Three-dimensional Large Landing Angle Guidance Based on Two-dimensional Guidance Laws

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Abstract

Both the design process and form of the three-dimensional (3D) suboptimal guidance law (3DSGL) are very complex. Therefore, we propose the use of two-dimensional (2D) guidance laws to meet the guidance requirements of 3D space. By analyzing the relationship between the flight-path angle and its projections on $OXY$ and $OXZ$ planes, we obtain the ideal design requirements of the guidance laws. Based on the requirements, we design a 2D suboptimal guidance law used in the horizontal plane; combining the 2D vertical suboptimal guidance law, we create a whole ballistic simulation of six degree-of-freedom. The results are compared with those using other three guidance modes in the case of large windage of the initial azimuth angle. When the proportional navigation guidance (PNG) law is used in the horizontal planes, the landing angle will obviously decrease. With the proposed guidance mode, the large landing angle can be realized and meet the guidance precision requirements. Moreover, the required overload can decrease to meet the control requirement. The effects of the proposed guidance mode are close to that of 3DSGL despite its very simple form.

Keywords: optimal control; guidance law; three-dimensional; multi-constraints; landing angle; satellite-guided

1. Introduction

Certain features of satellite-guided projectile, such as fire and forget, weatherproof and low cost per kill, have caused it to receive considerable attention \cite{1-6}. The vertical measurement error of a satellite positioning system is larger than its horizontal measurement error \cite{7-8}. To reduce this impact on guidance accuracy, a near-vertical descent is required in the terminal guidance phase. Ideally, the terminal flight-path angle should be close to $-90^\circ$. The control capability of guided projectiles is limited due to the constraints of volume and cost. It is for this reason that the terminal flight-path angle should be close to, but not equal to, $-90^\circ$. The large landing angle, defined as the absolute value of the flight-path angle at the landing moment, can also increase the damage effect and penetration capacity of guided projectiles. A trajectory with large landing angle is likewise required by other satellite-guided munitions.

Therefore, a two-dimensional (2D) vertical suboptimal guidance law (2DVSGL) was designed with constraints of landing angle, miss distance, and control energy consumption \cite{9}. Using 2DVSGL and the proportional navigation guidance (PNG) law in the vertical and horizontal planes respectively (2DVSGL&PNG), three-dimensional (3D) guidance of guided projectiles could be achieved.

Let the projections of the terminal flight-path angle
\( \theta_i \) on OXY, OXZ, and OZY planes be \( \theta_{i_{\text{XY}}} \), \( \theta_{i_{\text{XZ}}} \) and \( \theta_{i_{\text{YZ}}} \), respectively; using 2DVSL&PNG will allow the angle sizes of \( \theta_{i_{\text{XY}}} \) and \( \theta_{i_{\text{XZ}}} \) to be controlled, but this cannot control \( \theta_{i_{\text{YZ}}} \). However, \( \theta_{i_{\text{XY}}} \) and \( \theta_{i_{\text{XZ}}} \) decide the size of the landing angle. In the instance of small windage of the initial azimuth angle, \( \theta_{i_{\text{YZ}}} \) has little impact on the terminal flight-path angle; however, when the windage angle is large, \( \theta_{i_{\text{YZ}}} \) will significantly decrease the landing angle.

Designing a 3D guidance law (3DGL) is an effective approach in solving the problem; however, there are fewer studies on 3DGL than on 2D guidance law (2DGL) due to the complexity of the 3DGL design. Adler\cite{10} first studied 3D PNG. Most early researches are also related to PNG\cite{11-12} while multi-constraints constitutes a new research area. Naghash, et al.\cite{13} developed a guidance law that maximized terminal velocity using 3D Bezier curve. Dai\cite{14} designed a 3D guidance law that maximized terminal velocity using 3D Bezier curve. Mi, et al.\cite{15} designed a 3DGL with minimum-time interception trajectory using nonlinear programming. We present the design of the 2D horizontal suboptimal guidance law (2DHSGL) in Section 3.

On the basis of 2DVSL, a 3D suboptimal guidance law (3DGSGL) was developed using vector computation method\cite{16}. The proposed 3DGL can meet the requirements of both \( \theta_{i_{\text{XY}}} \) and \( \theta_{i_{\text{XZ}}} \), at the same time, consider the constraints of miss distance and control energy consumption, thereby ensuring satisfactory guidance effects.

From Refs.\cite{10}-\cite{16}, 3DGL designs are more complex than those of 2DGLs. The forms of 3DGLs are also more complex, and will consume more onboard computational resources. Onboard computing power is limited because of the limitations of projectile volume, cost, and power supply.

In practice, one 3DGL is often decomposed into two guidance laws applied to vertical and horizontal planes. However, these guidance laws cannot be called 2DGLs because each involves 3D information and has more complex forms.

The so-called 2DGL is concerned only with vertical or horizontal information, allowing for a simpler form. In this paper, we design a 2DGL and applied it, together with a 2DVSL, to the horizontal and vertical planes, respectively, to meet 3D guidance requirements. The guidance effects of the designed 2DGL and 2DVSL are similar to that of 3DGSGL, but their design processes and forms are much simpler.

2. Design Requirements for 2DGLs Used in Vertical and Horizontal Planes

To achieve a large landing angle using 2DGL, the relation between the 3D flight-path angle and its projections on OXY and OXZ planes should be analyzed first. In Fig. 1, \( V_X \), \( V_Y \) and \( V_Z \) are the projections of guidance projectile velocity on the X, Y and Z axes, respectively. \( \theta \) is the flight-path angle, and \( \theta_{i_{\text{XY}}} \) the angle between the projection of \( V \) on the OXY plane and \( X \) axis. \( \psi \), also called the heading angle, is the angle between the projection of \( V \) on the OXZ plane and \( X \) axis.

From Fig. 1, we have

\[
\tan \theta_{i_{\text{XY}}} = \frac{V_Y}{V_X} \quad (1)
\]

\[
\tan \psi = \frac{V_Z}{V_X} \quad (2)
\]

\[
\tan \theta = \frac{V_Y}{\sqrt{V_X^2 + V_Z^2}} \quad (3)
\]

From Eqs. (1)-(3), we have

\[
\tan \theta = \frac{\tan \theta_{i_{\text{XY}}}}{\sqrt{1 + \tan^2 \psi}} \quad (4)
\]

Eq. (4) shows that \( \theta \) can obtain a maximum value or \( \theta = \theta_{i_{\text{XY}}} \) when \( \theta_{i_{\text{XY}}} \) is unchanged and \( \psi \) is set to zero. Therefore, in the OXZ plane, a horizontal guidance law must be designed to make the heading angle approach zero when the guided projectile is close to the target. For the OXY plane, a vertical guidance law must be designed to make the flight-path angle approach -90° at the landing moment. In Ref. [9], a 2DVSL was designed that could be used to meet the requirements of landing angle, miss distance, and control energy consumption in a vertical plane. To facilitate the following discussion, we write 2DVSLG as follows:

\[
u = k_1(q_x - \theta_i) + k_2q_y \quad (5)
\]

where \( q_x \) is the line-of-sight (LOS) angle, \( \theta_i \) the expected flight-path angle at landing moment \( t_l \). In general, \( k_1 \) and \( k_2 \) can be set between 0.05-0.20 and 3.0-6.0, respectively. We present the design of the 2D horizontal suboptimal guidance law (2DHSGL) in Section 3.

3. 2DHSGL

Consider a 2D homing scenario shown in Fig. 2. Let \( M \) and \( T \) denote the satellite-guided projectile and target, respectively; they are regarded as particles and their movements are on a horizontal plane. Take \( M \), \( MF \) and \( r_{XZ} = MT \) as pole, polar axis and projec-
tile-target distance, respectively. \( V_{xz} \) is the projectile velocity and \( q_z \) is the LOS angle. Satellite-guided projectiles are used against stationary targets; thus, the target velocity is zero.

The polar equations of motion are given by

\[
\begin{align*}
\dot{r}_{xz} &= -V_{xz} \cos(q_z - \psi) \quad (6) \\
\dot{r}_{xz} q_z &= V_{xz} \sin(q_z - \psi) \quad (7)
\end{align*}
\]

Differentiating Eq. (7) with respect to time and substituting Eq. (6) into it, we obtain

\[
\dot{\psi} = \frac{\dot{V}_{xz} - 2 \dot{r}_{xz}}{r_{xz}} q_z + \frac{\dot{r}_{xz}}{r_{xz}} \psi \quad (8)
\]

Let \( z_1 = q_z - \psi_0 \), where \( \psi_0 \) is the expected heading angle at landing moment, and the expected value of \( \psi_0 \) is zero, making \( z_1 = q_z \). We then let \( z_2 = z_1 = \dot{q}_z \) and \( v = \dot{\psi} \). From Eq. (8), we have

\[
\begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= \left( \frac{\dot{V}_{xz}}{V_{xz}} - 2 \frac{\dot{r}_{xz}}{r_{xz}} \right) z_2 + \frac{\dot{r}_{xz}}{r_{xz}} v \quad (10)
\end{align*}
\]

In general, satellite-guided projectiles have little or no thrust. Therefore, they cannot keep constant axial velocity and their velocity changes slowly. Thus, \( \dot{V}_{xz} / V_{xz} << 1 \) and we can assume \( \dot{V}_{xz} / V_{xz} = 0 \). If we let \( f = -\dot{r}_{xz} / r_{xz} \), then Eqs. (9)-(10) can be written as

\[
Z = AZ + Bv \quad (11)
\]

where \( Z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \), \( A = \begin{bmatrix} 0 & 1 \\ 0 & 2f \end{bmatrix} \), and \( B = \begin{bmatrix} 0 \\ -f \end{bmatrix} \). Initial conditions: \( t = t_0 \), \( z_1(t_0) = q_z(t_0) = \dot{q}_z(t_0) \).

Performance index function is as follows:

\[
J = \frac{1}{2} Z^T C Z + \frac{1}{2} \gamma \int_{t_0}^{t} Rv^2 \, dt \quad (12)
\]

where \( C = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \) and \( R = 1 \). Based on the optimal control theory, the optimal guidance law for the system Eq. (11) is

\[
v = -R^{-1} B^T Pz \quad (13)
\]

where \( P \) satisfies the Riccati matrix differential equation given by

\[
P(t) = -P(t)A(t) - A(t)^T P(t) + P(t)B(t)R^{-1}B(t)^T P(t) - Q(t) \quad (14)
\]

where \( Q(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \), and the terminal condition of \( P(t) \) is

\[
P(t_f) = C \quad (15)
\]

From Eq. (13), we know that the optimal guidance law can be written as

\[
v = k_1 (f(t, t_{go}) z_1 + k_2 (f(t, t_{go}) z_2 \quad (16)
\]

where \( t_{go} \) is the time-to-go. If we take \( k_1(f(t, t_{go}) \) and \( k_2(f(t, t_{go}) \) as constants, the form of Eq. (16) would be the simplest, and we can refer to it as a suboptimal guidance law. When \( k_1(f(t, t_{go}) \) and \( k_2(f(t, t_{go}) \) are constants, \( f \) and \( t_{go} \) can be substituted with constants. We substitute \( f \) with a constant \( \bar{f} \). Then, in Eq. (11), \( A = \begin{bmatrix} 0 & 1 \\ 0 & 2\bar{f} \end{bmatrix} \) and \( B = \begin{bmatrix} 0 \\ -\bar{f} \end{bmatrix} \). Solving Eq. (14), we have the analytical expression of suboptimal guidance law as follows:

\[
v = \frac{4e^{\bar{f}t_{go}} \sinh(\bar{f}t_{go}) (\bar{f}z_1 + e^{\bar{f}t_{go}} z_2 \sinh(\bar{f}t_{go}))}{2\sinh(\bar{f}t_{go}) \cosh(\bar{f}t_{go}) \cosh(\bar{f}t_{go}) e^{\bar{f}t_{go}} - \sinh(\bar{f}t_{go}) \cosh(\bar{f}t_{go}) + \bar{f}t_{go} - 2e^{\bar{f}t_{go}} \sinh(\bar{f}t_{go})} \quad (17)
\]

Eq. (17) can then be written as

\[
v = k_1 z_1 + k_2 z_2 \quad (18)
\]

where

\[
k_1 = \frac{4e^{\bar{f}t_{go}} \sinh(\bar{f}t_{go})}{2\sinh(\bar{f}t_{go}) \cosh(\bar{f}t_{go}) \cosh(\bar{f}t_{go}) e^{\bar{f}t_{go}} - \sinh(\bar{f}t_{go}) \cosh(\bar{f}t_{go}) + \bar{f}t_{go} - 2e^{\bar{f}t_{go}} \sinh(\bar{f}t_{go})} \quad (19)
\]

and

\[
k_2 = \frac{4e^{\bar{f}t_{go}} \sinh(\bar{f}t_{go}) e^{\bar{f}t_{go}} \sinh(\bar{f}t_{go})}{2\sinh(\bar{f}t_{go}) \cosh(\bar{f}t_{go}) \cosh(\bar{f}t_{go}) e^{\bar{f}t_{go}} - \sinh(\bar{f}t_{go}) \cosh(\bar{f}t_{go}) + \bar{f}t_{go} - 2e^{\bar{f}t_{go}} \sinh(\bar{f}t_{go})} \quad (20)
\]

Substituting \( \sinh(\bar{f}t_{go}) = (e^{\bar{f}t_{go}} - e^{-\bar{f}t_{go}}) / 2 \) and \( \cosh(\bar{f}t_{go}) = (e^{\bar{f}t_{go}} + e^{-\bar{f}t_{go}}) / 2 \) into Eq. (19) and Eq. (20),
respectively, and substituting \( t_{go} \) with a constant \( t_{go} \), we can obtain

\[
k_1 = \frac{8(e^{2\gamma m} - 1)}{3 - 4e^{2\gamma m} + e^{4\gamma m} + 4f t_{go}}
\]

\[
k_2 = \frac{4(e^{2\gamma m} - 1)^2}{3 - 4e^{2\gamma m} + e^{4\gamma m} + 4f t_{go}}
\]

Because \( t_{go} \approx -r_{xz}/\dot{r}_{xz} \), we have

\[
f t_{go} = 1
\]

Therefore, we can take \( f t_{go} = 1 \), then, Eqs. (21)-(22) can be written as

\[
k_1 = 1.59 \tilde{f}
\]

\[
k_2 = 5.09
\]

Because \( f = -\dot{r}_{xz} / r_{xz} \approx 1/t_{go} \), the mean of \( f \) is 0.1. Considering the requirements of available overload of guidance process and heading angle at the landing moment, in general, \( k_1 \) and \( k_2 \) can be set between 0.05-0.20 and 3.0-6.0, respectively. The larger the value of \( k_1 \), the closer the heading angle is to the desired value at the landing moment; however, if \( k_1 \) is too large, the miss distance may increase. The value and effects of \( k_2 \) are similar to the navigation ratio of PNG.

4. Simulation Results

In Ref. [16], a 3DSGL was designed using the vector calculation method. To facilitate comparison, we write it here as

\[
\dot{\psi} = N\dot{q}_y
\]

\[
\psi = N\dot{q}_x
\]

Comparing with Eqs. (26)-(27), Eqs. (28)-(29) are simpler. However, a further comparison with Eqs. (28)-(29) show that the proposed guidance laws stated in Eq. (5) and Eq. (18) are much simpler; they only have one term more than PNG, as shown in Eqs. (30)-(31), where \( N = 3 \). Next, we shall analyze the proposed guidance mode using simulation and compare the results with other guidance modes.

For the ground coordinate system, let the muzzle velocity of guided projectile be 800 m/s, initial flight-path angle 30°, initial heading angle 0°, muzzle coordinate (0, 0) km, and target coordinate (20, 0, 4) km. Guidance and control begin when projectiles reach the highest point. Here, we take \( k_1 \) and \( k_2 \) as 0.07 and 3.5, respectively.

The whole ballistic simulations of six degree-of-freedom are made using four different guidance modes. We analyze the effects of the proposed guidance mode by comparing them with other three guidance modes. The four guidance modes are as follows:

1. 3DSGL, denoted by Eqs. (26)-(27). Here, \( k_1 \) and \( k_2 \) are taken as 0.07 and 3.5, respectively.
2. 2DV&HSGL, corresponding to 2DVSGL applied to the vertical plane and 2DHSGL applied to the horizontal plane as denoted by Eq. (5) and Eq. (18).
3. 2DVSGL&PNG, corresponding to 2DVSGL applied in the vertical plane and PNG applied in the horizontal plane as denoted by Eq. (18) and Eq. (31).
4. PNG&PNG, corresponding to PNGs applied to both the vertical and horizontal planes as denoted by Eqs. (30)-(31).

Fig. 3 shows the 3D trajectories of the guided projectiles. In Fig. 3. Lines 1, 2, 3, and 4 are the trajectories made by 3DSGL, 2DV&HSGL, 2DVSGL&PNG and PNG&PNG, respectively. When 3DSGL is applied, the terminal trajectory is at its steepest. Using PNG&PNG results in a very soft terminal trajectory. In addition, the landing angle is only 5.91% less than that of 3DSGL when the proposed guidance mode, 2DV&HSGL, is applied.

Fig. 3  3D trajectories of guided projectiles.
The lines in Fig. 4 are the projections of 3D trajectories on the $OZY$ plane, which clearly show the steep level of terminal trajectories. Terminal trajectory steep degree can be quantified through the terminal flight-path angles. Fig. 5 shows the flight-path angle histories using the four different guidance modes; here, the terminal absolute values are the landing angles. The steep terminal trajectory can decrease the required overload when guided projectiles hit the targets, thereby reducing the possibility that the required overload might exceed the available overload.

![Fig. 4 Projections of 3D trajectories on OZY plane.](image)

![Fig. 5 Flight-path angle history.](image)

Table 1 shows the results using the four guidance modes. System delay and disturbance are not taken into account, and miss distance is observed to be caused by two factors: time step and required overload exceeding the available overload. In these simulations, time step is set at 0.001 s.

<table>
<thead>
<tr>
<th>Guidance mode</th>
<th>Landing angle/(°)</th>
<th>Heading angle/(°)</th>
<th>Miss distance/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>3DSGL</td>
<td>73.05</td>
<td>−11.74</td>
<td>0.38</td>
</tr>
<tr>
<td>2DV&amp;HSGL</td>
<td>68.73</td>
<td>−12.38</td>
<td>0.56</td>
</tr>
<tr>
<td>2DV&amp;HSGL</td>
<td>52.73</td>
<td>−37.14</td>
<td>41.79</td>
</tr>
<tr>
<td>PNG&amp;PNG</td>
<td>19.18</td>
<td>−36.32</td>
<td>123.85</td>
</tr>
</tbody>
</table>

Using the four guidance modes, the change in landing angle versus the $z$ coordinate is shown in Fig. 6. From Line 4, we can see that the increase in the $z$ coordinate allows the guided projectiles to bounce after landing easily. In using 2DVSGL&PNG, the landing angles obviously decrease due to PNG with the increase in $z$ coordinate. However, for 2DV&HSGL, the landing angles only show little change and have values close to those obtained using 3DSGL. Fig. 7 shows that Lines 1 and 2 almost coincide, indicating that the miss distances using 2DV&HSGL are very close to that using 3DSGL, but much less than that using the other two guidance law modes.

![Fig. 6 Change of landing angle versus z coordinate.](image)

![Fig. 7 Change of miss distance versus z coordinate.](image)

Table 2 presents the impact of $k_1$ and $k_2$ on ballistic performance when 2DV&HSGL is used. Here, the purpose is to make a comparison of control energy consumption, in which we are only concerned with

<table>
<thead>
<tr>
<th>$k_1$</th>
<th>$k_2$</th>
<th>Heading angle/(°)</th>
<th>Absolute value integral of overload</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>3.5</td>
<td>52.52</td>
<td>12.96</td>
</tr>
<tr>
<td>0.06</td>
<td>3.5</td>
<td>60.68</td>
<td>16.15</td>
</tr>
<tr>
<td>0.07</td>
<td>3.5</td>
<td>68.73</td>
<td>19.50</td>
</tr>
<tr>
<td>0.08</td>
<td>3.5</td>
<td>68.91</td>
<td>27.00</td>
</tr>
<tr>
<td>0.09</td>
<td>3.5</td>
<td>69.63</td>
<td>29.27</td>
</tr>
<tr>
<td>0.10</td>
<td>3.5</td>
<td>69.98</td>
<td>36.29</td>
</tr>
<tr>
<td>0.07</td>
<td>3.0</td>
<td>79.37</td>
<td>26.77</td>
</tr>
<tr>
<td>0.07</td>
<td>3.2</td>
<td>76.02</td>
<td>23.86</td>
</tr>
<tr>
<td>0.07</td>
<td>3.4</td>
<td>71.01</td>
<td>20.89</td>
</tr>
<tr>
<td>0.07</td>
<td>3.6</td>
<td>66.61</td>
<td>18.21</td>
</tr>
<tr>
<td>0.07</td>
<td>3.8</td>
<td>62.78</td>
<td>15.99</td>
</tr>
<tr>
<td>0.07</td>
<td>4.0</td>
<td>59.46</td>
<td>14.39</td>
</tr>
</tbody>
</table>
relative comparisons. Therefore, we substitute the absolute value integral of the overload, which does not consider gravity acceleration, for the control energy consumption. Within a certain range, the heading angle and the absolute value integral of the overload increase as $k_1$ increases when $k_2$ is given, and they decrease as $k_2$ increases when $k_1$ is given. The value of $k_1$ cannot be taken too large because the control capability of guided munitions is limited, and very little control energy consumption is expected during the flight.

5. Conclusions

Based on the large landing angle requirement, we designed 2DHSGL and combine it with 2DVSGL used in the vertical plane. This allows us to achieve 3D guidance of guided projectiles. The proposed guidance mode, 2DV&HSGL, can provide a steep terminal trajectory even when the windage of initial azimuth angle is large. When the coordinates of the target is at (20, 0, 4) km, using 2DV&HSGL, the landing angle becomes 68.73°, which is only 4.32° smaller than that using 3DSGL. In addition, the miss distance is close to zero. All the simulations indicate that the guidance effects of the proposed mode are close to that of 3DSGL, and much better than that of 2DVSGL&PNG and PNG&PNG. Furthermore, the 2DVSGL and 2DHSGL designs are much simpler than that of 3DSGL. At the same time, their forms are very simple, and only have one term more than PNG. This result offers significant savings on computing resources.

References


Biography:

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