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Self-relative (or Machian) information: $S_{BH} = \frac{1}{4}A_{BH} - \frac{3}{2}\log A_{BH}$

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ABSTRACT

The entropy–area relation of black holes is one of the important results of theoretical physics. It is one of the few relations that is used to test theories of quantum gravity in the absence of any experimental evidence. It states that $4 \times \ell_P^2$ is the fundamental area that holds *one* bit of information. Consequently, a question arises: why $4 \times \ell_P^2$ and not $1 \times \ell_P^2$ is the fundamental holder of *one* bit of information? In any case it seems the latter choice is more natural. We show that this question can be answered with a more explicit counting of the independent states of a black hole. To do this we introduce a method of counting which we name self-relative information. It says that a bit alone does not have any information unless it is considered near other bits. Utilizing this approach we obtain the correct entropy–area relation for black holes with $1 \times \ell_P^2$ as the fundamental holder of *one* bit of information. This method also predicts, naturally, the existence of logarithmic corrections to the entropy–area relation.

1. Introduction

Black holes are very important in classical relativity as well as quantum gravity. In classical physics the notions of black hole and big bang play a crucial role in understanding their singular behavior [1]. It is generally believed that the gravitational field becomes dominant near the singularities resulting in breaking down of classical general relativity. And as a consequence, the quantum effects of gravity become worthy of consideration. As mentioned above, the behavior of singularities in classical general relativity in one hand and the foundations of quantum mechanics on the other, may lead to the resolution of the question of singularities in quantum general relativity theory [2,3]. From another viewpoint, in the theory of everything, the final theory should resolve all the existing problems in current theories such as singular behaviors. For example, string theory as a candidate for the theory of everything[1] should present some clear ideas on black holes. In addition, lack of any direct experimental data in quantum gravity regime causes an ambiguity on the correctness of proposed theories. Until such experimental evidence, the theoretical work plays the crucial role of verifying the correctness of such theories. One of these theoretical evidences is the comparison of different methods. The most trusted method is the semi-classical analysis.[2] Fortunately, for the black hole behavior there is some semi-classical analysis and predictions. The agreement of these predictions with the predictions of the proposed theories is an important[3] sign for the correctness of those theories. In the following, we will focus on the problem of black hole entropy.

The entropy of black holes is one of the very interesting problems in the theoretical physics. In calculations related to black holes, the fundamental constants (c, $\hbar$ and G) appear and tie to each other and interestingly, this is exactly the realm of quantum gravity. The more interesting feature is that the resulting entropy can be deduced in the absence of any full quantum gravity [4]. As mentioned above, it is very essential to check quantum gravity candidates. Because there exists a result in the quantum gravity regime that can be a tester for theories of quantum gravity e.g. string theory or canonical quantum gravity and so on. The story of black hole entropy began with the possible contradiction between the existence of black holes and the second law of thermodynamics. Avoiding this contradiction results in an analogy between thermodynamical quantities and black hole's properties [5]. It is worth to mention that the similar four laws of black hole mechanics are just some analogies in classical regime when introduced in [5]. To understand better the nature of these analogies and consequently find an interpretation for them it is necessary to enter quantum phenomena. The mentioned analogies make a generalization in the second law of thermodynamics that a black hole has an entropy. This entropy of a black hole is proportional to its area, $S_{BH} \propto A$, as conjectured by Bekenstein [6]. The factor of proportionality is

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[1] Or the other approaches of quantum gravity as a part of the everything theory.

[2] Since, at least, this method coincides with the classical results in the appropriate limits.

[3] Maybe the most important.

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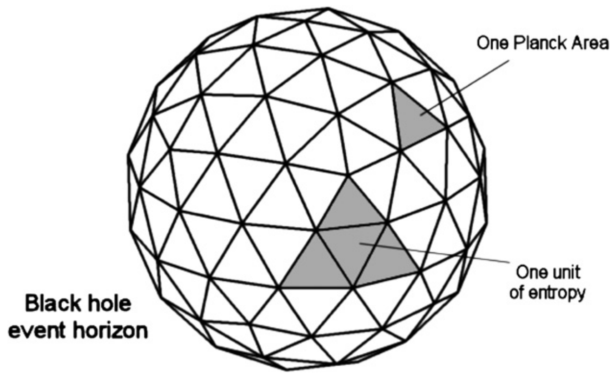

Fig. 1. This figure shows the standard interpretation of entropy–area relation for a black hole, schematically. In other words, the area is decomposed to fundamental areas, $1 \times \ell_P^2$, but the unit of entropy (information) is $4 \times \ell_P^2$. We have shown in the body of the Letter that both of area and entropy (information) units are same and equal to $1 \times \ell_P^2$ in the context of self-relative information procedure. The figure is borrowed from http://www.scholarpedia.org/article/Image:BHentropyF1.jpeg.

fixed by Hawking [4] such that $S_{BH} = \frac{1}{4}A$. As mentioned before, Hawking did the calculations with a semi-classical method. Nowadays different approaches show the same result for the entropy of black holes e.g. in string theory [7], in canonical quantum gravity [8] and in holographic perspective [9]. In addition, these quantum theories of the gravity predict a logarithmic correction term with a method dependent pre-factor.

On the other hand, the discrete structure of geometry is commonly believed as a consequence of quantum gravity [10]. This kind of structure makes it possible to find the entropy of a black hole due to counting the microstates [11] and calculate the entropy by Shanon law $S \propto \log P$ [12],[4] where P is the number of microstates. In more details, one can have an area proportional to the minimum area,[5] $A = N\ell_P^2$ where ℓ_P is the Planck length. Letting two possible values for each fundamental area[6] results in $P = 2^N$ possible states. By Shanon law the entropy becomes $S \propto N$ and then $S \propto A$ or $S = k\frac{A}{\ell_P^2}$. As has been mentioned in the literature there is no evidence in this approach to find the proportionality constant k [13].[7] Comparison of this information based method and other approaches [4] determines that $k = \frac{1}{4}$. It means that $4 \times \ell_P^2$ holds *one* bit of information, a 0 or a 1 (see Fig. 1). In our opinion this is a bizarre result, since naturally $1 \times \ell_P^2$ should hold *one* bit of information. In this Letter we will try to show that each $1 \times \ell_P^2$ holds *one* bit of information while still the same standard relation between area and entropy of a black hole is valid. In the following we will introduce the notion of self-relative information to establish physically meaningful information. Then we will use the suggested method of counting to obtain the black hole entropy–area relation. In addition we will show that the procedure imposes a logarithmic correction term, naturally. We will close the Letter with concluding remarks.

[4] In [12] the basis of the logarithmic function is 2 and it seems it is related to the definition of probability and possibility. This may resolve a log 2 factor in the final entropy–area relation.

[5] ℓ_P^2 is the only natural choice for the minimum area.

[6] This means that the fundamental area can only hold two different bits 0 or 1. The calculations do not depend to this special proposition and the method works properly for general cases, i.e. d-level systems, as it will be shown.

[7] In his lecture, Strominger expresses some difficulties in quantum gravity. Here it is worth to quote the first hint in his lecture: "If we tile the horizon with Planck-sized cells, and assign one degree of freedom to each cell, then the entropy, which is extensive, will go like the area. This suggests that the microstates can be described as living on the horizon itself. The hard part is to naturally get the $\frac{1}{4}$ from such a picture". Deducing this $\frac{1}{4}$ is the main part of current work.

2. Self-relative information

To commence this section it is worth mentioning briefly some points about the information-based viewpoint on black hole properties. As stated above, there is an analogy between black hole properties and thermodynamical quantities. On the other hand the thermodynamics can be seen by statistical mechanics' tools. In this form all the macroscopic thermodynamical quantities have a microscopic interpretation. For example the entropy relates to the variety of microstates that are constrained by a given macroscopic condition. Consequently, black hole macroscopic properties, i.e. its mass, angular momentum and electric charge, can be illustrated by some microscopic states. The next step is finding the microscopic states for a black hole which is done in the context of string theory, loop quantum gravity and also heuristic ways e.g. tiling the area of a black hole by the fundamental areas.[8] Attaching different states to each fundamental area makes this viewpoint very similar to information theory which contains a sequence of bits. This similarity results in natural usage of information theory concepts in black hole theory. One of these concepts is the relationship between the entropy and the information concept which can be characterized by Shanon law as mentioned previously. In this approach large entropy produces a large amount of information [14]. To meet this concept two dual approaches have been considered, a subjective picture versus an objective one [14]. In the subjective picture, information is known by the sender but is unknown for the receiver. However, in the objective viewpoint, the information is known for the receiver. To go further we will work in the subjective picture which is characterized by Shanon's entropy. Now let us introduce a question to enter more details.

How much information exist in a sequence of 0's and 1's? Or how much information can be stored in N bits of memory filled by 0 and 1? The straightforward answer is that since each bit has two different values then a sequence of N bits has 2^N distinguishable states. At first glance, it is correct but there is some ambiguities. To be clearer, let us try to answer how does computer understand what is the meaning of a sequence of bits? The computer compares the given sequence with its database to say for example in seven bit ASCII code, 10001011001110101010010100101001111101000010110001 means ENTROPY. To make this correspondence, access to the ASCII code table is necessary and without the table it is impossible. It means that to find the meaning of a sequence of bits, a dictionary is an essential requirement. As another example, in cryptography when the data is sent to someone, he must have the relevant database to understand the content of the message. Now what about the cases for which we do not have any access to the appropriate dictionary? What about the number of possible states on the area of a black hole?

As mentioned above, lack of a proper dictionary makes understanding of a given sequence of bits impossible. Since if we cannot study the meaning of a sequence, it is not physically understandable then we must ignore it. Now let we utilize the counting program to deduce entropy of a black hole. What is in front of us? Similar to the above discussions we do not have any dictionary to translate the data on the area of a black hole in an appropriate way. The essential question appears naturally, how one can solve this problem not only for a black hole but generally? We will show in a sequence of bits some information exist even in the absence of a dictionary and call it self-relative information. The heart of the idea is that when there is no definite translator in

[8] Historically, this way of thinking has been heuristic but nowadays there is a physical interpretation e.g. from loop quantum gravity.

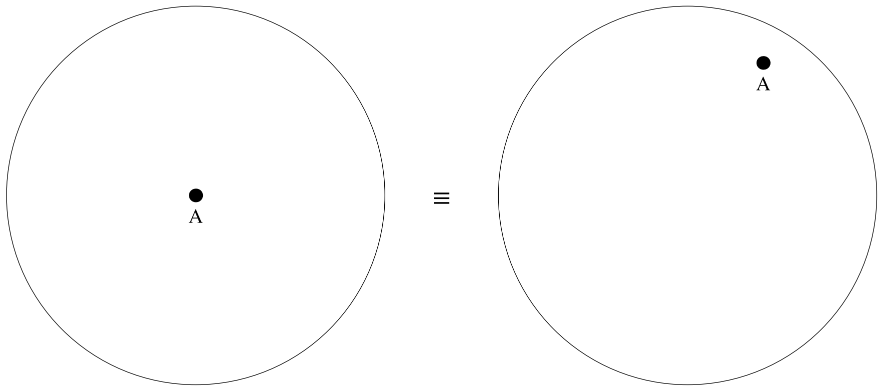

Fig. 2. To pick the first point, there is no differences between the points on the area. We will continue with the left figure without any loss of generality. It is worth mentioning that the first point, A, cannot understand the dimensionality of the area. This feature is crucial to reach to logarithmic correction term.

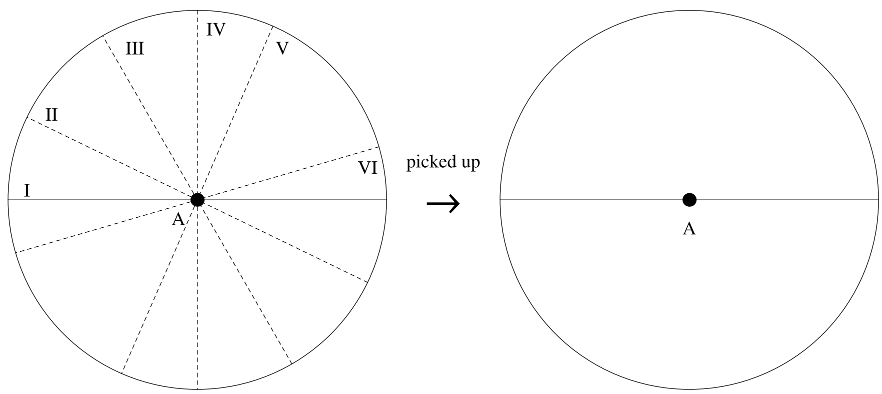

Fig. 3. The second point is in a definite distance to the first point, A. This additional second point and the first point can present only one dimension. But all the lines through A are same as each other due to the symmetry of the area. We picked the horizontal one without any loss of generality. Note, blindness of two points to the second dimension of the area plays a crucial role in appearing the logarithmic correction term.

nature then nature must choose a coding procedure which makes self-understanding possible. In a sequence of bits without a dictionary each bit does not mean by merely itself but its relative distance to other bits in the sequence can contain understandable and obtainable information. This idea is totally in agreement with the belief that there is no preferred observer in the universe and everything is relative.[9] In other words similar to the idea of Mach for geometry[10] and the heart of general relativity [2][11] there is no information for a sequence of only 0's.[12] It is easy to see that according to this kind of thinking on the notion of information, the amount of obtainable information in a sequence of bits is smaller in comparison to the standard viewpoint. Again, we stress that this new definition for the physical information is based on the relative relations of bits in the sequence e.g. relative distances. Now let us to count the number of physically understandable states on the area of a black hole in the next section.

[9] One can assume the dictionary exists but it has been lost. In this case there is two philosophically different choices, one stays on to find the dictionary and the other utilizes the relative information. We pick the second one in our discussion that is more usual in theoretical physics specially after special and general relativity theory.

[10] It says that there is no geometry in the presence of vacuum.

[11] The general relativity is a background independent theory i.e. only the relative quantities are physical quantities.

[12] Since there is no information in 0 or 1 and only their difference is a physical object, similar to the sign of electric charges, then it is true for a sequence of only 1's.

3. Black hole entropy

To calculate the entropy of a black hole we will use the counting method. In this method primarily we suppose that the structure of area is quantized and each quantum of area holds a bit of information. As mentioned before this method with the above assumptions breaks down because of the lack of the factor $\frac{1}{4}$ in the entropy–area relation. We show that this factor can be reproduced if one only attends to the understandable states. In other words, one must count only those states which are distinguishable. It is important to say that in this procedure we must note that we have not allowed any ambiguities to surface. The last phrase is essential in our calculations and we will see it in more details in the following. The existence of any ambiguities result in disability to recover information from a given sequence of bits. So, if we believe in recovery of information of nature then separating ambiguous sequences will be crucial and necessary not only in calculation but also in philosophy.

To distinguish different sequences two approaches exist, the first one is to compare two different sequences and state their equivalence or independence, the equivalent second method is constructing the independent sequences and then counting them. The second approach is more straightforward and we chose it here. Let the black hole be a sphere with area $A_{BH} = N^2 \ell_P^2$. In the first step all the bits contains 0 and we want to add 1's step by step and count the independent possibilities. Since sketching a sphere is not simple we use a circle but we know that the boundaries are imaginary. To change the first bit from 0 to 1 how many choices we

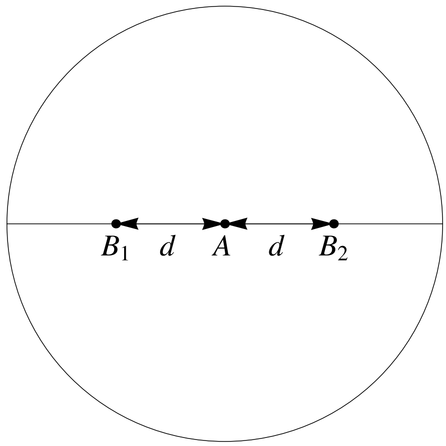

Fig. 4. To ignore any ambiguity we pick up two equivalent points for the second choice with same distances, d with respect to A.

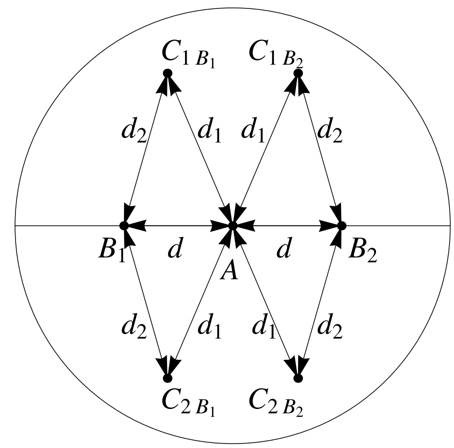

Fig. 5. The critical point is the third point since three points can understand the dimensionality of the area completely. There is four choices with d_1 and d_2 distances with respect to A and B respectively. Note that there is two equivalent points, B_1 and B_2. After this step there is four equivalent sets of points, $\{A, B_1, C_{1B_1}\}$, $\{A, B_1, C_{2B_1}\}$, $\{A, B_2, C_{1B_2}\}$ and $\{A, B_2, C_{2B_2}\}$. To pick up the fourth choice there is a unique point due to each set. The same is true for the latter choices that results in four absolutely same copies of a picture on the area in this figure.

have? The answer is N^2! But no, all our choices are equivalent because we cannot distinguish them, so for the first 1 we have only one choice which we name it point A in Fig. 2. What about the second one? $N^2 - 1$ choices? In this step different bits cause different sequences because of the existence of the first 1 (i.e. a point with a label, A). The relative distance between A and the second choice makes different sequences distinguishable. And since choosing the second point B with a distance d from A is not sensitive to the direction of equators pass through A then all these equators become equivalent and picking each of them up makes no independent sequence, Fig. 3. To reach the independent sequences one must pick up one of them and choose B with a distance d from A, Fig. 3. There exists still an ambiguity because of two choices for B on a line passing through A. To remove this ambiguity we can pick up both choices and make the third choice bearing in mind that B_1 and B_2 are equivalent, Fig. 4. It is important that these different B's are not distinguishable. So picking up both of them eliminates the worry about the ambiguity. Obviously choosing a point on an equator of a sphere with circumference $A_{BH} = N^2\ell_P^2$ has $\sim N$ possibilities. It is interesting to mention that the ignorance on such ambiguities reduces the $\sim N^4$ choices for both of the first points to $\sim N$. We will show that such dividing by $\sim N^3$ results in logarithmic correction to the entropy–area relation of a black hole. Turning to the third choice, it is the most important choice to get the correct $\frac{1}{4}$ factor in entropy–area relation. Suppose we want to choose the third point, C, in relative distances d_1 and d_2 with respect to A and B. We will continue the discussions in the two following subsections, in the first one we will show how $\frac{1}{4}$ appears naturally in the self-relative information proposal and in the second subsection, the appearance of the logarithmic correction term.

3.1. Picking up (counting) all distinguishable states

To do more on the third choice we will use the above proposal of removing ambiguities from the choices as mentioned for the second choice, B. Up to now, we have a point A, and two equivalent B_1 and B_2. Now to introduce the third point, C, with relative distances d_1 and d_2 with respect to A and B, there is four different choices as it is obvious in Fig. 5. All C's have a distance d_1 from A but C_{1B_1} and C_{2B_1} have distance d_2 from B_1 and C_{1B_2} and C_{2B_2} have distance d_2 from B_2. So now we have four indistinguishable states, $\{A, B_1, C_{1B_1}\}$, $\{A, B_1, C_{2B_1}\}$, $\{A, B_2, C_{1B_2}\}$ and $\{A, B_2, C_{2B_2}\}$. It means that these four sets have the same information and as a consequence they must be counted once in our method. Exactly, similar to picking up B to remove any ambiguities we will take all of these four indistinguishable states. Now for the fourth choice, D, with (allowable) relative distances d'_1, d'_2 and d'_3 to A, B and C respectively one must choose one of the equivalent sets.[13] But the interesting feature is that now for each set only one choice exists since there is only *one* intersection point for three circles generally. So for each set we have a D and in total we have four D's and similarly for next choices. It means that to ignore the ambiguities all the points must appear four times in the area or in other words, the area has four similar copies of one sequence. It means that the effective area is $A_{effective} = \frac{1}{4}A_{BH}$, Fig. 6. And since $Area \propto N^2$ then the effective information-filled bits are not N^2 but are $\frac{1}{4}N^2$. The entropy due to the relations $S = \log 2^{\frac{1}{4}N^2} = (\log 2) \times \frac{1}{4}N^2$ and $A_{BH} = N^2\ell_P^2$ becomes $S_{BH} = \frac{1}{4}\frac{A_{BH}}{\ell_P^2}$ with exactly[14] the correct factor $\frac{1}{4}$. Note that deducing of this factor is a direct consequence of self-relative viewpoint on the information.

3.2. The logarithmic correction term

Due to the above discussions, the idea of self-relative information can show why the entropy–area relation of black holes may be correct even with assuming ℓ_P^2 as a holder of *one* bit of information. But naturally this way of looking at the problem makes the correction terms appearance spontaneously and this is an advantage that this method has. As mentioned above, in choosing the first point A, there is not N^2 choices but only one choice because in the absence of any ticked bit there is no differences between the bits. To reduce this degeneracy the total number of states must be divided by N^2. And similarly for the second point, B, we are allowed to choose only $\sim N$ states instead of $\sim N^2$. Because for two points only the relative distance is important in self-relative information method. It means that the total number of distin-

[13] We stress on allowable to make the existence of D possible on the two-dimensional surface, i.e. three circles with origins A, B and C and radii d'_1, d'_2 and d'_3 respectively, must have an intersection to make the radii allowable.

[14] We ignore a $\log 2$ factor since it does not contribute to our discussion. As mentioned in the footnote 4 this factor is a natural factor due to our binary structure of bits.

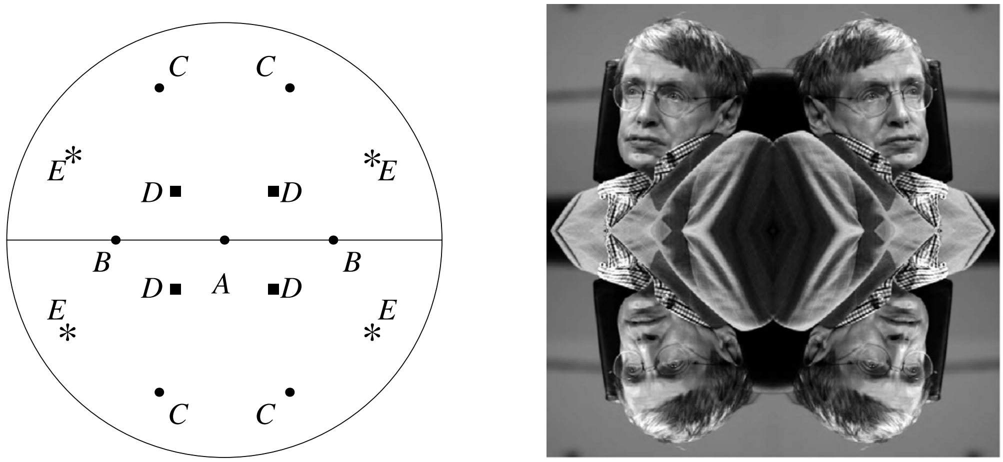

Fig. 6. There is four absolutely same copies of a picture on the area. This feature makes decreasing of the effective area containing the information, $A_{effective} = \frac{1}{4}A$. The direct consequence is that supposing each $1 \times \ell_P^2$ holds *one* bit of information predicts the correct entropy–area relation for black holes. This is indebted to self-relative information paradigm.

guishable states must be divided by N^3. So the entropy becomes $S_{BH} = \log \frac{2^{\frac{1}{4}N^2}}{N^3}$ that is $S_{BH} = \frac{1}{4}N^2 - \frac{3}{2}\log N^2$ and since $A_{BH} = N^2\ell_P^2$ then $S_{BH} = \frac{1}{4}\frac{A_{BH}}{\ell_P^2} - \frac{3}{2}\log\frac{A_{BH}}{\ell_P^2}$. This result is totally in agreement with the previous results. The constant factor of the logarithmic term is exactly in agreement with other approaches [7,15].

Note that the above results are not restricted to binary bit concept. For example for a d-level system, i.e. each bit can have d independent states, the entropy will be $S_{BH} = \log \frac{d^{\frac{1}{4}N^2}}{N^3}$ and consequently $S_{BH} = \frac{1}{4}N^2 \log d - \frac{3}{2}\log N^2$ or[15] $S_{BH} = \frac{1}{4}\frac{A_{BH}}{\ell_P^2} - \frac{3}{2}\log_d \frac{A_{BH}}{\ell_P^2}$. It is worth mentioning that the choice of the logarithmic function's basis does not affect the coefficient of this term. Since as assigned before this factor comes from avoiding any ambiguities to pick up the first and the second points and it is a natural factor due to two-dimensionality of black hole area.

4. Conclusions

In this Letter, in a model independent information based method we have deduced the entropy–area relation for a black hole which not only illustrates the correct coefficient in linear term, i.e. $\frac{1}{4}$, but also predicts a logarithmic correction term naturally. To do this, we have introduced a new interpretation of the concept of accessible information in a sequence of bits. The idea is based on the reality that in the absence of any reference dictionary or database, decoding a sequence of bits is impossible. And as a consequence, there is no pure knowledge about the information carried by that sequence. So to count the information in such a sequence the usual method of counting leads to incorrect results because the existence of the dictionary defines the meaning of the sequences. To remove this problem, we have proposed that all the information must be held by each sequence itself. That means the structure of the sequence itself must show all the information about that sequence. One possibility is that, the relative place of the bits in a same sequence are understandable and physical. This viewpoint on the information makes requiring to a dictionary unnecessary. In other words only the distinguishable internal relative structure of sequences leads to independent sequences. This feature is very important for counting the distinguishable sequences. We have named the method self-relative information viewpoint[16] since in this method all the requisites are the sequences themselves and only the relative positions of the bits in a given sequence have information.

It is shown in the context of self-relative information viewpoint, that the very famous relation of entropy–area for a black hole can be deduced i.e. $S_{BH} = \frac{1}{4}A_{BH}$. In standard viewpoint on the black hole entropy–area relation there is a $\frac{1}{4}$ proportionality factor that results in each $4 \times \ell_P^2$ element holding *one* bit of information. The natural question is that why $4 \times \ell_P^2$ and not $1 \times \ell_P^2$ is the fundamental holder of information? We have shown that by counting the distinguishable microstates using the self-relative method, not only is $1 \times \ell_P^2$ the fundamental holder of the information but also $S_{BH} = \frac{1}{4}A_{BH}$ is valid.[17] Note that for holding *one* bit of information, $1 \times \ell_P^2$ is more natural and credible than $4 \times \ell_P^2$. To do calculations we started with inverse method which constructed all the distinguishable microstates and then count them using the self-relative information paradigm. Another point is that during the calculations we must try to remove the possible ambiguities to obtain the correct final answer. The logarithmic correction term appears naturally as a direct consequence of the method that can be interpreted as an evidence for the legitimacy of this way of thinking. It is necessary to say some words about the first two chosen points. All points are on a two-dimensional surface i.e. the area of the black hole. To begin the counting we suppose that all the points are 0 i.e. a white two-dimensional space. Then we picked up[18] the first point, A, on this two-dimensional area. But the universe for this first black point in the white area is zero-dimensional because it cannot understand the dimensionality of the area with any experiments.[19] This feature is essential in calculating the logarithmic correction. Since to choose a point in zero-dimensional space there is no N^2 choices but one choice even if the space is two-dimensional.[20] This is because of the blindness of a sole point to

[15] As mentioned in [12] the basis of the logarithmic function in definition of entropy by Shanon law is an arbitrary and we have fixed it d itself.

[16] This method of thinking is exactly similar to Mach's thinking about the geometry.

[17] And even the logarithmic correction term appears naturally.

[18] I.e. changing the value from 0 to 1 or the color from white to black.

[19] Do not forget that all the physical quantities are relative and for the first point there is no other points for doing any physical comparison even understanding the dimension of space.

[20] Note that we have supposed $A_{BH} = N^2\ell_P^2$.

the dimensionality of its perimeter space. Or in other words, for the first choice there is no difference between all the points or all the points are equivalent. To continue the counting we picked up the second black point, B. Now there is only two black points, A and B. For these two points only their relative distance is meaningful and physical since this is the only relative quantity for a space with only two objects. Two points build a one-dimensional space and they are blind to any extra dimension, on the black hole's area i.e. the second dimension. It means that two points see the space, the area of interest, only in a one-dimensional form and not two-dimensional even if it is the dimensionality in reality. So the choices for the second point is not proportional to N^2 but it has only N choices due to one-dimensionality for only two points. The procedure for the next points becomes trivial since introducing the third point on the area of the black hole makes the space's dimensionality (the black hole's area) recognizable and therefore the dimensionality of the area becomes physical for the third point so the choices are proportional to $\sim N^2$. For the next points the area is two-dimensional since the third point has established the dimensionality of the space for all the next points. This is the reason for dividing the total number of choices by N^3 which causes the logarithmic correction term to appear.

The self-relative information proposal can be seen in the context of loop quantum gravity approach due to similar structures in some senses. The entropy–area relation has been considered in the latter approach as mentioned in [8]. In this scope as considered in [16] different states represent status of a black hole which are equivalent if and only if be indistinguishable by measurements outside the black hole region. That is, the information on the horizon and not inside it, is considerable [15,16]. It is exactly what is done in self-relative approach. Also, in comparison to [15], which is a quantum informational approach to black hole entropy–area relation, an interesting point is similar prediction for the coefficient factor of logarithmic correction term, $-\frac{3}{2}$, notwithstanding arbitrary level freedom for each bit. In [15] this universal factor is a consequence of entangled qubits but in our case it is a consequence of two-dimensionality of area. Another point is that the method in [15] cannot fix the coefficient of linear term if $1 \times \ell_P^2$ be assumed as the fundamental area that is an inevitable difference to the self-relative approach.

It is worth mentioning that to reconfirm the self-relative information paradigm it is possible to check it with the results from multi-dimensional models. They have shown that the entropy–area relation for black holes embedded in a D-dimensional geometry is same as the above result for the four-dimensional geometry with a $\frac{1}{4}$ factor. The application of the self-relative information proposal to this multi-dimensional configuration is not very straightforward. The first steps are same as four-dimensional one i.e. choosing the points is in the same manner as above. But there is a crucial interpretation that is the area holds information e.g. $1 \times \ell_P^2$ holds *one* bit of information. This concept is very crucial in the counting method. Unfortunately, this interpretation is not very obvious when selected points are not associated with area but with super-area.[21] It seems natural that when a fundamental area holds *one* bit of information then a fundamental super-area holds more.[22] In this sense this problem is still open to interpretation.[23] Also the application of self-relative information proposal should be used for other four-dimensional black holes i.e. rotating and charged black holes. Although the above open problems exist but perhaps considering the self-relative information paradigm as a way to understand better the entropy–area relation can help us find the rest of the iceberg of quantum gravity [13].

Finally, we would like to mention that among different approaches to quantization of general relativity like string theory or loop quantum gravity etc. there are some common features. As mentioned in [10], discreetness of geometrical objects (such as length, area and volume) and the holographic principle are common in different approaches to quantum gravity. The idea introduced in [10] says that to study the true quantum gravity, one must assume these features as the initial axioms and build the theory on these bases. Then in the semi-classical limits recover classical general relativity or quantum mechanics. We would like to suggest that self-relative thinking about the information can be another essential axiom about the nature. This idea is amplified by mentioning that there is no dictionary to decode the nature's information so the only way to think about it, is self-relative viewpoint. For the final words it seems good to note that if somebody believes in "it from bit" idea of Wheeler [11] then self-relative information plays a crucial role to interpret the quotation.

5. Summary

In this short section we will briefly present our paradigm in an axiomatic way.

Axiom 1. Discreetness of the area.

Evidence 1. Existence of the fundamental area, e.g. ℓ_P^2, is a common sense in quantum theories of gravity.[24]

Axiom 2. Each $1 \times \ell_P^2$ holds *one* bit of information.

Evidence 2. Maybe "it from bit".[25]

Axiom 3. Self-relative (Machian) information paradigm.

Evidence 3. There is no external knowledge about the information (i.e. there is no dictionary or database).

Evidence 3′. The notion of relativity is an essential concept in definition of physical quantities.

[21] By super-area we mean an object with more than two dimensions that plays the same role as the ordinary area in the calculation of entropy–area relation in four-dimensional geometry with a two-dimensional horizon.

[22] It makes a factor greater than 1 in the entropy–area relation that is in agreement with the self-relative information paradigm. It is very simple to show that this approach predicts $\frac{1}{2^{D-2}}$ for a black hole embedded in a D-dimensional geometry which is less than $\frac{1}{4}$. This brings some hope for self-relative information paradigm to be correct even for D-dimensional geometry with an explicit definition for how much information exists in a fundamental super-area.

[23] We would like to point that even if this approach does not work for D-dimensional geometries it still is interesting since our universe has four macroscopic dimensions. Maybe for calculation in extra dimensions those are not observables at least macroscopically and hence we need new definitions and notions for defining information. One suggestion can be quantum information since usually these extra dimensions correspond to quantum geometrical effects.

[24] There is a question that is ℓ_P the fundamental length? or for example $k \times \ell_P$ is the fundamental one? Where k is a proportionality factor. The answer is not straightforward. But in the absence of any knowledge on this problem proposition of ℓ_P as the fundamental length is natural. In other viewpoint, combination of the present self-relative information proposal and the entropy–area relation of black holes can be interpreted as an evidence for being ℓ_P as the fundamental length.

[25] For our proposal this proposition is only a way to interpret the concept of the entropy especially for black holes.

Black hole entropy–area theorem. *From the Axioms 1, 2 and 3*[26] *it can be shown that the following relation*[27] *in Planck's units exists between entropy of a black hole, S_{BH}, and its horizon area, A_{BH},*

$$S_{BH} = \frac{1}{4} A_{BH} - \frac{3}{2} \log A_{BH}.$$

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[26] It is worth mentioning that Axiom 3 (self-relative information) is indispensable to prove this theorem.

[27] Note that the existence of logarithmic correction term is a natural consequence of this method.