a way to measure gestalts. This elementary achievement has produced beautifully simple relations such as the Newman–Gerstman law in which intersymbol influences in printed English text are found to change inversely with the square of the number of intervening symbols. Work in a similar vein by Miller and Selfridge on the memory span for various orders of approximation to English, and by Attneave himself on statistical approximations to visual figures shows something of the range of successful applications of information measures to studies of organization and patterning.

Finally, Attneave reviews the largely successful efforts of psychologists to ferret out the statistical properties of information measures. Small samples are a persistent hazard to experimentation in psychology, and research workers are continually forced to study probability distributions of their measurements. Thus it was that psychologists were among the first to point out the relation between the $p \log_2 p$ formula and the likelihood ratio, and to connect information statistics with the large and important body of theory on chi-square. These efforts were rewarded by the discovery of an arsenal of new analytical devices in information theory. Similar analytical schemes have been developed slowly and painfully in the classical theory of chi-square, but they simply popped right out of the information measures.

Attneave's summary is short because much of the story is still to be written. Nevertheless, this little book leaves a convincing impression that terrestrial information-psychology can be as thought provoking as the astral variety, and a lot more palatable.

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Finite Markov Chains. By John G. Kemeny and J. Laurie Snell. van Nostrand, New York, 1960. viii + 210 pp., $5.00.

Ever since Shannon has shown the central role of Markov schemes in communication, and Mosteller and Bush their role in learning theory, the workers in this general area have been awaiting the appearance of a workmanlike and no-nonsense textbook devoted exclusively to this topic, which could be consulted without having to refer to other chapters of a broader book (which are likely to be mostly devoted to different topics). The book by Kemeny and Snell will be found, by already prepared readers, to be of very great use. But it is unfortunately not likely to be the final answer or a durable one. In particular, it does not supersede the corresponding chapters of Feller's classic "Introduction to Probability" and it cannot be at all recommended without serious reservations concerning Kemeny's philosophy which underlies Chapter 1 and Section 2.1 (prerequisites and basic concepts).

Let us first review briefly the contents of the remainder of the book. *Chapter II:* Matrix Theory; Classification of States and Chains. *Chapter III:* Absorbing Chains. *Chapter IV:* Regular Markov Chains (including the law of large numbers and first passage times). *Chapter V:* Ergodic Chains (including cyclic chains and reverse Markov chains). *Chapter VI*: Miscellaneous Further Results. *Chapter VII*: Some Applications (including sports but not including the finite state model of information theory). (It may be noted that Kemeny's "regular" chain is what is usually called by the more illustrative "acyclic" or "aperiodic" chains; a "Markov process" differs from a "Markov chain" in that it need not be stationary, but only as long as the nonstationarity depends on the time elapsed since the process started.) A feature of the treatment is that it involves no use of the method of characteristic roots of a stochastic matrix.

For a book intended as a "reference for workers outside of mathematics," one is very surprised by the deliberate choice to include no index and not even the sketchiest bibliography: for example, in the sections devoted to learning and input–output models, the only papers quoted are those of Kemeny and Snell; for background material, the reader is exclusively referred to other books of the authors or of their collaborators. It is also surprising that such an unusual terminology was chosen for a reference work.

This reviewer is quite impressed by Kemeny's enthusiasm, in his efforts to reform much of the elementary mathematical curriculum by enriching it with topics which used to be considered as advanced. This is the third successive version of his textbook formula, in which the ingredients remain the same and only the proportions vary. However, we happen not to like particularly Kemeny's deliberately chosen treatment of the probability ingredient (perhaps nobody can like a revolution which would make one's good skills *entirely* ununderstandable). Let us therefore return to Chapter 1 and to Section 2.1. We find it marred by a logician's partisan attitude about the nature of probability, which can only confuse issues in this context. For example, little help can be expected here from illustrations from horse races. Further, in the sections where Kemeny speaks of probability in connection with Markov chains, he speaks of the "probability of a state," as is usual; however, when the concept of probability was introduced, it was defined as applying to "statements about events." Similarly, the sample space has here become the "possibility space" and a random variable is here called a "function on the possibility space." We can of course fully sympathize with every effort to minimize the confusions linked with the traditional use of the word "variable" in the term "random variable"; but the whole literature uses this term. Further, since one has given a correct definition of a "random variable" as a function, one might have followed up by giving a definition of a "random function" as a function on the product of the sample space and the time axis. Actually, the book is content to "briefly describe" what a stochastic process is, referring for a better description to a few pages of the "Finite Mathematical Structures" of the authors.

On the other hand, one should stress that the leisurely pace and the great number of fully computed examples are great assets for the book and they undoubtedly will make it extremely useful for the already prepared and forewarned teacher.

Our appetite was whetted by a statement in the preface that the authors "have developed a pair of programs for the IBM 704 which will find a number of interesting quantities for a given process directly from the transition matrix," and which "were invaluable in the checking of conjectures for theorems." But perhaps books of this level are not the proper vehicles for new results not otherwise published.

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