# Physical meaning of two-particle HBT measurements in case of correlated emission 

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#### Abstract

It is shown that, in the presence of correlations in particle emission, the measured HBT radii are related to the correlation range rather than to the size of the interaction volume. Only in the case of weak correlations the standard interpretation may be applicable. The earlier discussion [Phys. Rev. Lett. 68 (1992) 1109; Phys. Rev. C 49 (1994) 2722] of the short-range correlations in configuration space is generalized to include also the correlations of particle momenta.


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1. Measurements of HBT correlations in multiparticle production provide important information on the production mechanism, in particular on the space-time structure of the particle emission region [2]. To obtain this information, however, it is necessary to rely on some specific theoretical interpretation of the observed phenomena. The results are model dependent: the physical meaning assigned to the measured quantities does depend on the theoretical input.

In the standard treatment of this problem one usually starts with a model where particles are uncorrelated (except for Bose-Einstein correlations) and then corrects the results by including final state interactions. This includes corrections for Coulomb interactions, low energy particle interaction parametrized by scattering lengths and effects of resonances [3]. In the present Letter we discuss correlations due to strong interactions in the production process. Some such correlations are known to occur [4], some others, still hypothetical, may be, hopefully, uncovered by the HBT measurements [1].

To simplify the presentation we consider only the two-dimensional (transverse) distributions, ${ }^{1}$ taken as Gaussians to avoid complicated integrations which only obscure the essential points of our argument. In this case Wigner functions $W\left(\mathbf{p}_{1}, \ldots, \mathbf{p}_{n} ; \mathbf{x}_{1}, \ldots, \mathbf{x}_{n}\right)$ can be used instead of the more complicated emission functions $S\left(p_{1}, \ldots, p_{n} ; x_{1}, \ldots, x_{n}\right)$. The Wigner functions are real functions of momenta and positions and are in a

[^0]well-defined sense [5] the best quantum analog of particle density in phase-space. Therefore the parameters characterizing the Wigner functions can be interpreted ${ }^{2}$ as the parameters characterizing the space distribution of sources and their momentum spectra [7].

The density matrix in momentum space is related to the Wigner function by the formula:

$$
\begin{equation*}
\rho\left(\mathbf{p}_{1}, \ldots, \mathbf{p}_{n} ; \mathbf{p}_{1}^{\prime}, \ldots, \mathbf{p}_{n}^{\prime}\right)=\int d^{2} x_{1} \cdots d^{2} x_{n} \exp \left[i\left(\mathbf{Q}_{1} \mathbf{x}_{1}+\cdots+\mathbf{Q}_{n} \mathbf{x}_{n}\right)\right] W\left(\mathbf{K}_{1}, \ldots, \mathbf{K}_{n} ; \mathbf{x}_{1}, \ldots, \mathbf{x}_{n}\right) \tag{1}
\end{equation*}
$$

where $\mathbf{K}_{i}=\left(\mathbf{p}_{i}+\mathbf{p}_{i}^{\prime}\right) / 2$ and $\mathbf{Q}_{i}=\mathbf{p}_{i}-\mathbf{p}_{i}^{\prime}$.
It follows that the momentum distribution of particles can be expressed as

$$
\begin{equation*}
\Omega_{0}\left(\mathbf{p}_{1}, \ldots, \mathbf{p}_{n}\right)=\rho\left(\mathbf{p}_{1}, \ldots, \mathbf{p}_{n} ; \mathbf{p}_{1}, \ldots, \mathbf{p}_{n}\right)=\int d^{2} x_{1} \cdots d^{2} x_{n} W\left(\mathbf{p}_{1}, \ldots, \mathbf{p}_{n} ; \mathbf{x}_{1}, \ldots, \mathbf{x}_{n}\right) \tag{2}
\end{equation*}
$$

Similarly, for the coordinate distribution we have

$$
\begin{equation*}
\Omega_{0}\left(\mathbf{x}_{1}, \ldots, \mathbf{x}_{n}\right)=\rho\left(\mathbf{x}_{1}, \ldots, \mathbf{x}_{n} ; \mathbf{x}_{1}, \ldots, \mathbf{x}_{n}\right)=\int d^{2} p_{1} \cdots d^{2} p_{n} W\left(\mathbf{p}_{1}, \ldots, \mathbf{p}_{n} ; \mathbf{x}_{1}, \ldots, \mathbf{x}_{n}\right) \tag{3}
\end{equation*}
$$

For the momentum distribution of identical bosons we have to symmetrize the production amplitudes. This modifies the momentum distribution (see, e.g., [8]) into

$$
\begin{equation*}
\Omega\left(\mathbf{p}_{1}, \ldots, \mathbf{p}_{n}\right)=\frac{1}{n!} \sum_{P, P^{\prime}} \rho\left(\mathbf{p}_{i_{1}}, \ldots, \mathbf{p}_{i_{n}} ; \mathbf{p}_{i_{1}^{\prime}}, \ldots, \mathbf{p}_{i_{n}^{\prime}}\right) \tag{4}
\end{equation*}
$$

where the sum runs over all permutations $P$ and $P^{\prime}$ of $\left(i_{1}, \ldots, i_{n}\right)$ and $\left(i_{1}^{\prime}, \ldots, i_{n}^{\prime}\right){ }^{3}$ This is the key formula which explains the main interest in the HBT measurements: the distribution of identical particles opens a window to the non-diagonal elements of the density matrix and thus also to the Wigner function. It is also clear, however, that this information is not sufficient to obtain full information about the distribution of sources. Thus further theoretical input is needed.

The purpose of the present Letter is to discuss the physical meaning of the measured two-particle HBT parameters in terms of the characteristics of the momentum and coordinate distribution of the sources as described by the Wigner function. The well-known case of uncorrelated emission (for recent reviews, see, e.g., [2]) is summarized briefly in Section 2. The emission of particles correlated in pairs is described in Section 3. In Section 4 a more realistic situation, when only a fraction of the particles is emitted in pairs while others remain uncorrelated, is considered. The experimental consequences are discussed in Sections 5 and 6. Our conclusions are listed in the last section.
2. The assumption of uncorrelated production means that the Wigner function factorizes into a product of single particle Wigner functions. Of course this factorization is then satisfied also for the unsymmetrized density matrix.

To illustrate the consequences of this Ansatz and to fix our notation, consider a single particle Wigner function in the most general Gaussian form ${ }^{4}$

$$
\begin{equation*}
W(\mathbf{p}, \mathbf{x})=\frac{1}{4 \pi^{2} \Delta_{u}^{2}\left(R_{u}^{2}-r_{u}^{2}\right)} \exp \left[-\frac{\mathbf{p}^{2}}{2 \Delta_{u}^{2}}-\frac{\left(\mathbf{x}-r_{u} \mathbf{p} / \Delta_{u}\right)^{2}}{2\left(R_{u}^{2}-r_{u}^{2}\right)}\right] \tag{5}
\end{equation*}
$$

One sees that the parameter $r_{u}$ is responsible for momentum-position correlation.

[^1]From (5), using (2) and (3), we derive for single particle distributions

$$
\begin{align*}
& \Omega_{0}(\mathbf{p})=\int d^{2} x W(\mathbf{p}, \mathbf{x})=\frac{1}{2 \pi \Delta_{u}^{2}} \exp \left[-\frac{\mathbf{p}^{2}}{2 \Delta_{u}^{2}}\right] \\
& \Omega_{0}(\mathbf{x})=\int d^{2} p W(\mathbf{p}, \mathbf{x})=\frac{1}{2 \pi R_{u}^{2}} \exp \left[-\frac{\mathbf{x}^{2}}{2 R_{u}^{2}}\right] \tag{6}
\end{align*}
$$

One sees that the parameter $\Delta_{u}$ describes the width of the distribution in momentum space whereas $R_{u}$ determines the size of the system in configuration space.

Using (1) and (4), we obtain the two-particle distribution for identical particles:

$$
\begin{equation*}
\Omega\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)=\frac{1}{4 \pi^{2} \Delta_{u}^{4}} \exp \left[-\frac{\mathbf{p}_{1}^{2}+\mathbf{p}_{2}^{2}}{2 \Delta_{u}^{2}}\right]\left\{1+\exp \left[-\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2} R_{\mathrm{HBT}}^{2}\right]\right\} \tag{7}
\end{equation*}
$$

where

$$
\begin{equation*}
R_{\mathrm{HBT}}^{2} \equiv R_{u}^{2}-r_{u}^{2}-\frac{1}{4 \Delta_{u}^{2}} \tag{8}
\end{equation*}
$$

One sees that in this simple case measurements of the single particle distribution and pair distribution allow to determine $\Delta_{u}$ and $R_{\mathrm{HBT}}$. One also sees from (8) that these two parameters are not sufficient to determine $R_{u}$, the size of the system in configuration space [10]. To this end it is necessary to know the correlation between the momentum and the position of the emission point of a particle, as expressed by the parameter $r_{u}$.
3. The most general Gaussian two-particle Wigner function, symmetric with respect to simultaneous exchange of the particle momenta and positions, can be written as

$$
\begin{align*}
W_{c}\left(\mathbf{p}_{1}, \mathbf{p}_{2} ; \mathbf{x}_{1}, \mathbf{x}_{2}\right)= & \frac{1}{16 \pi^{4} \Delta_{+}^{2} \Delta_{-}^{2}\left(R_{+}^{2}-r_{+}^{2}\right)\left(R_{-}^{2}-r_{-}^{2}\right)} \\
& \times \exp \left[-\frac{\mathbf{p}_{+}^{2}}{\Delta_{+}^{2}}-\frac{\mathbf{p}_{-}^{2}}{\Delta_{-}^{2}}\right] \exp \left[-\frac{\left(\mathbf{x}_{+}-r_{+} \mathbf{p}_{+} / \Delta_{+}\right)^{2}}{R_{+}^{2}-r_{+}^{2}}-\frac{\left(\mathbf{x}_{-}-r_{-} \mathbf{p}_{-} / \Delta_{-}\right)^{2}}{R_{-}^{2}-r_{-}^{2}}\right] \tag{9}
\end{align*}
$$

where $\mathbf{p}_{ \pm}=\left(\mathbf{p}_{1} \pm \mathbf{p}_{2}\right) / 2$ and $\mathbf{x}_{ \pm}=\left(\mathbf{x}_{1} \pm \mathbf{x}_{2}\right) / 2$. Note that if

$$
\begin{equation*}
\Delta_{-}=\Delta_{+}, \quad R_{+}=R_{-}, \quad r_{+}=r_{-} \tag{10}
\end{equation*}
$$

the Wigner function factorizes and the problem reduces to the one discussed in the previous section.
One sees from (9) that $r_{ \pm}$are responsible for the correlations between positions and momenta. To see the physical meaning of the other 4 parameters we calculate the distribution of momenta

$$
\begin{equation*}
\Omega_{0}\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)=\frac{1}{4 \pi^{2} \Delta_{+}^{2} \Delta_{-}^{2}} \exp \left[-\frac{\mathbf{p}_{1}^{2}+\mathbf{p}_{2}^{2}}{2 \Delta_{+}^{2}}-\frac{\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2}}{2 \omega^{2}}\right] \tag{11}
\end{equation*}
$$

and positions

$$
\begin{equation*}
\Omega_{0}\left(\mathbf{x}_{1}, \mathbf{x}_{2}\right)=\frac{1}{4 \pi^{2} R_{+}^{2} R_{-}^{2}} \exp \left[-\frac{\mathbf{x}_{1}^{2}+\mathbf{x}_{2}^{2}}{2 R_{+}^{2}}-\frac{\left(\mathbf{x}_{1}-\mathbf{x}_{2}\right)^{2}}{2 \xi^{2}}\right] \tag{12}
\end{equation*}
$$

where

$$
\begin{equation*}
\frac{1}{\omega^{2}}=\frac{1}{2 \Delta_{-}^{2}}-\frac{1}{2 \Delta_{+}^{2}}, \quad \frac{1}{\xi^{2}}=\frac{1}{2 R_{-}^{2}}-\frac{1}{2 R_{+}^{2}} \tag{13}
\end{equation*}
$$

From this we see that $\Delta_{+}^{2}$ describes the momentum distribution, whereas $\omega^{2}$ describes the correlations between the momenta in the pair. Similarly, $R_{+}^{2}$ describes the distribution of the particle positions while $\xi^{2}$ describes correlations between the positions of particles in the pair. Note that $\omega^{2}$ and $\xi^{2}$ are not necessarily positive. Note also that correlations do indeed disappear $(1 / \omega=1 / \xi=0)$ when condition $(10)$ is satisfied.

Using (9), (1) and (4), the two-particle momentum distribution is obtained:

$$
\begin{equation*}
\Omega\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)=\Omega_{0}\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)\left(1+\exp \left[-\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2} R_{c}^{2}\right]\right), \tag{14}
\end{equation*}
$$

where $\Omega_{0}$ is given by (11) and

$$
\begin{equation*}
R_{c}^{2}=R_{-}^{2}-r_{-}^{2}-1 / 4 \Delta_{-}^{2} \tag{15}
\end{equation*}
$$

One sees that $\Omega\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)$ depends only on three parameters: $\Delta_{+}^{2}, \Delta_{-}^{2}$, and $R_{-}^{2}-r_{-}^{2}$, whereas $R_{+}^{2}$ and $r_{+}^{2}$ do not have any impact on the momentum distribution.

Using (13) we obtain

$$
\begin{equation*}
R_{-}^{2}=\frac{\xi^{2} R_{+}^{2}}{\xi^{2}+2 R_{+}^{2}} \tag{16}
\end{equation*}
$$

which explicitly shows the effect of correlations in configuration space on the physical interpretation of the HBT measurements.

Note that for positive correlations $\left(\xi^{2}>0\right) R_{-}^{2}$ is always smaller than both $\xi^{2} / 2$ and $R_{+}^{2}$. In particular, when $\xi^{2} \ll R_{+}^{2}$ we have $R_{-}^{2} \approx \xi^{2} / 2$. In this case the HBT measurements give only information on correlations and not on the size of the system in configuration space.

One also sees that for negative correlations $R_{-}^{2}$ is always greater than $R_{+}^{2}$.
This discussion shows that correlations in configuration space can significantly influence the interpretation of the measured HBT parameters. Only if there are no correlations $\left(1 / \xi^{2}=0\right), R_{+}$and $R_{-}$are identical and by this "accident" one can obtain information about the total volume of the reaction.
4. In the previous section we have discussed the situation when all pairs of the emitted particles are correlated. This is an interesting theoretical exercise which, however, hardly corresponds to reality. The measured HBT correlations indicate that the data are in reasonable agreement with the assumption of uncorrelated production. This suggests that to discuss practical consequences of our formalism it is more appropriate to consider a situation when correlated emission affects only a fraction of all the particles, the others remaining uncorrelated.

The formalism developped in Sections 2 and 3 is well suited to cover this case. We write the Wigner function as a sum of two terms: one describing the uncorrelated emission and the other responsible for the correlations. Following the discussion of Sections 2 and 3 we write

$$
\begin{equation*}
W\left(\mathbf{p}_{1}, \mathbf{p}_{2} ; \mathbf{x}_{1}, \mathbf{x}_{2}\right)=w_{u} W_{u}\left(\mathbf{p}_{1}, \mathbf{x}_{1}\right) W_{u}\left(\mathbf{p}_{2}, \mathbf{x}_{2}\right)+w_{c} W_{c}\left(\mathbf{p}_{1}, \mathbf{p}_{2} ; \mathbf{x}_{1}, \mathbf{x}_{2}\right), \tag{17}
\end{equation*}
$$

where $W_{u}(\mathbf{p}, \mathbf{x})$ is given by (5) and $W_{c}\left(\mathbf{p}_{1}, \mathbf{p}_{2} ; \mathbf{x}_{1}, \mathbf{x}_{2}\right)$ by (9). $w_{u}$ is the probability that the considered particles are uncorrelated and $w_{c}=1-w_{u}$ is the probability that they were emitted as a correlated pair.

The density matrix is thus given by a sum of two terms, one constructed from $W_{u}$ and the other from $W_{c}$. This gives the single particle momentum distribution ${ }^{5}$

$$
\begin{equation*}
\Omega_{0}\left(\mathbf{p}_{1}\right)=\frac{1}{2 \pi \Delta_{u}^{2}} e^{-\mathbf{p}_{1}^{2} / 2 \Delta_{u}^{2}} \Phi_{0}\left(\mathbf{p}_{1}\right), \tag{18}
\end{equation*}
$$

[^2]where
\[

$$
\begin{equation*}
\Phi_{0}\left(\mathbf{p}_{1}\right)=w_{u}+w_{c} \frac{2 \Delta_{u}^{2}}{\Delta_{+}^{2}+\Delta_{-}^{2}} e^{-\mathbf{p}_{1}^{2} / \eta^{2}} \tag{19}
\end{equation*}
$$

\]

represents the modification of the single particle spectrum due to the correlated emission. Here

$$
\begin{equation*}
\frac{1}{\eta^{2}}=\frac{1}{\Delta_{+}^{2}+\Delta_{-}^{2}}-\frac{1}{2 \Delta_{u}^{2}} \tag{20}
\end{equation*}
$$

Using (17) and employing (1) and (4), the momentum distribution for identical particles $\Omega\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)$ can now be derived and thus one can construct the usually measured quantity

$$
\begin{equation*}
C\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right) \equiv \frac{\Omega\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)}{\Omega_{0}\left(\mathbf{p}_{1}\right) \Omega_{0}\left(\mathbf{p}_{2}\right)} \tag{21}
\end{equation*}
$$

where $\Omega_{0}\left(\mathbf{p}_{1}\right)$ is the single-particle distribution in the events with at least one pair of identical particles, given by (18). The result is

$$
\begin{equation*}
C\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)=w_{u} C_{u}\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)+w_{c} C_{c}\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right) \tag{22}
\end{equation*}
$$

with

$$
\begin{equation*}
C_{u}\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)=\frac{1+e^{-\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2} R_{\mathrm{HBT}}^{2}}}{\Phi_{0}\left(\mathbf{p}_{1}\right) \Phi_{0}\left(\mathbf{p}_{2}\right)} \tag{23}
\end{equation*}
$$

and

$$
\begin{equation*}
C_{c}\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)=\frac{\Delta_{u}^{4}}{\Delta_{+}^{2} \Delta_{-}^{2}} \frac{e^{-\left(\mathbf{p}_{1}+\mathbf{p}_{2}\right)^{2} / 2 \chi_{+}^{2}} e^{-\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2} / 2 \chi_{-}^{2}}}{\Phi_{0}\left(\mathbf{p}_{1}\right) \Phi_{0}\left(\mathbf{p}_{2}\right)}\left[1+e^{-\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2} R_{c}^{2}}\right] \tag{24}
\end{equation*}
$$

with

$$
\begin{equation*}
\frac{1}{\chi_{ \pm}^{2}}=\frac{1}{2 \Delta_{ \pm}^{2}}-\frac{1}{2 \Delta_{u}^{2}} \tag{25}
\end{equation*}
$$

5. The formulae (22)-(24) describe the HBT measurements for a general superposition of uncorrelated and correlated emission. They thus cover a wide range of possible physical situations.

To discuss their interpretation we have to consider the possible origin of these two contributions. The uncorrelated emission may stem either from directly produced pions or from the pions emitted from uncorrelated clusters (resonances). The correlated emission may reflect (i) a genuine structure of the source [1] or (ii) the interaction between pions. The attractive interactions lead to positive correlations $\left(\xi^{2}>0\right)$. They are usually represented as clusters of pions. The repulsive interactions (which were never observed ${ }^{6}$ ) would give negative correlations $\left(\xi^{2}<0\right)$.

As seen from (22)-(24), for positive correlations one may expect the two components, $C_{u}$ and $C_{c}$, to have different ranges in $\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2}$. The difference may be large, especially in heavy ion collisions. Indeed, in this case the range of the first one $\left(\sim 1 / R_{\mathrm{HBT}}^{2}\right)$ is determined by the size of the whole system, whereas the range of the second one $\left(\sim 1 / R_{c}^{2}\right)$ is determined by the geometrical size of clusters (and/or of local fluctuations) and by the momentum distributions.

We shall consider in detail the generic scenario when all particles are emitted from uncorrelated sources [1]. The single particle distribution is then fully determined by the distribution and decay properties of the emitting

[^3]sources. The condition
\[

$$
\begin{equation*}
\int d^{2} x_{2} d^{2} p_{2} W_{c}\left(\mathbf{p}_{1}, \mathbf{p}_{2} ; \mathbf{x}_{1}, \mathbf{x}_{2}\right)=W_{u}\left(\mathbf{p}_{1}, \mathbf{x}_{1}\right) \tag{26}
\end{equation*}
$$

\]

implies

$$
\begin{equation*}
2 \Delta_{u}^{2}=\Delta_{+}^{2}+\Delta_{-}^{2}, \quad 2 R_{u}^{2}=R_{+}^{2}+R_{-}^{2}, \quad 2 r_{u} \Delta_{u}=r_{-} \Delta_{-}+r_{+} \Delta_{+} \tag{27}
\end{equation*}
$$

and, naturally, $\Phi_{0}(\mathbf{p}) \equiv 1$.
A special case of this scenario (particle emission from independent granules) was discussed in [1] where it was furthermore assumed that (i) the distribution of sources is momentum-independent $\left(1 / \Delta_{+}^{2}=0\right)$ and (ii) the momentum dependence in source decay may be neglected with respect to dependence on difference of momenta $\left(1 / \Delta_{-}^{2} \ll R_{c}^{2}, R_{\mathrm{HBT}}^{2}\right)$. Under these conditions ${ }^{7}$ the expression for the correlation function considerably simplifies

$$
\begin{equation*}
C\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)=1+w_{u} e^{-\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2} R_{\mathrm{HBT}}^{2}}+w_{c} \frac{\Delta_{u}^{4}}{\Delta_{+}^{2} \Delta_{-}^{2}} e^{-\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2} R_{c}^{2}}, \tag{28}
\end{equation*}
$$

where $w_{c}=1 / n$ and $n$ is the total number of sources.
One sees clearly the two-component structure of the correlation function. ${ }^{8}$ As pointed out in [1], the observation of the second term may serve as an indication of the clustering and/or of the granular structure of the emission region in heavy ion collisions. The size of the granules (clusters) may be read off from the range of the second component.

The simple formula (28) illustrates very well the basic physics of the problem. As seen from our general expression (24), however, the actual shape of the second component may be significantly influenced by the momentum dependence of the emitting sources. It is true that $1 / \Delta_{+}^{2}$ and $1 / \Delta_{-}^{2}$, being of the order of 1 fermi ${ }^{2}$ or less, are small as compared to $R_{\mathrm{HBT}}^{2}$ which (in heavy ion collisions) is of the order of (several fermi) ${ }^{2}$. They may well be comparable, however, with $R_{c}^{2}$ which need not be much larger than 1 fermi ${ }^{2}$. Thus neglecting the momentum dependence of the emitting sources [1] may be a too drastic simplification.

Moreover, even in absence of the correlations in configuration space (i.e., for $R_{+}=R_{-} ; r_{+}=r_{-}=0$ ) the two component structure of the correlation function persists. Indeed, we obtain from (22)-(24)

$$
\begin{align*}
C\left(\mathbf{p}_{1}, \mathbf{p}_{2}\right)= & w_{u}+\left(w_{u}+w_{c} \frac{\Delta_{u}^{4}}{\Delta_{+}^{2} \Delta_{-}^{2}} e^{-\left(\mathbf{p}_{1}+\mathbf{p}_{2}\right)^{2} / 2 \chi_{+}^{2}}\right) e^{-\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2} R_{\mathrm{HBT}}^{2}} \\
& +w_{c} \frac{\Delta_{u}^{4}}{\Delta_{+}^{2} \Delta_{-}^{2}} e^{-\left(\mathbf{p}_{1}+\mathbf{p}_{2}\right)^{2} / 2 \chi_{+}^{2}} e^{-\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2} / 2 \chi_{-}^{2}} . \tag{29}
\end{align*}
$$

The two-component structure is recovered but now the momentum correlations and not the correlations in configuration space are responsible for it.

We conclude that, although the two-component structure of the HBT measurements seems a robust consequence of the correlated emission, the physical meaning of the measured parameters is by no means unique. Thus we feel that in the analysis of actual experiments our general approach, summarized in the formulae (22)-(24), may be needed to account for the observations and to give the correct physical meaning to the measured parameters.

## 6. Several comments are in order.

[^4](i) One may note that, since for positive correlations one naturally expects $\Delta_{+}^{2}>\Delta_{-}^{2}$, (27) implies that $\chi_{-}^{2}>0$ and $\chi_{+}^{2}<0$. This means that $C_{c}$ (cf. (24)) increases with increasing momentum of the pair. This effect may turn out helpful for identification of the second component; ${ }^{9}$
(ii) It is worth to remember that there are several reasons why the conditions (27), relating the correlated and the uncorrelated distributions, may be violated (also the probability $w_{c}$ of correlated emission need not be equal to $1 / n$ ). First, not all particles are emitted in clusters, some of them are produced directly. Second, most of the clusters observed in hadronic collisions are characterized by fairly small multiplicity (about three particles on the average) and rather small charge [4]. Therefore only a small fraction of all clusters emit two identical charged pions and there is no obvious reason why they should have the same properties as an average cluster. Thus although one may hope that the discussion of the previous section describes correctly the basic physics of the problem, the quantitative analysis may require the more flexible approach;
(iii) Finally, let us comment on the possibility of negative correlations, i.e., repulsive interaction $\left(\xi^{2}<0\right.$, $\omega^{2}<0$ ). In this case the cluster picture is not applicable. From (16) we deduce $R_{-}>R_{+}$. Since $R_{+}$is expected to be close to $R_{u}$, we conclude that $R_{c}>R_{\mathrm{HBT}}$, i.e., the range of the second component is shorter than that of the first one. Thus an observation of an abnormally narrow peak in the distribution of $\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)^{2}$ may be an indication of repulsive interactions in the system. It would be interesting to analyze the data keeping this perspective in mind. ${ }^{10}$
7. In conclusion, we have analyzed the effects of interparticle correlations in particle emission on the measurements of quantum interference. It has been shown that the physical interpretation of the measured parameters is significantly influenced by the presence of such correlations. In particular, for strongly correlated systems the measured range of the HBT effect is related to the correlation range rather than to the size of the interaction volume. Only in the case of weak correlations the standard interpretation may be applicable. The short-range positive correlations in configuration space were discussed in detail. The analysis given in [1] was generalized. A possibility to uncover negative interparticle correlations, if any, was pointed out.

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    ${ }^{1}$ I.e., distributions integrated over some interval of the longitudinal variables.

[^1]:    ${ }^{2}$ Given all the caveats related to the fact that we are dealing with quantum phenomena [2,6].
    ${ }^{3}$ For fermions there is an extra minus sign when $P$ and $P^{\prime}$ are odd with respect to each other.
    ${ }^{4}$ As already mentioned in the Introduction, all vectors are two-dimensional. This model is sometimes referred to as the Zajc model [9].

[^2]:    ${ }^{5}$ In (18) the corrections due to BE correlations are neglected. They are expected to be small at high energies.

[^3]:    ${ }^{6}$ As already stated in Section 1, we discuss here only correlations due to strong interactions in the production process.

[^4]:    ${ }^{7}$ They are too restrictive: to obtain (28) it is enough to assume $\Delta_{+}=\Delta_{+}=\Delta_{u}$, i.e., no correlations in momentum space.
    ${ }^{8}$ A sum of two Gaussians in the two-particle correlation function was also considered for another reason in [11].

[^5]:    9 This conclusion relies heavily on the condition (27) and thus needs not be generally valid.
    10 Another well-known reason for such a narrow peak is the presence of the long-living resonances [12].

