



$\rho D^* D^*$ vertex from QCD sum rules

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Abstract

We calculate the form factors and the coupling constant in the $\rho D^* D^*$ vertex in the framework of QCD sum rules. We evaluate the three point correlation functions of the vertex considering both ρ and D^* mesons off-shell. The form factors obtained are very different but give the same coupling constant: $g_{\rho D^* D^*} = 6.60 \pm 0.31$. This number is 50% larger than what we would expect from SU(4) estimates.

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1. Introduction

Charmonium production is a very useful source of information in heavy ion collisions. The knowledge of the J/ψ production rate can improve our understanding of these collisions and help us to know if there was a “color glass condensate” in the initial state. Charmonium production is very sensitive to the existence and to the properties of the intermediate “quark–gluon plasma” [1]. All the interesting effects happening in these initial and intermediate phases can be blurred by interactions in the final stage of these collisions, when charmonium states interact with other comovers such as pions, ρ mesons and nucleons, which form a hot hadronic gas. Since these interactions occur at an energy of the order of magnitude of the temperature ($\simeq 100$ – 150 MeV), their study has to be made with non-perturbative methods. These can be QCD sum rules [2], quark models and the effective Lagrangian approach [3,4]. This last approach has been developed for almost ten years now and a great progress in the understanding of the interactions of charmed mesons with light mesons and nucleons has been achieved. Part of this progress is due to a persistent study of the vertices involving charmed mesons, namely $D^* D\pi$ [5,6], $DD\rho$ [7], DDJ/ψ [8],

$D^* DJ/\psi$ [9,10], $D^* D^* \pi$ [11,12], $D^* D^* J/\psi$ [13], $D_s D^* K$, $D_s^* DK$ [14] and $DD\omega$ [15]. More specifically, it is very important to know the precise functional form of the form factors in these vertices and even to know how this form changes when one or the other (or both) mesons are off-shell. This careful determination of the charm form factors has been done bit by bit over the last seven years in the framework of QCD sum rules, which are the best tool to give a first principles answer to this problem.

Understanding charmonium production in heavy ion collisions would be already a good reason to study of hadronic charm form factors. However, since 2003, due the precise measurements of B decays performed by Belle, BES and BaBar Collaborations, this subject gained a new relevance. In B decays new particles have been observed, such as the $D_{sJ}(2317)$ and the $X(3872)$. These particles very often decay into an intermediate two body state, which then undergoes final state interactions, with the exchange of one or more virtual mesons. As an example of specific situation where a precise knowledge of the $\rho D^* D^*$ form factor is required, we may consider the decay $X(3872) \rightarrow J/\psi + \rho$. As suggested in [16], this decay proceeds in two steps. First the X decays into a $D-D^*$ intermediate state and then these two particles exchange a D^* producing the final J/ψ and ρ . This is shown in Figs. 1b and 1f of [16]. In order to compute the effect of these interactions in the final decay rate we need the $\rho D^* D^*$ form factor.

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In the present Letter we calculate this form factor with QCDSR. The $\rho D^* D^*$ vertex is similar to the $J/\psi D^* D^*$ vertex treated in [13]. As before, because there are three vector particles involved, the number of Lorentz structures is very large and we have to choose a reliable one to perform the calculations. Here we introduce the pole-continuum analysis and impose the pole dominance as a criterion to reduce the freedom in the choice of the Borel parameter. In the next section, for completeness we describe the QCDSR technique and in Section 3 we present the results and compare them with results obtained in other works.

2. The sum rule for the $\rho D^* D^*$ vertex

Following our previous works and especially Ref. [13], we write the three-point function associated with the $\rho D^* D^*$ vertex, which is given by

$$\Gamma_{\nu\alpha\mu}^{(\rho^+)}(p, p', q) = \int d^4x d^4y e^{ip'x} e^{-iqy} \times \langle 0|T\{j_\mu^{D^{*0}}(x)j_\alpha^{\rho^+\dagger}(y)j_\nu^{D^{*-}\dagger}(0)\}|0\rangle \quad (1)$$

for an off-shell ρ^+ meson, where $q = (p' - p)$, and

$$\Gamma_{\nu\alpha\mu}^{(D^{*-})}(p, p', q) = \int d^4x d^4y e^{ip'x} e^{-iqy} \times \langle 0|T\{j_\nu^{D^{*0}}(x)j_\alpha^{D^{*-}\dagger}(y)j_\mu^{\rho^+\dagger}(0)\}|0\rangle, \quad (2)$$

for an off-shell D^{*-} meson. Since there are two independent momenta and three Lorentz indices, the general expression for the vertices (1) and (2) has fourteen independent Lorentz structures. Therefore, we can write $\Gamma_{\nu\alpha\mu}$ in terms of fourteen invariant amplitudes associated with each one of these structures in the following form:

$$\begin{aligned} \Gamma_{\nu\alpha\mu}(p, p', q) &= \Gamma_1(p^2, p'^2, q^2)g_{\mu\nu}p_\alpha + \Gamma_2(p^2, p'^2, q^2)g_{\mu\alpha}p_\nu \\ &+ \Gamma_3(p^2, p'^2, q^2)g_{\nu\alpha}p_\mu + \Gamma_4(p^2, p'^2, q^2)g_{\mu\nu}q_\alpha \\ &+ \Gamma_5(p^2, p'^2, q^2)g_{\mu\alpha}q_\nu + \Gamma_6(p^2, p'^2, q^2)g_{\nu\alpha}q_\mu \\ &+ \Gamma_7(p^2, p'^2, q^2)p_\mu p_\nu p_\alpha + \Gamma_8(p^2, p'^2, q^2)q_\mu p_\nu p_\alpha \\ &+ \Gamma_9(p^2, p'^2, q^2)p_\mu q_\nu p_\alpha + \Gamma_{10}(p^2, p'^2, q^2)p_\mu p_\nu q_\alpha \\ &+ \Gamma_{11}(p^2, p'^2, q^2)q_\mu q_\nu p_\alpha + \Gamma_{12}(p^2, p'^2, q^2)q_\mu p_\nu q_\alpha \\ &+ \Gamma_{13}(p^2, p'^2, q^2)p_\mu q_\nu q_\alpha + \Gamma_{14}(p^2, p'^2, q^2)q_\mu q_\nu q_\alpha. \end{aligned} \quad (3)$$

However, not all of these fourteen invariant amplitudes, $\Gamma_i(p^2, p'^2, q^2)$, are independent. Due to current conservation $p^\mu \Gamma_{\nu\alpha\mu}(p, p', q) = 0$ for an off-shell D^{*-} meson, and $q^\alpha \Gamma_{\nu\alpha\mu}(p, p', q) = 0$ for an off-shell ρ^+ meson. Current conservation introduces five constraints among these fourteen invariant amplitudes, such that only nine of them are independent. As an example, in the case of an off-shell D^{*-} meson, the invariant amplitudes Γ_5 and Γ_9 are related through: $\Gamma_9 = -\Gamma_5/p^2$, in such a way that these two terms in Eq. (3) can be written in a form manifestly current conserving as:

$\Gamma_5(p^2, p'^2, q^2)q_\nu(g_{\mu\alpha} - p_\alpha p_\mu/p^2)$. However, since the fourteen Lorentz structures appearing in Eq. (3) are indeed independent, we can write one sum rule for each one of these fourteen Lorentz structures. The kind of relation given above tell us that from these fourteen sum rules, only nine of them are independent, but these relations do not change the sum rules.

Eqs. (1) and (2) can be calculated in two different ways: using quark degrees of freedom (the theoretical or QCD side) or using hadronic degrees of freedom (the phenomenological side). In the QCD side the correlators are evaluated using the Wilson operator product expansion (OPE). The OPE incorporates the effects of the QCD vacuum through an infinite series of condensates of increasing dimension. On the other hand, the representation in terms of hadronic degrees of freedom is responsible for the introduction of the form factors, decay constants and masses. Both representations are matched invoking the quark–hadron global duality.

2.1. The OPE side

In the OPE or theoretical side each meson interpolating field appearing in Eqs. (1) and (2) can be written in terms of the quark field operators in the following form:

$$j_\nu^{\rho^+}(x) = \bar{d}(x)\gamma_\nu u(x) \quad (4)$$

and

$$j_\mu^{D^{*-}}(x) = \bar{c}(x)\gamma_\mu d(x), \quad (5)$$

where u , d and c are the up, down and charm quark field, respectively. Each one of these currents has the same quantum numbers of the associated meson.

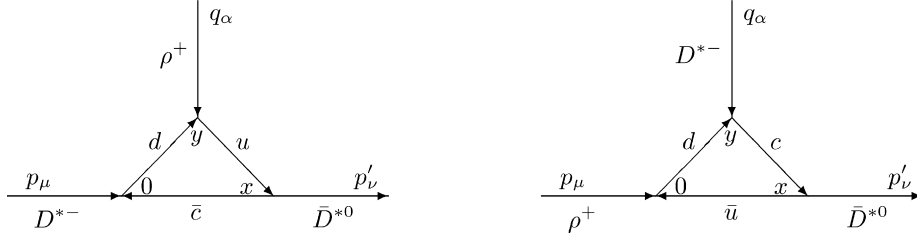
For each one of the invariant amplitudes appearing in Eq. (3), we can write a double dispersion relation over the virtualities p^2 and p'^2 , holding $Q^2 = -q^2$ fixed:

$$\Gamma_i(p^2, p'^2, Q^2) = -\frac{1}{\pi^2} \int_{s_{\min}}^{\infty} ds \int_{u_{\min}}^{\infty} du \frac{\rho_i(s, u, Q^2)}{(s - p^2)(u - p'^2)}, \quad (6)$$

$i = 1, \dots, 14,$

where $\rho_i(s, u, Q^2)$ equals the double discontinuity of the amplitude $\Gamma_i(p^2, p'^2, Q^2)$, calculated using the Cutkosky's rules. The invariant amplitudes receive contributions from all terms in the OPE. The first one of those contributions comes from the perturbative term and it is represented in Fig. 1.

We can work with any structure appearing in Eq. (3), but we must choose those which have less ambiguities in the QCD sum rules approach, which means, less influence from the higher dimension condensates and a better stability as a function of the Borel mass. We have chosen the $g_{\mu\alpha}q_\nu$ structure. In this structure the quark condensate (the condensate of lower dimension) contributes in the case of D^* meson off-shell.


Fig. 1. Perturbative diagrams for the ρ off-shell (left) and D^* off-shell (right) correlators.

The corresponding perturbative spectral densities which enter in Eq. (6) are

$$\rho^{(\rho)}(s, u, Q^2) = \frac{3}{2\pi\sqrt{\lambda}} \left[\left(\frac{s-u-t}{2} - (2m_c^2) \right) \left(\frac{B-A}{2} \right) + 2(J-I) + \frac{\pi}{2}(2m_c^2 - u - s) - D \right] \quad (7)$$

for ρ off-shell, and

$$\rho^{(D^*)}(s, u, Q^2) = -\frac{3}{2\pi\sqrt{\lambda}} \left[\left(\frac{u-s-t}{2} \right) \left(\frac{B-A}{2} \right) - 2(I+J) + \frac{\pi}{2}(u+s+2m_c^2) + D \right] \quad (8)$$

for D^* off-shell. Here $\lambda = \lambda(s, u, t) = s^2 + t^2 + u^2 - 2st - 2su - 2tu$, $s = p^2$, $u = p'^2$, $t = -Q^2$ and A, B, D, I and J are functions of (s, t, u) , given by the following expressions:

$$A = \frac{2\pi}{\sqrt{s}} \left(\bar{k}_0 - \frac{|\bar{k}|p'_0}{|\bar{p}'|} \cos\bar{\theta} \right); \quad B = 2\pi \frac{|\bar{k}|}{|\bar{p}'|} \cos\bar{\theta}; \quad (9)$$

$$D = -\pi |\bar{k}|^2 (1 - \cos^2\bar{\theta}); \quad (10)$$

$$I = \frac{\pi |\bar{k}|^2}{\sqrt{s}} (1 - \cos^2\bar{\theta}) \left(\frac{|\bar{k}|p'_0}{|\bar{p}'|} \cos\bar{\theta} - \bar{k}_0 \right); \quad (11)$$

$$J = -\frac{\pi |\bar{k}|^3}{|\bar{p}'|} (1 - \cos^2\bar{\theta}) \cos\bar{\theta}; \quad (12)$$

where

$$p'_0 = \frac{s+u-t}{2\sqrt{s}}; \quad |\bar{p}'|^2 = \frac{\lambda}{4s}; \quad \bar{k}_0 = \frac{s-m^2}{2\sqrt{s}}; \quad (13)$$

$$|\bar{k}|^2 = \bar{k}_0^2 - m^2; \quad \cos\bar{\theta} = -\frac{u + \eta m_c^2 - 2p'_0 \bar{k}_0}{2|\bar{p}'||\bar{k}|}; \quad (14)$$

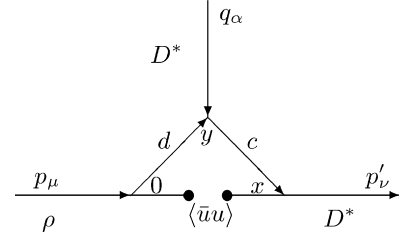
with $\eta = 1$ for ρ off-shell and $\eta = -1$ for D^* off-shell.

The contribution of the quark condensate which survives after the double Borel transform is represented in Fig. 2 for the D^* off-shell case and is given by

$$\Gamma_c^{(D^*)} = -\frac{m_c \langle \bar{q}q \rangle}{p^2(p'^2 - m_c^2)}, \quad (15)$$

where $\langle \bar{q}q \rangle$ is the light quark condensate. For the ρ off-shell there is no quark condensate contribution.

We expect the perturbative contribution to dominate the OPE, because we are dealing with heavy quarks. In Refs. [5,10] the gluon condensate contribution to the $D^*D\pi$ and $J/\psi D^*D$


Fig. 2. Contribution of the $u\bar{u}$ condensate to the D^* off-shell correlator.

vertices was evaluated, and the authors showed that its contribution was negligible as compared with the perturbative one. For this reason, we do not include the gluon and quark–gluon condensates in the present work.

The resulting vertex functions in the QCD side for the structure $g_{\mu\alpha}q_\nu$ are written as

$$\Gamma^{(\rho)}(p, p') = -\frac{1}{4\pi^2} \int_{m_c^2}^{s_0} ds \int_{m_c^2+t}^{u_0} du \frac{\rho^{(\rho)}(s, u, Q^2)}{(s-p^2)(u-p'^2)} \quad (16)$$

for ρ off-shell and

$$\Gamma^{(D^*)}(p, p') = -\frac{1}{4\pi^2} \int_0^{s_0} ds \int_t^{u_0} du \frac{\rho^{(D^*)}(s, u, Q^2)}{(s-p^2)(u-p'^2)} + \Gamma_c^{(D^*)} \quad (17)$$

for D^* off-shell, where, as usual, we have already transferred the continuum contribution from the hadronic side to the QCD side, through the introduction of the continuum thresholds s_0 and u_0 [17].

2.2. The phenomenological side

The $\rho D^* D^*$ vertex can be studied with hadronic degrees of freedom. The corresponding three-point functions, Eqs. (1) and (2), will be written in terms of hadron masses, decay constants and form factors. This is the so-called phenomenological side of the sum rule and it is based on the interactions at the hadronic level, which are described here by the following effective Lagrangian [3]

$$\begin{aligned} \mathcal{L}_{\rho D^* D^*} = & ig_{\rho D^* D^*} [((\partial_\mu D^{*\nu}) \vec{\tau} \cdot \vec{D}_\nu^* - D^{*\nu} \vec{\tau} \cdot \partial_\mu \vec{D}_\nu^*) \cdot \vec{\rho}^\mu \\ & + (D^{*\nu} \vec{\tau} \cdot \partial_\mu \vec{\rho}_\nu - \partial_\mu D^{*\nu} \vec{\tau} \cdot \vec{\rho}_\nu) \vec{D}^{*\mu} \\ & + D^{*\mu} (\vec{\tau} \cdot \vec{\rho}^\nu \partial_\mu \vec{D}_\nu^* - (\vec{\tau} \cdot \partial_\mu \vec{\rho}^\nu) \vec{D}_\nu^*)], \end{aligned} \quad (18)$$

from where one can extract the matrix element associated with the $\rho D^* D^*$ vertex. The meson decay constants, f_ρ and f_{D^*} ,

are defined by the following matrix elements:

$$\langle 0 | j_\rho^\mu | \rho(p) \rangle = m_\rho f_\rho \epsilon_\rho^\mu(p) \quad (18)$$

and

$$\langle 0 | j_v^{D*} | D^*(p) \rangle = m_{D^*} f_{D^*} \epsilon_{D^*}^\nu(p), \quad (19)$$

where ϵ_ρ^μ and $\epsilon_{D^*}^\nu$ are the polarization vectors of the ρ and D^* mesons, respectively. Saturating Eqs. (1) and (2) with the ρ and two D^* states and using Eqs. (18) and (19) we arrive at

$$\begin{aligned} \Gamma_{\mu\nu\alpha}^{(\rho)} = & -g_{\rho D^* D^*}^{(\rho)}(Q^2) \sqrt{2} \frac{f_{D^*}^2 f_\rho m_{D^*}^2 m_\rho}{(P^2 + m_{D^*}^2)(Q^2 + m_\rho^2)(P'^2 + m_{D^*}^2)} \\ & \times \left(-g_{\mu\mu'} + \frac{p_\mu p_{\mu'}}{m_{D^*}^2} \right) \left(-g_{\nu\nu'} + \frac{p'_\nu p'_{\nu'}}{m_{D^*}^2} \right) \\ & \times \left(-g_{\alpha\alpha'} + \frac{q_\alpha q_{\alpha'}}{m_\rho^2} \right) \\ & \times [(p + p')^{\alpha'} g^{\mu\nu'} + (2p' - p)^{\nu'} g^{\alpha'\mu'} \\ & - (2p - p')^{\mu'} g^{\alpha'\nu'}], \quad (20) \end{aligned}$$

when the ρ is off-shell, with a similar expression for the D^* off-shell:

$$\begin{aligned} \Gamma_{\mu\nu\alpha}^{(D^*)} = & -g_{\rho D^* D^*}^{(D^*)}(Q^2) \sqrt{2} \\ & \times \frac{f_{D^*}^2 f_\rho m_{D^*}^2 m_\rho}{(P^2 + m_\rho^2)(Q^2 + m_{D^*}^2)(P'^2 + m_{D^*}^2)} \\ & \times \left(-g_{\mu\mu'} + \frac{p_\mu p_{\mu'}}{m_\rho^2} \right) \left(-g_{\nu\nu'} + \frac{p'_\nu p'_{\nu'}}{m_{D^*}^2} \right) \\ & \times \left(-g_{\alpha\alpha'} + \frac{q_\alpha q_{\alpha'}}{m_{D^*}^2} \right) \\ & \times [(p + p')^{\alpha'} g^{\mu\nu'} - (2p - p')^{\nu'} g^{\alpha'\mu'} \\ & - (2p' - p)^{\mu'} g^{\alpha'\nu'}]. \quad (21) \end{aligned}$$

The contractions of μ' , ν' and α' in the above equation will lead to the fourteen Lorentz structures appearing in Eq. (3). We can see from Eq. (20) that the form factor $g_{\rho D^* D^*}^{(\rho)}(Q^2)$ is the same for all the structures and thus can be extracted from sum rules written for any of these structures. The resulting phenomenological invariant amplitudes associated with the structure $g_{\alpha\mu} q_\nu$ are

$$\begin{aligned} \Gamma_{\text{ph}}^{(\rho)}(P^2, P'^2, Q^2) \\ = g_{\rho D^* D^*}^{(\rho)}(Q^2) \frac{\sqrt{2} f_{D^*}^2 f_\rho m_{D^*}^2 m_\rho \left(2 - \frac{m_\rho^2}{2m_{D^*}^2}\right)}{(P^2 + m_{D^*}^2)(Q^2 + m_\rho^2)(P'^2 + m_{D^*}^2)} \quad (22) \end{aligned}$$

for the ρ off-shell, and

$$\begin{aligned} \Gamma_{\text{ph}}^{(D^*)}(P^2, P'^2, Q^2) \\ = g_{\rho D^* D^*}^{(D^*)}(Q^2) \frac{f_{D^*}^2 f_\rho m_{D^*}^2 m_\rho (Q^2 + 4m_{D^*}^2)}{(P^2 + m_{D^*}^2)(Q^2 + m_{D^*}^2)(P'^2 + m_{D^*}^2) \sqrt{2} m_{D^*}^2} \quad (23) \end{aligned}$$

for D^* off-shell.

In order to improve the matching between the two sides of the sum rules we perform a double Borel transformation [17] in the variables $P^2 = -p^2 \rightarrow M^2$ and $P'^2 = -p'^2 \rightarrow M'^2$, on both invariant amplitudes Γ and Γ_{ph} . Equating the results we get the final expressions for the sum rules which allow us to obtain the form factors $g_{\rho D^* D^*}^{(T)}(Q^2)$ appearing in Eqs. (22)–(23), where T is ρ or D^* . In this work we use the following relations between the Borel masses M^2 and M'^2 : $\frac{M^2}{M'^2} = \frac{m_\rho^2}{m_{D^*}^2}$ for a D^* off-shell and $M^2 = M'^2$ for a ρ off-shell.

3. Results and discussion

Table 1 shows the values of the parameters used in the present calculation. We used the experimental value for $f_\rho = \frac{m_\rho}{g_\rho}$, with $g_\rho = 4.79$ [18], and took f_{D^*} from Ref. [19]. The continuum thresholds are given by $s_0 = (m + \Delta_s)^2$ and $u_0 = (m + \Delta_u)^2$, where m is the mass of the incoming meson. Using $\Delta_s = \Delta_u = 0.5$ GeV for the continuum thresholds and fixing $Q^2 = 1$ GeV², we found a good stability of the sum rule for $g_{\rho D^* D^*}^{(\rho)}$ for M^2 in the interval $1 < M^2 < 9$ GeV², as can be seen in Fig. 3. Within this interval we need to choose the best value of the Borel mass to extract the coupling constant of the vertex. It is well known in QCDSR that if we choose a too small value of the Borel variable M^2 , then the sum rule will be dominated by the pole, but the convergence of the OPE is poor. On the other hand, if M^2 is too large, then the OPE convergence is good but the sum rule is dominated by the continuum. The best value of the Borel mass is the one with which both criteria are reasonably satisfied.

In Fig. 4 we show the pole contribution (solid line) and the continuum contribution (dashed line) divided by their sum as a

Table 1
Parameters used in the calculation

m_c (GeV)	m_{D^*} (GeV)	m_ρ (GeV)	f_{D^*} (GeV)	f_ρ (GeV)	$\langle \bar{q}q \rangle$ (GeV) ³
1.35	2.01	0.778	0.240	0.161	$(-0.23)^3$

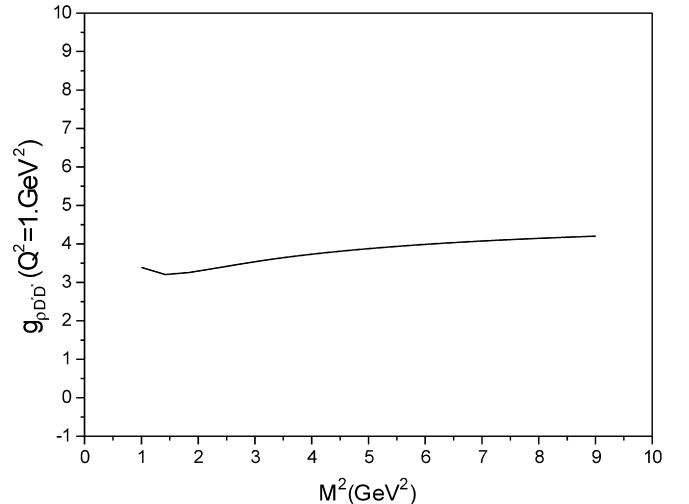


Fig. 3. $g_{\rho D^* D^*}^{(\rho)}(Q^2 = 1.0 \text{ GeV}^2)$ as a function of the Borel mass M^2 .

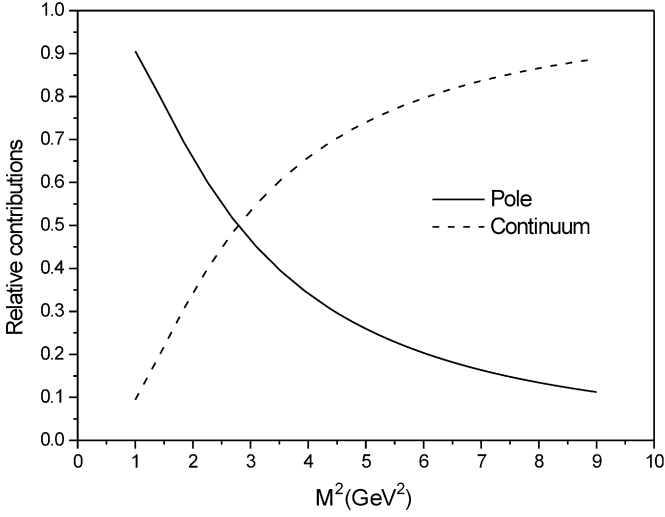


Fig. 4. Pole (solid line) and continuum (dashed line) contribution to $g_{\rho D^* D^*}^{(\rho)}(Q^2 = 1 \text{ GeV}^2, M^2)$, as a function of the Borel mass M^2 .

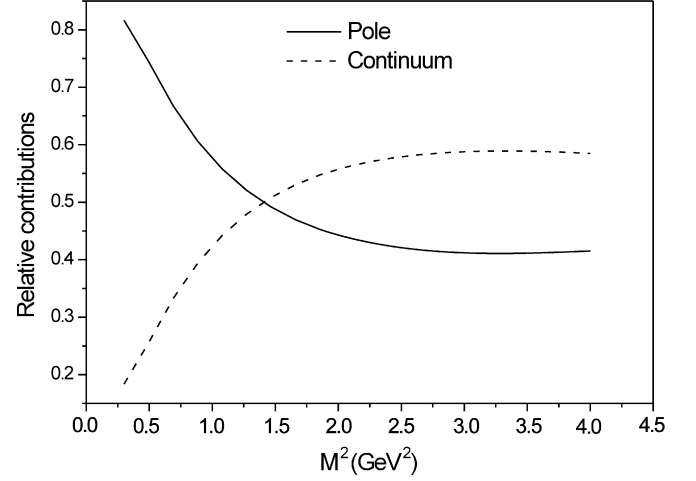


Fig. 6. Pole versus continuum contributions to $g_{\rho D^* D^*}^{(D^*)}(Q^2 = 1 \text{ GeV}^2)$ as a function of the Borel mass M^2 .

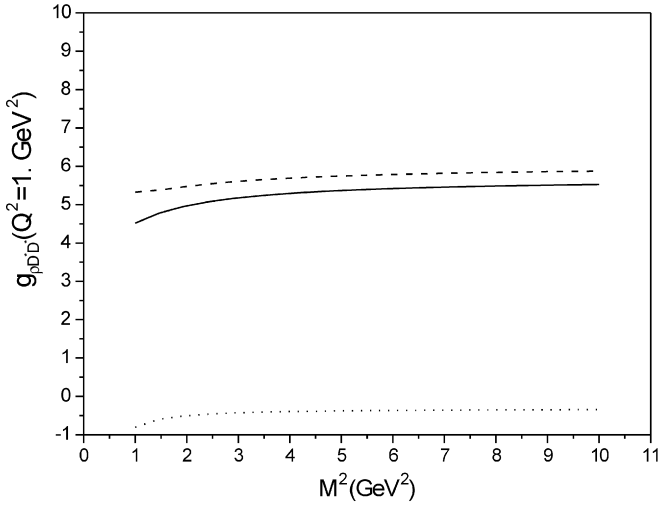


Fig. 5. $g_{\rho D^* D^*}^{(D^*)}(Q^2 = 1 \text{ GeV}^2)$ as a function of the Borel mass. We show the perturbative contribution (dashed line), quark condensate contribution (dotted line) and total (solid line).

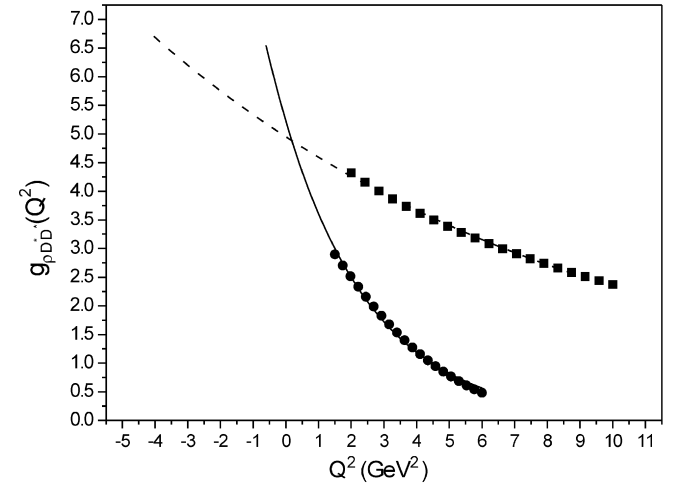


Fig. 7. $g_{\rho D^* D^*}^{(\rho)}$ (circles) and $g_{\rho D^* D^*}^{(D^*)}$ (squares) QCDSR form factors as a function of Q^2 . The solid and dashed lines correspond to the exponential parametrizations of the QCDSR data with the two forms mentioned in the text.

function of Borel mass. In the case of ρ off-shell, we see that the pole contribution is bigger than the continuum one in the Borel window $1 < M^2 < 3 \text{ GeV}^2$. The best Borel mass seems to be $M^2 = 2.0 \text{ GeV}^2$. Also for this value of M^2 the coupling α_s is approximately 0.2 and this suggests that perturbative corrections are small.

In the case of $g_{\rho D^* D^*}^{(D^*)}$ the interval for stability is $1 < M^2 < 10 \text{ GeV}^2$, as can be seen in Fig. 5. In order to choose the Borel mass we proceed in the same way as before and we analyze the pole and continuum contributions. As indicated in Fig. 6 in the window $0.5 < M^2 < 1.5 \text{ GeV}^2$ the pole contribution dominates. We choose $M^2 = 1.5 \text{ GeV}^2$.

Having determined M^2 we calculated the Q^2 dependence of the form factors. We present the results in Fig. 7, where the circles correspond to the $g_{\rho D^* D^*}^{(\rho)}(Q^2)$ form factor in the interval where the sum rule is valid. The squares are the result of the sum rule for the $g_{\rho D^* D^*}^{(D^*)}(Q^2)$ form factor.

In the case of an off-shell ρ meson, our numerical results can be fitted by the following exponential parametrization (shown by the solid line in Fig. 7):

$$g_{\rho D^* D^*}^{(\rho)}(Q^2) = 5.22e^{-Q^2/2.70}. \quad (24)$$

As in our previous works [7–10,12,13], we define the coupling constant as the value of the form factor at $Q^2 = -m_m^2$, where m_m is the mass of the off-shell meson. Therefore, using $Q^2 = -m^2$ in Eq. (24), the resulting coupling constant is:

$$g_{\rho D^* D^*} = 6.55. \quad (25)$$

For an off-shell D^* meson, our sum rule results can also be fitted by an exponential parametrization, which is represented by the dashed line in Fig. 7:

$$g_{\rho D^* D^*}^{(D^*)}(Q^2) = 4.95e^{-Q^2/13.33}. \quad (26)$$

Using $Q^2 = -m_{D^*}^2$ in Eq. (26) we get:

$$g_{\rho D^* D^*} = 6.70, \quad (27)$$

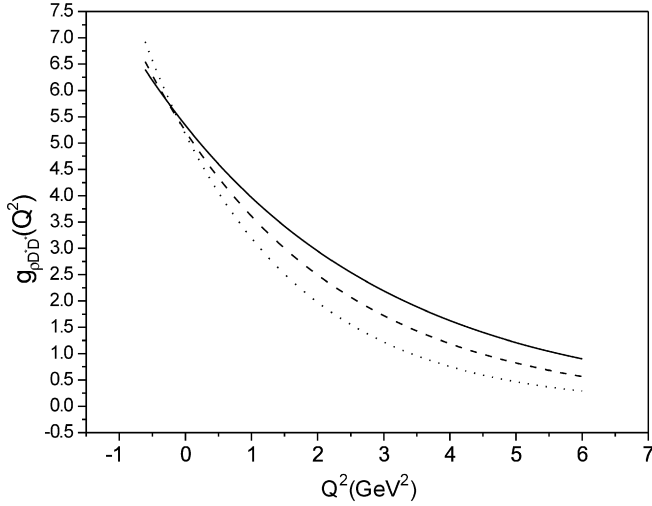


Fig. 8. Dependence of the form factor on the continuum threshold for the ρ off-shell case. The dashed curve corresponds to $\Delta_{s,u} = 0.5$ GeV, the solid one to $\Delta_{s,u} = 0.6$ GeV and the dotted curve to $\Delta_{s,u} = 0.4$ GeV.

in a good agreement with the result of Eq. (25).

In order to study the dependence of our results with the continuum threshold, we vary $\Delta_{s,u}$ between $0.4 \text{ GeV} \leq \Delta_{s,u} \leq 0.6 \text{ GeV}$ in the parametrization corresponding to the case of an off-shell ρ . As can be seen in Fig. 8, this procedure gives us an uncertainty interval of $6.40 \leq g_{\rho D^* D^*} \leq 6.92$ for the coupling constant.

Concluding, the two cases considered here, off-shell ρ or D^* , give compatible results for the coupling constant, evaluated using the QCDSR approach. Considering the uncertainties in the continuum thresholds we obtain:

$$g_{\rho D^* D^*} = 6.60 \pm 0.31. \quad (28)$$

This generic value of the coupling constant can be easily related to the couplings of the specific charge states. From Eq. (17) we arrive at:

$$g_{\rho D^* D^*} = \frac{g_{\rho^- D^{*0} D^{*+}}}{\sqrt{2}} = \frac{g_{\rho^+ D^{*0} D^{*+}}}{\sqrt{2}} \\ = -g_{\rho^0 D^{*+} D^{*+}} = g_{\rho^0 D^{*0} D^{*0}}. \quad (29)$$

From Eqs. (24) and (26) we can also extract the cut-off parameter, Λ , associated with the form factors. We get $\Lambda \sim 1.64 \text{ GeV}$ for an off-shell ρ meson and $\Lambda \sim 3.65 \text{ GeV}$ for an off-shell D^* . The cut-off values obtained here follow the same trend as observed in Refs. [7–10]: the value of the cut-off is directly associated with the mass of the off-shell meson probing the vertex. The form factor is harder if the off-shell meson is heavier.

As for the value of $g_{\rho D^* D^*}$, this coupling has not been discussed in the literature as much as those involving the J/ψ and there are only few works presenting estimates for it. The starting point in these estimates is always the SU(4) symmetry. According to SU(4) we should expect:

$$g_{J/\psi D^* D^*} = g_{J/\psi D D}, \quad (30)$$

$$g_{\rho D^* D^*} = g_{\rho D D} \quad (31)$$

and

$$g_{\rho D^* D^*} = \frac{\sqrt{6}}{4} g_{J/\psi D^* D^*}. \quad (32)$$

From our previous works [8,13] we find that in QCDSR Eq. (30) is satisfied. However, from [7] and from the present work we conclude that Eq. (31) is not satisfied, since $g_{\rho D^* D^*} = 6.60 \pm 0.31$ whereas $g_{\rho D D} = 3.04 \pm 0.30$.¹ Eq. (32) is not true either because $g_{J/\psi D^* D^*} = 6.2 \pm 0.9$. These relations are violated at the level of 50%. This is not surprising since the mass difference starts to play an important role when we go from the heavier vector mesons to ρ . As for the absolute value, the existing estimates, used in [3,4], lead to $g_{\rho D^* D^*} = 2.52$. Our result is a factor two larger.

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¹ Due to a difference in definitions, this number is a factor $\sqrt{2}$ smaller than the one quoted in [7].