results could depend on the variation of A_{lens} that is indeed an effective parameter with an unclear origin.

Our main dataset consists of CMB temperature and polarization anisotropies from the Planck 2015 data release ([17]). In what follows, we refer to this dataset simply as "*Planck*".

Together with the R16 constraint on the Hubble constant, that we treat as an external gaussian prior of $H_0 = 73.20 \pm 1.74$ km/s/Mpc at 68% c.l., we also consider the following additional datasets:

- The collection of Baryonic Acoustic Observations (BAO) (6dFGS [18], SDSS-MGS [19], BOSS LOWZ [20] and CMASS-DR11 [20] *BAO*);
- The luminosity distances of supernovae type Ia from the Joint Light-curve Analysis catalog (JLA) [21];
- *Planck* measurements of the CMB *lensing* potential power spectrum $C_\ell^{\phi\phi}$ [22];
- Weak lensing (WL) data from the CFHTLenS survey [23,24], taking wavenumbers $k \leq 1.5$ h Mpc^{-1} [3,25];

3. Results

Our main results are reported in Table 1 where we report the constraints at 68% c.l. on the 12 parameters of our extended scenario. As discussed in the previous section, we consider the Planck dataset (temperature and polarization) plus the new R16 prior in combination with BAO, JLA, CFHTLenS and Planck CMB lensing data sets. For comparison, we also consider the Planck data set alone. See Fig. 1.

We found that the Planck + R16 data set provides a reasonable increase in the effective chi-square value of $\Delta\chi^2_{eff} \sim 0.9$ with respect to the Planck data set alone, with one single additional data point. In other words, the R16 prior is fully compatible with the Planck data in our extended ΛCDM scenario. It is therefore interesting to understand which of the extra parameters contributes to restoring the agreement between Planck and R16. By looking at the extra parameters, we notice that while the neutrino effective number is compatible with its standard value of $N_{eff} = 3.046$, the dark energy equation of state is below -1 at the level of ~ 2 sigma, hinting at new physics in the dark energy sector. We also see that the A_L is larger than its standard value at more than 2 standard deviations. However this anomaly is driven by the Planck dataset and the inclusion of the R16 prior does not significantly affect its statistical significance.

Table 1
68% c.l. constraints on cosmological parameters in our extended 12 parameters scenario from different combinations of datasets.

	Planck	Planck + R16	Planck + R16 + BAO	Planck + R16 + JLA	Planck + R16 + WL	Planck + R16 + lensing
$\Omega_b h^2$	0.02239 ± 0.00030	0.02239 ± 0.00029	$0.02258^{+0.00026}_{-0.00032}$	0.02270 ± 0.00025	0.02253 ± 0.00029	0.02214 ± 0.00027
$\Omega_c h^2$	0.1186 ± 0.0035	0.1187 ± 0.0036	$0.1209^{+0.0032}_{-0.0036}$	0.1218 ± 0.0034	0.1188 ± 0.0036	0.1176 ± 0.0035
τ	0.058 ± 0.021	$0.058^{+0.021}_{-0.023}$	0.058 ± 0.021	0.059 ± 0.021	$0.050^{+0.019}_{-0.022}$	0.058 ± 0.021
n_S	0.967 ± 0.013	0.967 ± 0.013	0.976 ± 0.12	0.981 ± 0.011	0.973 ± 0.012	0.959 ± 0.012
$log(10^{10}A_S)$	3.048 ± 0.043	$3.048^{+0.043}_{-0.048}$	3.053 ± 0.043	3.056 ± 0.043	3.030 ± 0.041	3.043 ± 0.043
H_0	> 67.1	73.5 ± 1.9	71.3 ± 1.6	70.9 ± 1.5	73.6 ± 1.9	73.7 ± 2.0
σ_8	$0.81^{+0.16}_{-0.12}$	$0.804^{+0.056}_{-0.044}$	0.788 ± 0.036	$0.785^{+0.056}_{-0.037}$	$0.786^{+0.053}_{-0.042}$	0.827 ± 0.039
$\sum m_\nu$ [eV]	< 0.53	< 0.512	$0.35^{+0.16}_{-0.25}$	< 0.384	$0.43^{+0.12}_{-0.41}$	$0.32^{+0.14}_{-0.24}$
w	$-1.32^{+0.47}_{-0.67}$	$-1.29^{+0.15}_{-0.12}$	$-1.14^{+0.12}_{-0.10}$	$-1.079^{+0.072}_{-0.057}$	$-1.25^{+0.13}_{-0.11}$	$-1.33^{+0.15}_{-0.12}$
N_{eff}	$3.08^{+0.26}_{-0.30}$	$3.09^{+0.26}_{-0.31}$	$3.26^{+0.24}_{-0.28}$	$3.37^{+0.24}_{-0.28}$	$3.17^{+0.26}_{-0.31}$	2.94 ± 0.25
A_{lens}	$1.21^{+0.09}_{-0.14}$	$1.18^{+0.09}_{-0.11}$	1.210 ± 0.095	$1.22^{+0.09}_{-0.11}$	$1.233^{+0.085}_{-0.099}$	1.031 ± 0.062
$\frac{dn_S}{d\ln k}$	-0.0034 ± 0.0098	$-0.003^{+0.010}_{-0.011}$	-0.0003 ± 0.0091	$0.001^{+0.009}_{-0.011}$	-0.0003 ± 0.0097	-0.0054 ± 0.0090
r	< 0.0911	< 0.0934	< 0.0974	< 0.0943	< 0.099	< 0.0856
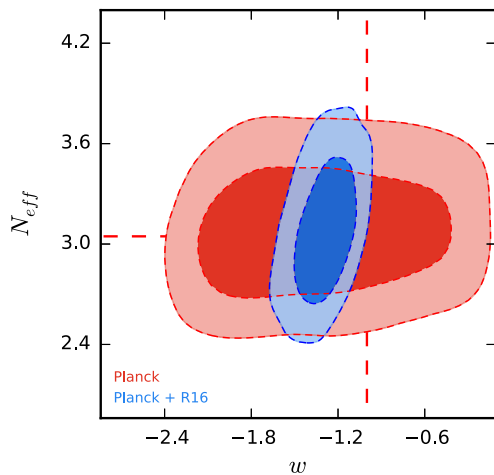
Fig. 1. Constraints at 68% and 95% c.l. on the N_{eff} vs w plane assuming the Planck data set with and without the R16 prior on H_0. As one can see, when the R16 prior is included, a preference for $w < -1$ is clearly present, while N_{eff} is consistent with the standard expectations. An extended ΛCDM theoretical framework of 12 parameters is assumed in the analysis. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
68% c.l. constraints on cosmological parameters in our extended 11 parameters scenario that includes variations in Ω_k from different combinations of datasets.

	Planck	Planck + R16 + lensing	Planck + R16 + BAO
$\Omega_b h^2$	0.02238 ± 0.00018	0.02221 ± 0.00018	$0.02232^{+0.00019}_{-0.00018}$
$\Omega_c h^2$	0.1183 ± 0.0016	0.1191 ± 0.0015	$0.1195^{+0.0015}_{-0.0015}$
τ	0.054 ± 0.021	$0.056^{+0.021}_{-0.020}$	0.083 ± 0.019
n_S	0.9675 ± 0.0055	0.9641 ± 0.0055	0.9646 ± 0.0053
$log(10^{10}A_S)$	3.039 ± 0.042	$3.043^{+0.042}_{-0.041}$	3.101 ± 0.037
H_0	51^{+6}_{-10}	73.7 ± 2.0	71.3 ± 1.6
σ_8	$0.724^{+0.063}_{-0.13}$	$0.845^{+0.026}_{-0.026}$	$0.861^{+0.036}_{-0.025}$
$\sum m_\nu$ [eV]	$0.29^{+0.13}_{-0.24}$	0.32 ± 0.16	< 0.172
w	$-0.99^{+0.72}_{-0.45}$	$-1.45^{+0.25}_{-0.19}$	$-1.193^{+0.088}_{-0.10}$
Ω_k	$-0.067^{+0.053}_{-0.025}$	$-0.0046^{+0.0053}_{-0.0064}$	$-0.0018^{+0.0026}_{-0.0034}$
$\frac{dn_S}{d\ln k}$	-0.0022 ± 0.0074	$-0.0019^{+0.0078}_{-0.0077}$	-0.0073 ± 0.0076
r	< 0.0977	< 0.0834	< 0.0722

When the BAO dataset is included, the indication for $w < -1$ is much less significant. Moreover, we note an increase in the value of N_{eff}, even if is still in agreement with its standard value. More interestingly, the inclusion of the BAO dataset shows an indication at one sigma for a neutrino mass with $\Sigma m_\nu = 0.35^{+0.16}_{-0.25}$ at 68% c.l. The inclusion of the R16 prior in the Planck + BAO dataset increases the effective chi-square by $\Delta \chi^2 \sim 4.5$, suggesting a tension between the R16 prior and Planck + BAO even in a 12 parameter extension. This is clearly driven by the value of the Hubble constant from the Planck + BAO dataset that is constrained to be $H_0 = 68.4^{+4.3}_{-4.1}$ at 95% c.l. [13], i.e. lower than the R16 prior.

The inclusion of the JLA dataset, on the other hand, suggests at about one standard deviation a value for $N_{eff} > 3.046$ and a dark equation of state $w < -1$. In this case, the effective chi-square value when a R16 prior is included in a Planck + JLA analysis increases by $\Delta \chi^2 \sim 4.1$, indicating, as in the case of BAO, a tension between the Planck + JLA dataset and the R16 prior.

Vice versa, when the WL and CMB lensing datasets are included, we have again an indication for $w < -1$ (at 1.7 sigma for WL and 2.4 sigma for CMB lensing) while the χ^2 is not significantly affected by the inclusion of the R16 prior. We indeed found an increase in the effective chi-square of $\Delta \chi^2 \sim 0.8$ when the R16 prior is included in the analysis of the Planck + WL dataset and

$\Delta \chi^2 \sim 1$ when it is included in the analysis of the Planck + lensing dataset.

In order to further test the stability of our results under a different choice of the parameter space, we have also considered the possibility of a "less extended" parameter space of 11 parameters. In this case, we fix the neutrino effective number and the lensing amplitude to their LCDM values of $N_{eff} = 3.046$ and $A_{lens} = 1$, but letting this time the curvature parameter Ω_k to vary freely. Our results are reported in Table 2. As we can see from the first column, in this parameter space the Hubble constant is constrained from Planck to be $H_0 = 51^{+6}_{-10}$ at 68% c.l. The Planck dataset alone is therefore not compatible anymore with the R16 prior despite the significant increase in the parameter space. Indeed, while the effect of introducing variations in the neutrino number N_{eff} and the lensing amplitude A_L is to allow a better compatibility of larger values of H_0, the introduction of curvature produces exactly the opposite effect. We can therefore claim that a positive curvature, as suggested by Planck data alone, does not solve the tension between Planck and R16 on the value of the Hubble parameter, even in a 11 parameters space. It is interesting to study the compatibility with R16 when additional datasets as BAO or lensing are included, since their main effect, as discussed in [3], is to constrain curvature to be very close to zero. We have indeed found (always in this new 11 parameters space) that a Planck + BAO or Planck + Lensing analysis constrain the Hubble constant to $H_0 = 73.7 \pm 2.0$ km/s/Mpc and $H_0 = 67^{+10}_{-20}$ km/s/Mpc respectively, at 68% c.l., i.e. to values that are now in agreement with the R16 prior. We report in the second and third columns of

Table 2 the constraints on the 11 cosmological parameters for the Planck + lensing + R16 and Planck + BAO + R16 datasets. We can notice that in both cases the curvature is always extremely close to zero and in both cases the equation of state w is below -1 at about 95% c.l. In the Planck + lensing + R16 case we have an indication at about 95% c.l. for a neutrino mass, while the optical depth is significantly larger for Planck + BAO + R16. We can therefore conclude that when restricted to a 11 parameters space and after fixing the curvature anomaly including the BAO or lensing dataset, we found that the combined datasets suggest, again, $w < -1$ at about 95% c.l.

4. Conclusions

The recent determination of the local value of the Hubble constant by R16 is now 3.3 sigma higher than the value determined by measurements of CMB anisotropies made by the Planck satellite mission in a ΛCDM model. While the presence of systematics is not yet excluded, it is interesting to investigate what kind of new physics could solve the discrepancy. In this brief paper, we have performed a combined analysis of the Planck and R16 result in an extended parameter space, varying simultaneously 12 cosmological parameters instead of the usual 6 of ΛCDM, since in this scenario a higher value of H_0 is naturally allowed. We found that in this 12 parameter space, the tension is reduced with $N_{eff} = 3.09^{+0.26}_{-0.31}$ at 68% c.l., in very good agreement with the standard expectations, $H_0 = 73.5 \pm 2.9$ km/s/Mpc at 68% c.l., and $w = -1.29^{+0.15}_{-0.12}$, suggesting a phantom-like dark energy component at the level of 2 sigma. Moreover, this extended scenario prefers a lower value of the reionization optical depth $\tau = 0.058 \pm 0.021$, in complete agreement with the new value provided by Planck HFI data [11]. This result and the indication for $w < -1$ are confirmed when cosmic shear data from the CFHTLenS survey or CMB lensing data from the Planck maps are included in the analysis. However, when BAO measurements are included we get $N_{eff} = 3.26^{+0.24}_{-0.28}$ at 68% c.l., $H_0 = 71.3 \pm 1.6$ km/s/Mpc at 68% c.l., and $w = -1.14^{+0.12}_{-0.10}$, with the indication for $w < -1$ now present at just ~ 1.1 sigma. The inclusion of the R16 prior in the Planck + BAO dataset produces a worse fit of $\Delta\chi^2 \sim 4.5$. This is due to the tension at the level 1.7 sigma existing between the H_0 value provided by Planck + BAO; also in this extended 12 parameter space ($H_0 = 68.4^{+4.3}_{-4.1}$ at 95% c.l. [13]), and R16.

Including the supernova type Ia luminosity distances from the JLA catalog gives $N_{eff} = 3.37^{+0.24}_{-0.28}$ at 68% c.l., $H_0 = 70.9 \pm 1.5$ km/s/Mpc at 68% c.l., and $w = -1.079^{+0.072}_{-0.057}$, showing non-standard values for *both* w and N_{eff} at one sigma level. The chi-square value of the best fit increases by $\Delta\chi^2 \sim 4.1$ when a R16 prior is included in a Planck + JLA analysis, again due to a tension existing between the datasets. In fact, Planck + JLA prefers $H_0 = 67.4^{+4.4}_{-4.2}$ at 95% c.l. [13] in this extended scenario, almost two sigma lower with respect to R16.

Finally, we have considered a new, slightly different, extended parameter space letting curvature to vary. While curvature does not solve the tension between Planck and R16 on the Hubble constant, we have found that a combination of datasets as Planck + BAO and Planck + lensing can be put in agreement with the R16 prior by letting, once again, the equation of state w to be < -1.

We can therefore conclude that a variation in w can solve the current tension between the Planck dataset and the R16 prior in an extended ΛCDM scenario and that this result is confirmed when including the WL and CMB lensing datasets. Clearly, this indication for $w < -1$ could hide a more complicated dark energy model. Indeed, since we assumed w as constant with time, this can smear out information about w and its time variation (see e.g. [26]). Apart from phantom dark energy models with a genuine equation of state $w < -1$ (see e.g. [27]), models with a time-varying equation of state as interacting dark energy could also provide an effective value (averaged over redshift) of $w_{eff} < -1$ as obtained here ([28,29]). Interestingly, modified gravity models, such as, for example, the Hu and Sawicky model [30], could also provide a value for $w_{eff} < -1$. Modified gravity could also account for the A_{lens} anomaly (see e.g. [31]).

However the tension with the R16 value persists when the Planck + BAO or Planck + JLA datasets are considered, suggesting an even more complicated extension might be needed for ΛCDM, or, maybe more likely, systematic errors between the data sets. Since the increase in the number of parameters considered here is already significant, the presence of systematics in the datasets provides, in our opinion, a more conservative explanation. However, even if not all the datasets considered point in this direction, most of them indicate that the LCDM model may still be incorrect and several tensions are solved by introducing new physics. Future data from CMB experiments and galaxy surveys as DESI and EUCLID will certainly clarify the issue.

Acknowledgements

References

[1] P.A.R. Ade, et al., Planck Collaboration, Astron. Astrophys. 571 (2014) A16, http://dx.doi.org/10.1051/0004-6361/201321591, arXiv:1303.5076 [astro-ph.CO].

[2] A.G. Riess, et al., Astrophys. J. 730 (2011) 119, http://dx.doi.org/10.1088/0004-637X/730/2/119, Astrophys. J. 732 (2011) 129, http://dx.doi.org/10.1088/0004-637X/732/2/129 (Erratum), arXiv:1103.2976 [astro-ph.CO].

[3] P.A.R. Ade, et al., Planck Collaboration, arXiv:1502.01589 [astro-ph.CO].

[4] A. Heavens, R. Jimenez, L. Verde, Phys. Rev. Lett. 113 (24) (2014) 241302, http://dx.doi.org/10.1103/PhysRevLett.113.241302, arXiv:1409.6217 [astro-ph.CO];
E. Di Valentino, E. Giusarma, O. Mena, A. Melchiorri, J. Silk, arXiv:1511.00975 [astro-ph.CO];
E. Di Valentino, E. Giusarma, M. Lattanzi, O. Mena, A. Melchiorri, J. Silk, Phys. Lett. B 752 (2016) 182, http://dx.doi.org/10.1016/j.physletb.2015.11.025, arXiv:1507.08665 [astro-ph.CO];
M. Archidiacono, E. Giusarma, S. Hannestad, O. Mena, Adv. High Energy Phys. 2013 (2013) 191047, http://dx.doi.org/10.1155/2013/191047, arXiv:1307.0637 [astro-ph.CO].

[5] V. Salvatelli, A. Marchini, L. Lopez-Honorez, O. Mena, Phys. Rev. D 88 (2) (2013) 023531, http://dx.doi.org/10.1103/PhysRevD.88.023531, arXiv:1304.7119 [astro-ph.CO];
A.A. Costa, X.D. Xu, B. Wang, E.G.M. Ferreira, E. Abdalla, Phys. Rev. D 89 (10) (2014) 103531, http://dx.doi.org/10.1103/PhysRevD.89.103531, arXiv:1311.7380 [astro-ph.CO];
A. Pourtsidou, C. Skordis, E.J. Copeland, Phys. Rev. D 88 (8) (2013) 083505, http://dx.doi.org/10.1103/PhysRevD.88.083505, arXiv:1307.0458 [astro-ph.CO].

[6] K. Ichiki, C.M. Yoo, M. Oguri, Phys. Rev. D 93 (2) (2016) 023529, http://dx.doi.org/10.1103/PhysRevD.93.023529, arXiv:1509.04342 [astro-ph.CO].

[7] I. Ben-Dayan, R. Durrer, G. Marozzi, D.J. Schwarz, Phys. Rev. Lett. 112 (2014) 221301, http://dx.doi.org/10.1103/PhysRevLett.112.221301, arXiv:1401.7973 [astro-ph.CO].

[8] V. Marra, L. Amendola, I. Sawicki, W. Valkenburg, Phys. Rev. Lett. 110 (24) (2013) 241305, http://dx.doi.org/10.1103/PhysRevLett.110.241305, arXiv:1303.3121 [astro-ph.CO];

I. Odderskov, S. Hannestad, T. Haugbølle, J. Cosmol. Astropart. Phys. 1410 (10) (2014) 028, http://dx.doi.org/10.1088/1475-7516/2014/10/028, arXiv:1407.7364 [astro-ph.CO];
I. Odderskov, S.M. Koksbang, S. Hannestad, J. Cosmol. Astropart. Phys. 1602 (02) (2016) 001, http://dx.doi.org/10.1088/1475-7516/2016/02/001, arXiv:1601.07356 [astro-ph.CO].

[9] G. Efstathiou, Mon. Not. R. Astron. Soc. 440 (2) (2014) 1138, http://dx.doi.org/10.1093/mnras/stu278, arXiv:1311.3461 [astro-ph.CO].

[10] A.G. Riess, et al., arXiv:1604.01424 [astro-ph.CO].

[11] N. Aghanim, et al., Planck Collaboration, arXiv:1605.02985 [astro-ph.CO].

[12] S. Grandis, D. Rapetti, A. Saro, J.J. Mohr, J.P. Dietrich, arXiv:1604.06463 [astro-ph.CO].

[13] E. Di Valentino, A. Melchiorri, J. Silk, Phys. Rev. D 92 (12) (2015) 121302, http://dx.doi.org/10.1103/PhysRevD.92.121302, arXiv:1507.06646 [astro-ph.CO].

[14] E. Calabrese, A. Slosar, A. Melchiorri, G.F. Smoot, O. Zahn, Phys. Rev. D 77 (2008) 123531.

[15] A. Lewis, S. Bridle, Phys. Rev. D 66 (2002) 103511, arXiv:astro-ph/0205436.

[16] A. Lewis, Phys. Rev. D 87 (10) (2013) 103529, arXiv:1304.4473 [astro-ph.CO].

[17] N. Aghanim, et al., Planck Collaboration, arXiv:1507.02704 [astro-ph.CO].

[18] F. Beutler, et al., Mon. Not. R. Astron. Soc. 416 (2011) 3017, arXiv:1106.3366 [astro-ph.CO].

[19] A.J. Ross, L. Samushia, C. Howlett, W.J. Percival, A. Burden, M. Manera, Mon. Not. R. Astron. Soc. 449 (1) (2015) 835, arXiv:1409.3242 [astro-ph.CO].

[20] L. Anderson, et al., BOSS Collaboration, Mon. Not. R. Astron. Soc. 441 (1) (2014) 24, arXiv:1312.4877 [astro-ph.CO].

[21] M. Betoule, et al., SDSS Collaboration, Astron. Astrophys. 568 (2014) A22, http://dx.doi.org/10.1051/0004-6361/201423413, arXiv:1401.4064 [astro-ph.CO].

[22] P.A.R. Ade, et al., Planck Collaboration, arXiv:1502.01591 [astro-ph.CO].

[23] C. Heymans, et al., Mon. Not. R. Astron. Soc. 427 (2012) 146, arXiv:1210.0032 [astro-ph.CO].

[24] T. Erben, et al., Mon. Not. R. Astron. Soc. 433 (2013) 2545, arXiv:1210.8156 [astro-ph.CO].

[25] T.D. Kitching, et al., CFHTLenS Collaboration, Mon. Not. R. Astron. Soc. 442 (2) (2014) 1326, arXiv:1401.6842 [astro-ph.CO].

[26] I. Maor, R. Brustein, P.J. Steinhardt, Phys. Rev. Lett. 86 (2001) 6, http://dx.doi.org/10.1103/PhysRevLett.86.6, Phys. Rev. Lett. 87 (2001) 049901 (Erratum), arXiv:astro-ph/0007297.

[27] R.R. Caldwell, Phys. Lett. B 545 (2002) 23, http://dx.doi.org/10.1016/S0370-2693(02)02589-3, arXiv:astro-ph/9908168;
S.M. Carroll, M. Hoffman, M. Trodden, Phys. Rev. D 68 (2003) 023509, http://dx.doi.org/10.1103/PhysRevD.68.023509, arXiv:astro-ph/0301273;
A. Melchiorri, L. Mersini-Houghton, C.J. Odman, M. Trodden, Phys. Rev. D 68 (2003) 043509, http://dx.doi.org/10.1103/PhysRevD.68.043509, arXiv:astro-ph/0211522;
V. Sahni, Y. Shtanov, J. Cosmol. Astropart. Phys. 0311 (2003) 014, http://dx.doi.org/10.1088/1475-7516/2003/11/014, arXiv:astro-ph/0202346;
U. Alam, S. Bag, V. Sahni, arXiv:1605.04707 [astro-ph.CO].

[28] S. Das, P.S. Corasaniti, J. Khoury, Phys. Rev. D 73 (2006) 083509, http://dx.doi.org/10.1103/PhysRevD.73.083509, arXiv:astro-ph/0510628.

[29] Y.F. Cai, E.N. Saridakis, M.R. Setare, J.Q. Xia, Phys. Rep. 493 (2010) 1, http://dx.doi.org/10.1016/j.physrep.2010.04.001, arXiv:0909.2776 [hep-th].

[30] W. Hu, I. Sawicki, Phys. Rev. D 76 (2007) 064004, http://dx.doi.org/10.1103/PhysRevD.76.064004, arXiv:0705.1158 [astro-ph].

[31] E. Di Valentino, A. Melchiorri, J. Silk, Phys. Rev. D 93 (2) (2016) 023513, http://dx.doi.org/10.1103/PhysRevD.93.023513, arXiv:1509.07501 [astro-ph.CO].