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Enhancement of CP violating terms for neutrino oscillation in Earth matter

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Abstract

We investigate the $\nu_e \to \nu_\mu$ oscillation in the framework of three generations when neutrinos pass through the Earth. The oscillation probability is represented by the form, $P(\nu_e \to \nu_\mu) = A\cos\delta + B\sin\delta + C$ in arbitrary matter profile by using the leptonic CP phase δ. We compare our approximate formula in the previous paper with the formula which includes second order terms of $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$ and $s_{13} = \sin\theta_{13}$. Non-perturbative effects of α and s_{13} can be taken into account in our formula and the precision of the formula is rather improved around the MSW resonance region. Furthermore, we compare the Earth matter effect of A and B with that of C studied by other authors. We show that the magnitude of A and B can reach a few ten % of C around the main three peaks of C in the region $E > 1$ GeV by numerical calculation. We give the qualitative understanding of this result by using our approximate formula. The mantle–core effect, which is different from the usual MSW effect, appears not only in C but also in A and B, although the effect is weakened.

1. Introduction

The first evidence of neutrino oscillation have been discovered in the atmospheric neutrino experiments and the mass squared difference $|\Delta m^2_{31}|$ and the 2–3 mixing angle θ_{23} [1] have been measured. Also the deficit of solar neutrino strongly suggests the neutrino oscillation with the Large Mixing Angle (LMA) solution for Δm^2_{21} and θ_{12} [2]. This has been confirmed by the KamLAND experiment by using the artificial neutrino beam emitted from several reactors [3]. On the other hand, only the upper bound $\sin^2 2\theta_{13} \leqslant 0.1$ is obtained for the 1–3 mixing angle [4]. Thus, the values of the mass differences and the mixing angles are gradually clarified. Our aim in the future is to determine the unknown parameters like the sign of Δm^2_{31}, θ_{13} and the leptonic CP phase δ.

The simple analytic formula for estimating the matter effects is useful in order to study these parameters because neutrinos pass through the earth in most ex-

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periments and receive the matter potential represented by $a = \sqrt{2}G_F N_e$, where N_e is the electron number density and G_F is the Fermi constant. In the case of short baseline length, we can approximate the density as constant because the variation of N_e is small. However, the longer the baseline is, the larger the matter effect is. In previous papers, several approximate formulas have been proposed in order to include the effect of varying density. Classified by the neutrino energy E, there are following approximate formulas: low energy formulas by the expansion in the small parameter $2Ea/|\Delta m^2_{31}| \ll 1$ or $s_{13} = \sin\theta_{13} \ll 1$ [5], high energy formulas by the expansion in $\Delta m^2_{21}/2Ea \ll 1$ or $\alpha = \Delta m^2_{21}/\Delta m^2_{31} \ll 1$ [6], and the formulas by the expansion in $2E\delta a/\Delta m^2_{31} \ll 1$ [7], where δa is the deviation from the average matter potential.

On the other hand, there is the method to approximate the Earth matter density as three constant layers in the case of mantle–core–mantle [8]. It was discussed in Ref. [9] how the probability is enhanced when neutrinos pass through periodically varying density. Then, it was pointed out in Ref. [10] that the mantle–core effect, which is different from the usual Mikheyev–Smirnov–Wolfenstein (MSW) effect [11], appears in the oscillation probability. More detailed analysis has been in Refs. [12,13]. This effect is interesting because the large enhancement of the probability can occur even if both the effective mixing angles in the mantle and the core are small. In recent papers [14], the possibility for measuring the θ_{13} in atmospheric neutrino experiments has been discussed by using this mantle–core effect. They concluded that the value of θ_{13} can be measured in some cases.

In our previous papers, we have shown that the oscillation probability for $\nu_e \to \nu_\mu$ transition is represented by the following form,

$$P(\nu_e \to \nu_\mu) = A\cos\delta + B\sin\delta + C \tag{1}$$

in constant matter [15] and also in arbitrary matter [16]. By using this general feature for the CP dependence, the method for solving the parameter ambiguity problem pointed out in Ref. [17] is discussed in Ref. [18]. Each coefficients has an order $A = O(s_{13}\alpha)$, $B = O(s_{13}\alpha)$ and $C = O(s^2_{13}) + O(\alpha^2)$ on the two small parameters $\alpha = \Delta m^2_{21}/\Delta m^2_{31} \sim 0.04$ and $s_{13} = \sin\theta_{13} < 0.2$. In the case of $\alpha < s_{13}$, the ratio of A, B to C are given by $A/C = O(\alpha/s_{13})$ and $B/C = O(\alpha/s_{13})$. So, it is expected that the CP violating effect due to A and B becomes large and can reach a few ten % of C even for the case that neutrinos pass through the earth core. However, the effect due to the CP phase has not been taken into account in previous works.

In this Letter, as the preparation of studying Earth matter effect, we review our approximate formula introduced in Ref. [19] as

$$A \simeq 2c_{23}s_{23}\,\mathrm{Re}\big[S^{\ell *}_{\mu e}S^h_{\tau e}\big], \tag{2}$$

$$B \simeq -2c_{23}s_{23}\,\mathrm{Im}\big[S^{\ell *}_{\mu e}S^h_{\tau e}\big], \tag{3}$$

$$C \simeq \big|S^\ell_{\mu e}\big|^2 c^2_{23} + \big|S^h_{\tau e}\big|^2 s^2_{23}, \tag{4}$$

where $S^\ell_{\mu e}$ and $S^h_{\tau e}$ are the amplitudes calculated from two-generation Hamiltonians H_ℓ and H_h. H_ℓ is represented by Δm^2_{21} and θ_{12} and H_h is represented by Δm^2_{31} and θ_{13}. We show that our formula includes the non-perturbative effect of α and s_{13} and the precision is rather improved around the MSW resonance regions compared to the well-known simple formula [20,21], which includes up to second order terms of α and s_{13}. Furthermore, we compare the Earth matter effect of A and B with that of C by using the Preliminary Reference Earth Model (PREM) [22] in the case of two reference baseline length. We show that the magnitude of A and B can reach a few ten % of C around the main three peaks of C in the region $E > 1$ GeV by numerical calculation. This means that the above perturbative estimation is valid even in the case of including non-perturbative effect. We give the qualitative understanding of this result by using our approximate formula. The mantle–core effect, which is different from the usual MSW effect, appears not only in C but also in A and B, although the effect is weakened.

2. General formulation for neutrino oscillation probabilities

In this section, we review the exact formulation of neutrino oscillation in arbitrary matter profile based on Ref. [16]. At first, let us parametrize the Maki–Nakagawa–Sakata (MNS) matrix U [23], which connects the flavor eigenstate ν_α with the mass eigenstate ν_i, by the standard parametrization [24]

$$U = O_{23}\Gamma_\delta O_{13}\Gamma^\dagger_\delta O_{12}, \tag{5}$$

where $\Gamma_\delta = \mathrm{diag}(1, 1, e^{i\delta})$ and O_{ij} is the rotation matrix between i and j generation. As the matter potential only appears in the (ee) component of the Hamiltonian, O_{23} and Γ_δ can be factored out. So, we can rewrite the Hamiltonian in matter as $H = O_{23}\Gamma_\delta H'(O_{23}\Gamma_\delta)^\dagger$. H' is the reduced Hamiltonian defined on the basis $\nu' = (O_{23}\Gamma_\delta)^\dagger\nu$. H' is a real symmetric matrix and the concrete expression is given by

$$H' = O_{13}O_{12}\,\mathrm{diag}(0, \Delta_{21}, \Delta_{31})(O_{13}O_{12})^T + \mathrm{diag}\big(a(t), 0, 0\big), \tag{6}$$

where $\Delta_{ij} = \Delta m^2_{ij}/2E$. The number of parameters in the Hamiltonian H' is fewer than that in the original Hamiltonian H by two. This is useful to calculate the oscillation probability simply. If we define the amplitude $S'_{\alpha\beta}$ of $\nu'_\alpha \to \nu'_\beta$ transition as $(\alpha\beta)$ component of time ordered product

$$S' = \mathrm{T}\exp\left[-i\int_0^L H'(t)\,dt\right], \tag{7}$$

the oscillation probability is given by

$$P(\nu_e \to \nu_\mu) = A\cos\delta + B\sin\delta + C, \tag{8}$$

$$A = 2c_{23}s_{23}\,\mathrm{Re}\big[S'^*_{\mu e}S'_{\tau e}\big], \tag{9}$$

$$B = -2c_{23}s_{23}\,\mathrm{Im}\big[S'^*_{\mu e}S'_{\tau e}\big], \tag{10}$$

$$C = |S'_{\mu e}|^2 c^2_{23} + |S'_{\tau e}|^2 s^2_{23}, \tag{11}$$

as in [16].

From the Eqs. (9)–(11), one can see that the probability for $\nu_e \to \nu_\mu$ transition is represented by two components of the reduced amplitude, $S'_{\mu e}$ and $S'_{\tau e}$. Namely, the matter effect for the oscillation probability is only contained in the two components.

3. Non-perturbative effect in our approximate formula

In this section, we numerically calculate the amplitudes $S'_{\mu e}$ and $S'_{\tau e}$ introduced in the previous section by using the PREM. Then, it is explained how we obtain the hint for the basic concept on deriving our approximate formula. As an example, the approximate formula in constant matter is derived explicitly and is compared with the formula in Refs. [20,21], which includes up to second order of the small parameters $\alpha = \Delta_{21}/\Delta_{31}$ and s_{13}. As a result, it is shown that our approximate formula includes the non-perturbative effect which becomes important around the MSW resonance.

3.1. Behavior of reduced amplitudes in Earth matter

Let us calculate the amplitudes $S'_{\mu e}$ and $S'_{\tau e}$ for the case that neutrinos pass through the earth. We use the PREM as the Earth density model and we choose two reference baselines, 6000 and 12 000 km. Fig. 1 shows how the matter density changes along the path of neutrinos. In Fig. 2, we plot the values of the amplitudes $S'_{\mu e}$ and $S'_{\tau e}$ corresponding to the neutrino energy 0.03–20 GeV. Here, we use the parameters $\Delta m^2_{21} = 7\times 10^{-5}\ \mathrm{eV}^2$ and $\sin^2 2\theta_{12} = 0.8$ as indicated from the solar neutrino experiments and the KamLAND experiment, $\Delta m^2_{31} = 2\times 10^{-3}\ \mathrm{eV}^2$ from the atmospheric neutrino experiments and the K2K experiment, and $\sin^2 2\theta_{13} = 0.1$ within the upper limit of the CHOOZ experiment.

It is found from Fig. 2 that $S'_{\mu e}$ and $S'_{\tau e}$ become large in low energy and high energy, respectively, for both baselines and the regions, where $S'_{\mu e}$ and $S'_{\tau e}$ dominantly contribute, are separated to each other. In other words, the MSW effect related to the 1–2 mixing and 1–3 mixing angles are mainly included in $S'_{\mu e}$ and $S'_{\tau e}$, respectively. We have derived the approximate formula for arbitrary matter profile by using this feature in Ref. [19]. Concretely, $S'_{\mu e}$ and $S'_{\tau e}$ are calculated from two kinds of different Hamiltonians, which are given by the 1–2 and 1–3 subsystems, respectively.

3.2. Procedure of deriving approximate formula

The idea introduced in the previous subsection is actually realized as follows. We use the two small parameters $\alpha = \Delta_{21}/\Delta_{31}$ and s_{13}. Then, our approximate formula is calculated by the following three steps.

(1) We define two Hamiltonians in the 1–2 and 1–3 subsystems taking the limit of $s_{13} \to 0$ and $\alpha \to 0$ in (6) as

$$H_\ell = \begin{pmatrix} \Delta_{21}s^2_{12} + a(t) & \Delta_{21}c_{12}s_{12} & 0 \\ \Delta_{21}c_{12}s_{12} & \Delta_{21}c^2_{12} & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix},$$

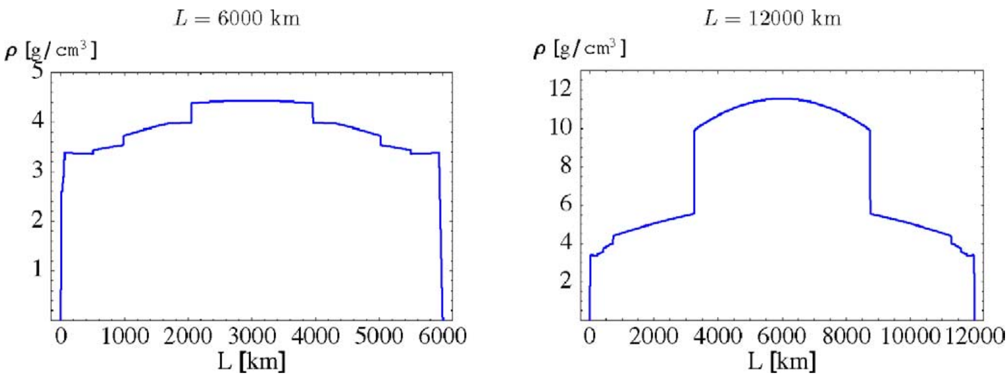

Fig. 1. Matter density in the PREM with baseline length 6000 and 12000 km from left to right.

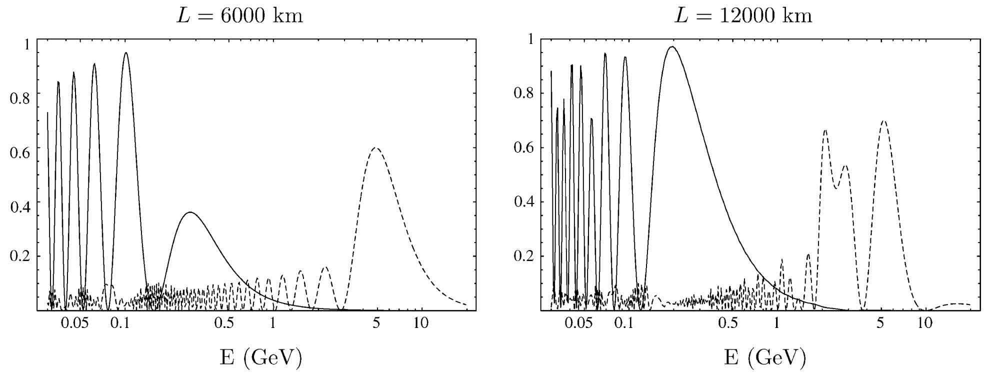

Fig. 2. Energy dependence of $S'_{\mu e}$ and $S'_{\tau e}$ with baseline length 6000 and 12 000 km by using the PREM. The solid and dashed lines represent $S'_{\mu e}$ and $S'_{\tau e}$, respectively.

$$H_h = \begin{pmatrix} \Delta_{31} s_{13}^2 + a(t) & 0 & \Delta_{31} c_{13} s_{13} \\ 0 & 0 & 0 \\ \Delta_{31} c_{13} s_{13} & 0 & \Delta_{31} c_{13}^2 \end{pmatrix}. \tag{12}$$

(2) We calculate two amplitudes S^ℓ and S^h from the Hamiltonians H_ℓ and H_h by the equations

$$S^\ell = \mathrm{T}\exp\left[-i\int_0^L H_\ell(t)\,dt\right],$$
$$S^h = \mathrm{T}\exp\left[-i\int_0^L H_h(t)\,dt\right]. \tag{13}$$

(3) We replace the amplitudes in (9)–(11) as $S'_{\mu e} \to S^\ell_{\mu e}$ and $S'_{\tau e} \to S^h_{\tau e}$.

3.3. Approximate formula in constant matter

Next, let us review the approximate formula in constant matter based on Ref. [19]. According to the procedure in the previous subsection, we substitute the Hamiltonian H_ℓ in constant matter given by (12) into (13) and we obtain $S^\ell_{\mu e}$ as

$$S^\ell_{\mu e} = \left[\exp(-iH_\ell L)\right]_{\mu e}$$
$$= -i \sin 2\theta_\ell \sin\phi_\ell \exp\left(-i\frac{\Delta_{21}+a}{2}L\right), \tag{14}$$

where $\phi_\ell \equiv \Delta_\ell L/2$ and the subscript ℓ represents the quantities calculated from H_ℓ. The concrete expressions for the mass squared difference and the 1–2 mixing angle in matter are given by

$$\frac{\Delta_\ell}{\Delta_{21}} = \frac{\sin 2\theta_{12}}{\sin 2\theta_\ell} = \sqrt{\left(\cos 2\theta_{12} - \frac{a}{\Delta_{21}}\right)^2 + \sin^2 2\theta_{12}}. \tag{15}$$

These are well-known expressions in the framework of two generations. The contribution of the low energy MSW effect, which is dominant around the energy region determined by $a \sim \Delta_{21}\cos 2\theta_{12}$, is included in mainly $S^\ell_{\mu e}$. The phase factor in (14) does not contribute when we calculate the probability in two generations. However, this gives important contribution on the calculation of the terms dependent on the CP phase in three generations.

Similarly, we obtain $S^h_{\tau e}$ by substituting H_h in constant matter given by (12) into (13) as

$$S^h_{\tau e} = \left[\exp(-iH_h L)\right]_{\tau e} = -i\sin 2\theta_h \sin\phi_h \exp\left(-i\frac{\Delta_{31}+a}{2}L\right), \tag{16}$$

where $\phi_h \equiv \Delta_h L/2$ and the subscript h represents the quantities calculated from H_h. The concrete expressions are given by

$$\frac{\Delta_h}{\Delta_{31}} = \frac{\sin 2\theta_{13}}{\sin 2\theta_h} = \sqrt{\left(\cos 2\theta_{13} - \frac{a}{\Delta_{31}}\right)^2 + \sin^2 2\theta_{13}}. \tag{17}$$

One can see that these expressions correspond to those obtained by the replacement $\Delta_{21} \to \Delta_{31}$ and $\theta_{12} \to \theta_{13}$ in (14) and (15). The contribution of high energy MSW effect, which is dominant around the energy region determined by $a \sim \Delta_{31}\cos 2\theta_{13}$, is included in mainly $S^h_{\tau e}$. We can calculate A, B and C in constant matter as

$$P(\nu_e \to \nu_\mu) = A\cos\delta + B\sin\delta + C, \tag{18}$$

$$A \simeq \sin 2\theta_\ell \sin 2\theta_{23} \sin 2\theta_h \sin\phi_\ell \sin\phi_h \cos\frac{\Delta_{32}L}{2}, \tag{19}$$

$$B \simeq \sin 2\theta_\ell \sin 2\theta_{23} \sin 2\theta_h \sin\phi_\ell \sin\phi_h \sin\frac{\Delta_{32}L}{2}, \tag{20}$$

$$C \simeq c_{23}^2 \sin^2 2\theta_\ell \sin^2\phi_\ell + s_{23}^2 \sin^2 2\theta_h \sin^2\phi_h, \tag{21}$$

from these expressions. These approximate formulas are similar to the following well-known formulas

$$A \simeq \frac{\Delta_{21}\Delta_{31}}{a(a-\Delta_{31})} c_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \times \sin\frac{aL}{2} \sin\frac{(a-\Delta_{31})L}{2} \cos\frac{\Delta_{31}L}{2}, \tag{22}$$

$$B \simeq \frac{\Delta_{21}\Delta_{31}}{a(a-\Delta_{31})} c_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \times \sin\frac{aL}{2} \sin\frac{(a-\Delta_{31})L}{2} \sin\frac{\Delta_{31}L}{2}, \tag{23}$$

$$C \simeq \frac{\Delta_{21}^2}{a^2} c_{23}^2 \sin^2 2\theta_{12} \sin^2\frac{aL}{2} + \frac{\Delta_{31}^2}{(a-\Delta_{31})^2} s_{23}^2 \sin^2 2\theta_{13} \sin^2\frac{(a-\Delta_{31})L}{2}. \tag{24}$$

These formulas are often used in order to analyze the property of neutrino oscillation because they have very simple form and approximate the exact values with a good precision.

In the following, we compare the probability calculated from our approximate formula (19)–(21) with that from the formula (22)–(24) in the case of constant matter. We calculate $P(\nu_e \to \nu_\mu)$ for two kinds of baselines, 3000 and 6000 km. We use the parameters $\sin^2 2\theta_{23} = 1$ and $\delta = 0^\circ$ in addition to those introduced in Fig. 2. Furthermore, $\rho = 4.7\ \mathrm{g/m^3}$ and $Y_e = 0.494$ are used as the matter density and the electron fraction. The result is given in Fig. 3.

It is found that our approximate formula has good coincidence to the exact one even around the MSW resonance, compared with that from the formula (22)–(24). We consider the reason for the difference in the next section.

3.4. Comparison of approximate formulas

Let us explain the order counting of α and s_{13} in our approximate formula. In the limit $\alpha \to 0$, we obtain $S'_{\mu e} = 0$. Therefore, we can write down the order of $S'_{\mu e}$ as

$$\begin{aligned} S'_{\mu e} &= O(\alpha) + O(\alpha^2) + O(\alpha^3) + \cdots \\ &+ O(s_{13}\alpha) + O(s_{13}\alpha^2) + O(s_{13}\alpha^3) + \cdots \\ &+ O(s_{13}^2\alpha) + O(s_{13}^2\alpha^2) + O(s_{13}^2\alpha^3) + \cdots \\ &+ \cdots. \end{aligned} \tag{25}$$

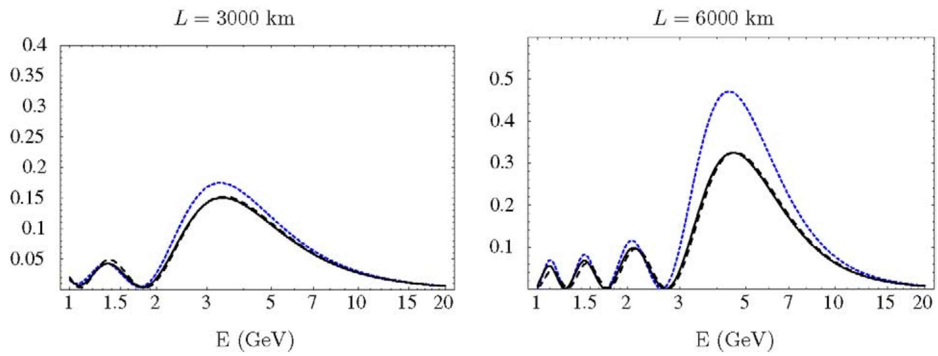

Fig. 3. Comparison between the probabilities $P(\nu_e \to \nu_\mu)$ calculated from our approximate formula (19)–(21) and the formula (22)–(24). Two baseline length are chosen as 3000 and 6000 km from left to right. The solid, dashed and dotted lines are the exact, our approximate formula and that from (22)–(24).

Note that α is included in all terms. Here, if we take the $s_{13} \to 0$, only the first line is remaining. In these terms, all orders of α are included and the first line considered to have a larger contribution compared to the following lines, because of the increasing exponent of s_{13}. This is confirmed by the comparison with the exact formula in Fig. 3. In the same way, we obtain $S'_{\tau e} = 0$ in the limit $s_{13} \to 0$. Therefore, we can write down the order of $S'_{\tau e}$ as

$$\begin{aligned} S'_{\tau e} = {} & O(s_{13}) + O(s_{13}^2) + O(s_{13}^3) + \cdots \\ & + O(s_{13}\alpha) + O(s_{13}^2\alpha) + O(s_{13}^3\alpha) + \cdots \\ & + O(s_{13}\alpha^2) + O(s_{13}^2\alpha^2) + O(s_{13}^3\alpha^2) + \cdots \\ & + \cdots . \end{aligned} \tag{26}$$

Here, if we take the limit $\alpha \to 0$, only the first line is remaining. In these terms, all orders of s_{13} are included and the first line is considered to have a larger contribution compared to the following lines, because of the increasing exponent of α.

Our method includes both, the terms of higher order of α in (25) and also those of s_{13} in (26). So, this new approach is not a systematic expansion. However, our method is not in contradiction to the well-known formula (22)–(24), which takes only the first order term of α and s_{13} in (25), (26) regarding them as small parameters. In addition, higher order terms of the perturbative expansion, which are not included in the formula (22)–(24), are now also included in our formula.

In the following, let us investigate the difference between these two methods more concretely. As seen in Fig. 2, the contribution of $S'_{\tau e}$ is dominant in the energy region $E > 1$ GeV. So, we can roughly consider as

$$P(\nu_e \to \nu_\mu) \simeq C \simeq s_{23}^2 \sin^2 2\theta_h \sin^2\left(\frac{\Delta_h L}{2}\right). \tag{27}$$

Note that we have the relation (17) between the mass squared differences and the mixing angles in vacuum and in matter. Expanding the right-hand side of (17) on the mixing angle in vacuum, we obtain

$$\begin{aligned} \frac{\Delta_h}{\Delta_{31}} &= \frac{\sin 2\theta_{13}}{\sin 2\theta_h} \\ &\simeq \left|1 - \frac{a}{\Delta_{31}}\right| \left(1 + \frac{2a\Delta_{31}}{(\Delta_{31}-a)^2} s_{13}^2 \right. \\ &\qquad \left. + \frac{a^2\Delta_{31}^2}{2(\Delta_{31}-a)^4} s_{13}^4 + \cdots \right). \end{aligned} \tag{28}$$

The condition for convergence is given by

$$\frac{4a\Delta_{31}s_{13}^2}{(\Delta_{31}-a)^2} < 1. \tag{29}$$

This condition is not satisfied around the MSW resonance region, where $\Delta_{31} \sim a$. Namely, the perturbative expansion becomes inconvergent. However, substituting the above expression (28) into (27) of the oscillation probability and taking only the first term, we obtain

$$\begin{aligned} & P(\nu_e \to \nu_\mu) \\ & \quad \simeq s_{23}^2 \frac{\Delta_{31}^2 \sin^2 2\theta_{13}}{(\Delta_{31}-a)^2} \sin^2 \frac{\Delta_{31}-a}{2} L. \end{aligned} \tag{30}$$

It gives the finite value in the limit $\Delta_{31} \to a$ as

$$P(\nu_e \to \nu_\mu) \simeq s_{23}^2 c_{13}^2 (s_{13}\Delta_{31}L)^2. \tag{31}$$

This is due to the product of the infinity of effective mixing angle and the zero of the effective mass squared difference in the probability (30). If we take the limit $\Delta_{31} \to a$ directly in the non-perturbative expression (27), we obtain

$$P(\nu_e \to \nu_\mu) = s_{23}^2 c_{13}^2 \sin^2(s_{13}\Delta_{31} L). \tag{32}$$

We find that the difference between the perturbative formula (31) and our formula (32) is the sin factor. In the case of the short baseline length L, the perturbation gives a good approximation, but the longer the baseline L is, the worse the perturbation becomes, as shown in Fig. 3, although the probability has a finite value. Concretely, if the condition

$$L < \frac{1}{s_{13}\Delta_{31}} \tag{33}$$

is satisfied, the perturbation gives a good approximation. Around the MSW resonance region, the perturbation breaks down because the coefficients of the higher order terms α or s_{13} become large. Therefore, it is needed to involve the higher order terms of α and s_{13}, in order to make a good approximate formula. Our method partially realizes this request.

At the end of this section, let us give a brief comment. In Refs. [20,21], the formula calculated by single expansion on α is also given and this includes all order terms of s_{13}. So, this approximate formula gives a good approximation in the high energy MSW resonance region compared with the formula (22)–(24), while the difference between the single expansion formula and numerical calculation becomes large in the low energy region as commented also in Ref. [21].

4. Earth matter effect for A, B and C

In this section, we perform numerical calculations of A, B and C by using the PREM. We give a qualitative understanding of the behavior of the coefficients A, B and C by matter effects of the mantle and the core.

4.1. Numerical calculation of A, B and C

In the previous section, the order of the reduced amplitudes are estimated as $S'_{\mu e} = O(\alpha)$ and $S'_{\tau e} = O(s_{13})$ in the case that we takes only the first order term of α and s_{13} in (25), (26). The order of coefficients are also obtained as $A = O(s_{13}\alpha)$, $B = O(s_{13}\alpha)$ and $C = O(s_{13}^2) + O(\alpha^2)$ by substituting $S'_{\mu e} = O(\alpha)$ and $S'_{\tau e} = O(s_{13})$ into (11)–(13). In the case of $\alpha < s_{13}$, the magnitude of ratios is give by $A/C = O(\alpha/s_{13})$ and $B/C = O(\alpha/s_{13})$. Therefore, it is expected from the perturbative point of view that the CP violating effect due to A and B becomes large and can reach a few ten % of C. However, because non-perturbative effect becomes important in MSW region as shown in Fig. 3, the CP violating effect should be investigated more carefully.

At first, let us numerically calculate how the coefficients A, B and C are enhanced by the Earth matter effect, in the case of the baseline length $L = 6000$ and 12 000 km for $\sin^2 2\theta_{13} = 0.10$ and 0.04, respectively. These values of $\sin^2 2\theta_{13}$ correspond to the values within the upper bound of the CHOOZ experiments. The PREM is used as the Earth matter density and the same mass squared differences and the mixing angles given in Section 3 are also used.

In the case of $L = 6000$ km, the behavior can be understood by using the formulation (19)–(21) in constant matter. The value of C becomes large around $E = 5$ GeV, which comes from the enhancement of the effective mixing angle $\sin\theta_h$ for $a \simeq \Delta_h$, and then oscillates depending on the factor $\sin\phi_h$. The values of A and B become small compared with that of C in high energy region, as the suppression factor $S^\ell_{\mu e} \propto 1/E$. See details in Refs. [15,20,25], for example, of the constant matter density.

In the case of $L = 12\,000$ km, three main peaks appear in C. On the other hand, the pattern of the enhancement for A and B seems to become more complicated than that of $L = 6000$ km. In the next subsection, we give a qualitative understanding of the above results by using our approximate formula for A, B and C.

We also represent the values of A, B and C around the energy of the three peaks of C in Table 1. These values are computed by the numerical calculation using the PREM.

Table 1 shows that the coefficients A and B can be rather large at the three main peaks of C. The absolute values of A and B become about 0.06 at the peak of the core. Furthermore, the ratios $|A/C|$ and $|B/C|$ also become a few ten % even for the case

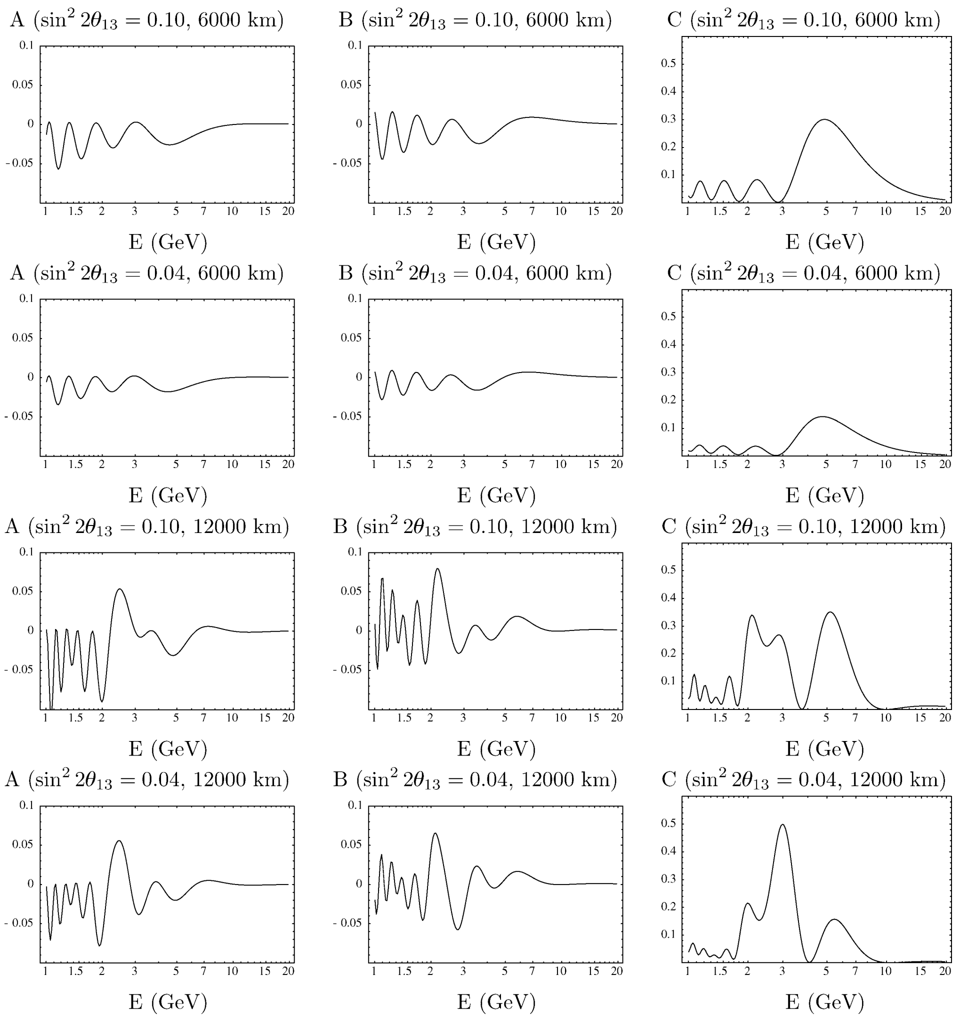

Fig. 4. Energy dependence of A, B and C by numerical calculation. We use the PREM as Earth matter density model with two baseline length 6000 and 12 000 km, and we choose the 1–3 mixing angle $\sin^2 2\theta_{13} = 0.10$ and 0.04 as representative values.

Table 1
Resonance values of A, B and C calculated numerically by using the PREM with baseline length 6000 and 12 000 km, $\sin^2 2\theta_{13} = 0.10$ and 0.04

L (km)	$\sin^2 2\theta_{13}$	E (GeV)	A	$(\lvert A/C\rvert)$	B	$(\lvert B/C\rvert)$	C	Peak type
6000	0.10	4.9	−0.025	(8.3%)	−0.003	(1.0%)	0.301	MSW (mantle)
6000	0.04	4.8	−0.017	(12.0%)	−0.001	(0.7%)	0.142	MSW (mantle)
12000	0.10	2.1	−0.061	(17.9%)	0.066	(19.4%)	0.340	MSW (core)
12000	0.10	2.8	0.017	(6.3%)	−0.028	(10.4%)	0.269	Mantle–core
12000	0.10	5.2	−0.026	(7.4%)	0.013	(3.7%)	0.352	MSW (mantle)
12000	0.04	2.0	−0.068	(31.6%)	0.038	(17.7%)	0.215	MSW (core)
12000	0.04	3.0	−0.030	(6.0%)	−0.037	(7.4%)	0.500	Mantle–core
12000	0.04	5.4	−0.014	(8.9%)	0.015	(9.6%)	0.157	MSW (mantle)

of including non-perturbative effect. These energy regions $E = 2$–6 GeV are explored by the atmospheric neutrino and the long baseline experiments. In actual experiments averaging of various parameters are necessary for example, energy, zenith-angle distribution, the sum of particle and antiparticle, and so on. Therefore, the CP phase effect may be weakened to some extent, but we consider that the CP phase effect should be estimated precisely in order to determine the value of θ_{13} in future experiments.

4.2. Approximate formula in matter with three layers

In order to give a qualitative understanding of the results obtained in the previous subsection, let us approximate the Earth matter density with baseline $L = 12\,000$ km as three constant layers such that the first and the third layers have the same density and length.

At first, we calculate the amplitude $S^{\ell}_{\mu e}$ from the low energy Hamiltonian H_ℓ. We use the superscript m and c for representing the amplitude in the first and third layer (mantle), and the second layer (core). Taking the limit $s_{13} \to 0$, the amplitudes $S^m_{\tau e}$, $S^m_{\tau\mu}$ and so on vanish. Only four terms contribute to the amplitude in three layers $S^{\ell}_{\mu e}$ as

$$S^{\ell}_{\mu e} = S^m_{\mu e} S^c_{ee} S^m_{ee} + S^m_{\mu\mu} S^c_{\mu e} S^m_{ee} + S^m_{\mu e} S^c_{e\mu} S^m_{\mu e} + S^m_{\mu\mu} S^c_{\mu\mu} S^m_{\mu e}. \tag{34}$$

Substituting (14) and

$$S^m_{ee} = \left[\exp\left(-i H^m_\ell L\right)\right]_{ee} = \left(\cos\phi^m_\ell + i\cos 2\theta^m_\ell \sin\phi^m_\ell\right) \times \exp\left(-i\frac{\Delta_{21} + a^m}{2} L^m\right), \tag{35}$$

$$S^m_{\mu\mu} = \left[\exp\left(-i H^m_\ell L\right)\right]_{\mu\mu} = \left(\cos\phi^m_\ell - i\cos 2\theta^m_\ell \sin\phi^m_\ell\right) \times \exp\left(-i\frac{\Delta_{21} + a^m}{2} L^m\right), \tag{36}$$

into (34), we obtain

$$S^{\ell}_{\mu e} = -i\exp\left(-i\frac{\Delta_{21} L + 2a^m L^m + a^c L^c}{2}\right) \times F\left(\phi^m_\ell, \phi^c_\ell; \theta^m_\ell, \theta^c_\ell\right), \tag{37}$$

where the function F is defined by

$$F\left(\phi^m, \phi^c; \theta^m, \theta^c\right) = \sin 2\phi^m \cos\phi^c \sin 2\theta^m + \cos^2\phi^m \sin\phi^c \sin 2\theta^c + \sin^2\phi^m \sin\phi^c \sin\left(2\theta^c - 4\theta^m\right). \tag{38}$$

We can easily extract the physical meaning from this expression, although this function F becomes the same one as given in Refs. [12,13] after a short calculation. We describe the meaning of each term later.

Next, we calculate the amplitude $S^h_{\tau e}$ taking the limit $\alpha \to 0$. In this limit, $S^m_{\mu e}$, $S^m_{\mu\tau}$ and so on vanish, so the amplitude $S^h_{\tau e}$ in three layer is calculated as

$$S^h_{\tau e} = S^m_{\tau e} S^c_{ee} S^m_{ee} + S^m_{\tau\tau} S^c_{\tau e} S^m_{ee} + S^m_{\tau e} S^c_{e\tau} S^m_{\tau e} + S^m_{\tau\tau} S^c_{\tau\tau} S^m_{\tau e}. \tag{39}$$

Substituting (16) and

$$S^m_{ee} = \left[\exp\left(-i H^m_h L\right)\right]_{ee} = \left(\cos\phi^m_h + i\cos 2\theta^m_h \sin\phi^m_h\right) \times \exp\left(-i\frac{\Delta_{31} + a^m}{2} L^m\right), \tag{40}$$

$$S^m_{\tau\tau} = \left[\exp\left(-i H^m_h L^m\right)\right]_{\tau\tau} = \left(\cos\phi^m_h - i\cos 2\theta^m_h \sin\phi^m_h\right) \times \exp\left(-i\frac{\Delta_{31} + a^m}{2} L^m\right), \tag{41}$$

into (39), we obtain

$$S^h_{\tau e} = -i\exp\left(-i\frac{\Delta_{31} L + 2a^m L^m + a^c L^c}{2}\right) \times F\left(\phi^m_h, \phi^c_h; \theta^m_h, \theta^c_h\right), \tag{42}$$

which corresponds to Eq. (37) by replacing the subscript and superscript as $(\ell) \to (h)$.

Substituting (37) and (42) into (9)–(11), the coefficients A, B and C in three layers are given by

$$A \simeq \sin 2\theta_{23} \cos\left(\frac{\Delta_{32} L}{2}\right) F\left(\phi^m_\ell, \phi^c_\ell; \theta^m_\ell, \theta^c_\ell\right) \times F\left(\phi^m_h, \phi^c_h; \theta^m_h, \theta^c_h\right), \tag{43}$$

$$B \simeq \sin 2\theta_{23} \sin\left(\frac{\Delta_{32} L}{2}\right) F\left(\phi^m_\ell, \phi^c_\ell; \theta^m_\ell, \theta^c_\ell\right) \times F\left(\phi^m_h, \phi^c_h; \theta^m_h, \theta^c_h\right), \tag{44}$$

$$C \simeq c_{23}^2 F\big(\phi_\ell^m, \phi_\ell^c; \theta_\ell^m, \theta_\ell^c\big)^2 + s_{23}^2 F\big(\phi_h^m, \phi_h^c; \theta_h^m, \theta_h^c\big)^2. \tag{45}$$

Thus, we can calculate the coefficients A and B, which are related to the magnitude of the CP effect, by using our approximate formula. We can see the following from the expressions of A, B and C. The expression of C is given as the sum of F_h and F_ℓ, where we use the abbreviation F_h and F_ℓ as $F(\phi_h^m, \phi_h^c; \theta_h^m, \theta_h^c)$ and $F(\phi_\ell^m, \phi_\ell^c; \theta_\ell^m, \theta_\ell^c)$. On the other hand, the expressions A and B are both given as the product of F_h and F_ℓ and furthermore multiplied by the oscillating factor related to Δ_{32}. This is the main difference between A, B and C. However, all the coefficients depend on the function F. In the following, we study the behavior of this function F.

At first, we divide F given in (38) into three parts as

$$F\big(\phi^m, \phi^c; \theta^m, \theta^c\big) = F_1 + F_2 + F_3, \tag{46}$$

$$F_1 = \sin 2\phi^m \cos\phi^c \sin 2\theta^m, \tag{47}$$

$$F_2 = \cos^2\phi^m \sin\phi^c \sin 2\theta^c, \tag{48}$$

$$F_3 = \sin^2\phi^m \sin\phi^c \sin\big(2\theta^c - 4\theta^m\big). \tag{49}$$

This separation of the function F is useful to understand, which contribution becomes large in the amplitude because F_1, F_2 and F_3 correspond to the MSW effect in the mantle, and in the core, and the mantle–core effect, respectively. By using the above expressions, the following interpretation in Refs. [12,13] can be understood more clearly.

(1) $\cos 2\phi^m = 0$ and $\sin\phi^c = 0$. Only F_1 remains and the function takes the form $F = \pm\sin 2\theta^m$ because $F_2 = F_3 = 0$ due to $\sin\phi^c = 0$. In the case that the above conditions are approximately satisfied around the MSW resonance region of the mantle, namely, around the energy determined by $\sin 2\theta^m = \pm 1$, the function F is enhanced.

(2) $\sin\phi^m = 0$ and $\cos\phi^c = 0$. Only F_2 remains and the function takes the form $F = \pm\sin 2\theta^c$ because $F_1 = F_3 = 0$ due to $\sin\phi^m = 0$ and $\cos\phi^c = 0$. In the case that the above conditions are approximately satisfied around the MSW resonance region of the core, namely, around the energy determined by $\sin 2\theta^c = \pm 1$, the function F is enhanced.

(3) $\cos\phi^m = 0$ and $\cos\phi^c = 0$. Only F_3 remains and the function takes the form $F = \pm\sin(2\theta^c - 4\theta^m)$ because $F_1 = F_2 = 0$ due to $\cos\phi^m = 0$ and $\cos\phi^c = 0$. Around the energy determined by $\sin(2\theta^c - 4\theta^m) = \pm 1$, the function F is enhanced. This can be large, even if both effective mixing angles in the mantle and in the core, θ^m and θ^c, are small. It is considered as the mantle–core effect. It is realized in the case that Δ_{31} takes the intermediate value of the matter potentials a^m and a^c, respectively, for the mantle and the core.

4.3. Interpretation of numerical results

In this subsection, the numerical result for $L = 12\,000$ km can be understood, by using the analytical expression derived in the previous subsection. All the coefficients A, B and C are determined by the functions F_ℓ and F_h. Here, we study the behavior of F_ℓ and F_h in the energy region larger than $E = 1$ GeV. We can approximate F_ℓ by using the fact $\Delta_{21} \ll a$ at $E > 1$ GeV. That is, the oscillation part and the mixing angle are approximated by

$$\phi_\ell = \frac{\Delta_\ell L}{2} \simeq \frac{aL}{2} \sim \text{const}, \tag{50}$$

$$\sin 2\theta_\ell = \frac{\Delta_{21}\sin 2\theta_{12}}{\sqrt{(\Delta_{21}\cos 2\theta_{12} - a)^2 + \Delta_{21}^2 \sin^2 2\theta_{12}}} \simeq \frac{\Delta_{21}\sin 2\theta_{12}}{a} \propto \frac{1}{E}, \tag{51}$$

from (15). As a result, we can also approximate F_ℓ from (46)–(49) as

$$F_\ell \propto \frac{1}{E}. \tag{52}$$

Thus, the value of F_ℓ decreases proportional to the inverse of the neutrino energy.

On the other hand, some of the peaks appear in F_h corresponding to F_{h1}, F_{h2} and F_{h3}, since F_h includes the 1–3 MSW effect in the considered energy range. Fig. 5 shows the component of F_h by using our analytical expression (46), where we use the matter densities in the mantle and the core as $\rho^m = 4.7\ \text{g/cm}^3$ and $\rho^c = 11.0\ \text{g/cm}^3$, the electron fraction as $Y_e^m = 0.494$ and $Y_e^c = 0.466$, calculated by the PREM in the case of the baseline $L = 12\,000$ km.

In these figures, the solid line shows the magnitude of F_h^2, and the dashed, dash-dotted and dotted lines show the magnitude of F_{h1}^2, F_{h2}^2 and F_{h3}^2, respectively.

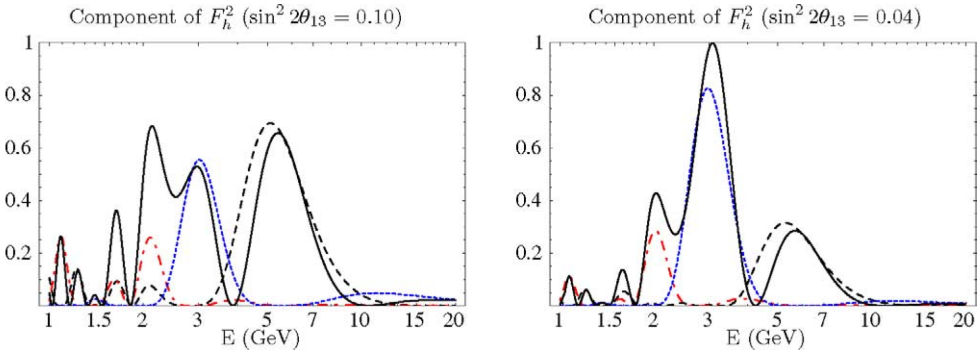

Fig. 5. Components of F_h^2 calculated from our analytical formula with $\sin^2 2\theta_{13} = 0.10$ and 0.04 from left to right. Solid, dashed, dash-dotted and dotted lines correspond to F_h^2, F_{h1}^2, F_{h2}^2 and F_{h3}^2, respectively.

Table 2
Components of F_h calculated from our analytical formula with $\sin^2 2\theta_{13} = 0.10$ and 0.04

$\sin^2 2\theta_{13}$	E (GeV)	F_h	F_{h1}	F_{h2}	F_{h3}
0.10	2.1	0.821	0.278	0.509	0.034
0.10	3.0	0.726	−0.029	0.011	0.744
0.10	5.4	−0.810	−0.821	0.015	−0.004
0.04	2.0	0.648	0.093	0.532	0.023
0.04	3.1	0.999	0.051	0.055	0.893
0.04	5.7	−0.535	−0.545	0.013	−0.003

Furthermore, we represent the values of each component F_{h1}, F_{h2} and F_{h3} at three peaks in Table 2.

Fig. 5 and Table 2 show that the peak in the right-hand side is dominated by F_{h1} and mainly depends on the MSW effect in the mantle. The MSW resonance in the mantle is realized at the condition $a^m = \Delta_{31} \cos 2\theta_{13}$. The energy determined by this condition is $E \sim \frac{\Delta_{31} \cos 2\theta_{13}}{2\sqrt{2} G N_e^m} \sim 5.7$ GeV. The peak in the left-hand side is dominated by F_{h2} and mainly depends on the MSW effect in the core. The MSW resonance in the core is realized at the condition $a^c = \Delta_{31} \cos 2\theta_{13}$. Noticing the relation $a^c \simeq 2.5 \times a^m$, the peak energy is given by around $E \sim 5.7/2.5 \sim 2.3$ GeV. The energy of these peaks do not largely depend on the value θ_{13} in the case of $\sin^2 2\theta_{13} \ll 1$. Furthermore, it is shown that the mantle–core effect mainly contributes to the peak at the center, when F_{h3} becomes large. The energy determined by the condition $\sin(2\theta^c - 4\theta^m) \sim 1$ is about $E = 3$–4 GeV for $\sin^2 2\theta_{13} = 0.04$. In the case of $\sin^2 2\theta_{13} = 0.10$, this condition cannot be satisfied in any energy region and as a result the enhancement is weakened. This phenomena is interesting because the value of F_h for $\sin^2 2\theta_{13} = 0.04$ (small mixing) is larger than that for $\sin^2 2\theta_{13} = 0.10$ (large mixing). It is interpreted as the total neutrino conversion pointed out by Petcov et al. [12].

Next, let us study how we can understand the behavior of A, B and C.

From (49), C is approximated by

$$C = \frac{1}{2}\left(F_\ell^2 + F_h^2\right) \simeq \frac{1}{2} F_h^2, \tag{53}$$

where we neglect F_ℓ, because of its smallness compared to F_h as shown in Fig. 5. Actually, the C-function has almost half of the size of the F_h^2-function. Therefore, C has three peaks as F_h^2. $P(\nu_e \to \nu_\mu)$ in Ref. [14] corresponds to C in this Letter. It means that the terms A and B, related to the CP phase, were not considered in previous papers.

Next, we obtain the expressions for A and B from (47) and (48) as

$$A \simeq \cos\left(\frac{\Delta_{32} L}{2}\right) F_\ell F_h \propto \frac{1}{E} \cos\left(\frac{\Delta_{32} L}{2}\right) F_h, \tag{54}$$

$$B \simeq \sin\left(\frac{\Delta_{32} L}{2}\right) F_\ell F_h \propto \frac{1}{E} \sin\left(\frac{\Delta_{32} L}{2}\right) F_h. \tag{55}$$

From these expressions, we can see the following. First, the mantle–core effect, which is different from the usual MSW effect, appears not only in C but also in A and B because of the multiplication of F_h. Second, A and B are suppressed compared with C in the energy range $E > 1$ GeV because of $F_\ell \propto 1/E$. Third, A and B depend on the oscillation part $\Delta_{32} L/2$ additionally to L/E dependence included in F. Because of this factor, the oscillation phases of A and B have a difference of about a quarter of the wavelength.

5. Summary

In this Letter, we investigate the matter effect included in the terms related to the CP phase, particularly in the case that neutrinos pass through the Earth core. The results are summarized as follows:

(1) Our approximate formulas (2)–(4) include non-perturbative effect of the small parameters $\alpha = \Delta_{21}/\Delta_{31}$ and s_{13}. As a result, the precision of the formula is rather improved compared to the formula which includes up to second order of α and s_{13} around the MSW resonance regions.

(2) We numerically calculate the coefficients A, B and C for the baseline length $L = 6000$ and 12 000 km by using the PREM as the Earth matter density. As a result, the magnitude of A and B can reach a few ten % of C around the three main peaks of C even for the case of including non-perturbative effect.

(3) We give the qualitative understanding of the behavior for A, B and C by using our approximate formula. The mantle–core effect, which is different from the usual MSW effect, appears not only in C but also in A and B, although the effect is weakened.

From the results of this Letter, it has been found that the effects of the leptonic CP phase can be comparatively large in the oscillation probability, when neutrinos pass through the Earth. We should consider the CP phase effects more seriously in order to extract the information on θ_{13} and the sign of Δm^2_{31} in future experiments.

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