Model of elements of the hydraulic control system for biaxial tensile test

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Abstract

The article deals with the modernization of existing equipment for evaluation of flexible-plastic properties of metal sheet in plane-strain. Design of the hydraulic section of the stress equipment for cross-examination was made in order to automate the whole process of experimental evaluation. Similarly mathematical models of individual components of the control system, which can be implemented into the control diagram, have been drawn up.

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Keywords: mathematical model, control system

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>V</td>
<td>cylinder capacity</td>
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<tr>
<td>Q</td>
<td>general flow rate of liquid</td>
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<tr>
<td>K</td>
<td>bulk modulus</td>
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<tr>
<td>L</td>
<td>constant of overflow of oil</td>
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<tr>
<td>m</td>
<td>mass of the cylinder</td>
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<tr>
<td>x</td>
<td>path of load</td>
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<tr>
<td>b</td>
<td>viscous damping</td>
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<tr>
<td>k_i</td>
<td>sensibility of the valve</td>
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<tr>
<td>k_p</td>
<td>compliance of the valve</td>
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Greek symbols

<table>
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<tr>
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<tbody>
<tr>
<td>ρ</td>
<td>consistence</td>
</tr>
<tr>
<td>ε</td>
<td>deformation</td>
</tr>
<tr>
<td>ε_d</td>
<td>desired deformation</td>
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1. Induction

Rolling of metallic materials results to anisotropy of their mechanical properties during plastic deformation. [6] Identification of the beginning of plastic deformation of metal sheets in plane stress is important in cold pressing of metal sheets and in assessment of load bearing of thin-walled structures. The development of numerical computational methods

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and methods of mathematical modeling requires knowing the behavior of metal sheets in the transition from flexible to plastic area in different stress states. The most versatile method of experimental determination of the beginning of metal sheets plastic deformation in plane stress is based on biaxial stress of the cruciform specimen. Further, control structure of hydraulic stress equipment, for both the functional and the organ level, and also a detailed description of the regulatory circuit on the organ level is proposed. This article contains derived mathematical models of electrohydraulic actuators, proportional reduction valve, cruciform specimen, model of hydraulic pipeline and models of tensiometric sensors. Models are presented in form of transfer functions and are expressed by block diagrams. At the end of the chapter there is shown the resulting image transfer of the regulated system.

2. Design of control system structure

For experimental evaluation of flexible-plastic properties of metal sheet in plane stress using cross samples at the Department of Applied Mechanics and Mechatronics (KAMaM) SjF TU in Košice is used a workplace consisting of (Fig.1):

- hydraulic equipment for biaxial stress of cross samples
- dynamic tensiometric apparatus SPIDER 8 for capturing the course of stress forces in the shoulders of the cross sample using dynamometers with resistive tensiometers,
- extensometer for measuring of deformation in middle section of cross sample during biaxial tensile stress.

![Fig. 1. Hydraulic scheme of stress equipment [1].](image)

The structure of the control system in Figure 2 consists of two autonomous one-dimensional control systems. The role of these control systems is to ensure:

- force actuation on the tested sample
- deformation and mechanical stress ratio regulation
- that the time response of deformation is not greater than what is specified in the relevant standard
Function 1 is ensured by both control systems, function 2 is ensured by control system 2 and function 3 by control system 1. In reality, these control systems are not autonomous, because each of the controlled variables is changing according to relation to both action variables \( F_1 \) and \( F_2 \).

![Fig. 2. The structure of control system of the functional level.](image1)

Impact of the action variable of one control system on the other control system and vice versa is taken into account here via fault variables. In case of control system 2 both deformations \( \varepsilon_1, \varepsilon_2 \) are measured. Deformation \( \varepsilon_1 \) is multiplied in proportion member by dimensional constant and after this modification it is established as desired variable \( \varepsilon_{2d} \) into control system 2. Function 3 is provided by control system 1 in a way, so that input to desired variable is fed with such course of desired value of deformation that the temporal change of desired variable \( \varepsilon_{1d} \) throughout the testing process is not greater than what is required by standard.

### 2.1. Design of control system on organ level

On the organ level regulated system (Fig. 3) is represented by tested sample (1). Electro-hydraulic actuator is represented for control system 1 by hydraulic cylinders (2), reduction valve (4) and measuring card (7). Control system 2 is represented by hydraulic cylinders (3), reduction valve (5) and measuring card (7).

![Fig. 3. Scheme of the control system on organ level](image2)
The regulator is realized using a PC-class computer for both control systems, which provides the function of control algorithm of the transformation members (2, 3) and proportion member. All these features were created in Matlab-Simulink environment using Real Time Toolbox. Tensiometers will be used as deformation sensors; SPIDER 8 measuring system will be also used.

3. Mathematical models of individual elements of the control system

The aim is to draw up mathematical models of individual components of the control system. Knowledge from literature sources [2, 3, 4] was used for drawing up these models. Specific constants shown in these models were determined by calculations.

Fig. 2 shows that the elements of the control system for axis x, or y are identical. Meaning that considering only one of them is sufficient (Fig. 4).

3.1. Mathematical model of the hydraulic subsystem of the hydraulic engine

The hydraulic subsystem of the engine (Fig. 5) is described by the relationship between the pressure in the cylinder chamber and the volume flow of oil into the linear hydraulic engine. In terms of control it is an important relationship because the compressibility of oil creates in the cylinder chamber of the hydraulic engine so called “spring” effect, which interacts with the weight of the piston, which results in resonance of the low frequency areas [3]. This effect is present in all hydraulic systems and in many cases unexpectedly limits the usable bandwidth.
Immediate mass of pressure fluid in the volume of cylinder is given by:

\[ m_k = \rho V \]  

(1)

By differentiating equation (1) by time we get the temporal change of weight expressed as:

\[ \frac{dm_k}{dt} = \frac{d\rho}{dt} V + \rho \frac{dV}{dt} \]  

(2)

Based on the law of conservation of mass the left side of equation (2) can be written:

\[ \frac{dm_k}{dt} = \rho Q \]  

(3)

State equation of fluids says that:

\[ \frac{d\rho}{dt} = \frac{\rho}{K} \frac{dp}{dt} \]  

(4)

By substituting equation (3) and (4) into equation (2), from the subsequent arrangement we get the relationship:

\[ \frac{dp(t)}{dt} = K \left( \frac{Q(t)}{V} - \frac{dV}{dt} \right) \]  

(5)

which is a mathematical model of the hydraulic subsystem of hydraulic engine. The block expression of this model is shown in Figure 6:

![Fig. 6. Block scheme of hydraulic engine.](image)

The initial proposals of control system sometimes do not consider effects of leakage. However, this factor can have a significant dampening effect on the response of hydraulic engine. When considering leakage, equation (17) changes:

\[ \frac{dp(t)}{dt} = K \left( \frac{Q(t)}{V} - \frac{dV}{dt} - Lp(t) \right) \]  

(6)

After including the effect of leakage the block diagram of the hydraulics section changes, as is shown in Figure 7:

![Fig. 7. Block scheme of subsystem of the hydraulic engine.](image)
### 3.2. Mathematical model of the mechanical subsystem of hydraulic engine

The motion of the piston of hydraulic engine due to the applied hydrostatic pressure of the oil is described within this subsystem. This equation can be expressed by relation (Fig. 8):

\[ m \frac{d^2x(t)}{dt^2} + b \frac{dx(t)}{dt} = Ap(t) \]  

(7)

![Block scheme of mechanical subsystem.](image)

### 3.3. Mathematical model of proportional reduction valve PDR08P-01

This valve is a member that, using feedback from the operator, or another automatic control source, adequately sets the system output (i.e., pressure or flow). It is this feedback that provides controllable output for hydraulic system, or provides safe function, which is essential when working with high-performance devices that are the typical application of hydraulics [2].

We base the notions on the manufacturer's catalog features, which are shown in Fig. 9:

![P-Q characteristic of proportional reduction valve PDR08P-01.](image)

We may notice that these features represent a set of parallel lines with the same slope (Fig. 10):

\[ Q = k_i i - k_p p \]  

(8)

where

\[ k_i = \frac{dQ}{di} = \frac{\Delta Q}{\Delta i} \]  

(9)
3.4. Mathematical model of cruciform specimen

The real model of cruciform specimen (Fig. 11):

For the purpose of controller design, we propose the following model of cruciform specimen:

\[ F = k_z \Delta l \]  \hspace{1cm} (11)

where

\[ k_z = \frac{E S_V}{l} \]  \hspace{1cm} (11)
It is possible deduce the final transfer function from scheme on Fig. 12:

\[
\frac{\Delta I(s)}{U(s)} = \frac{k_i K A k_k}{m V s^3 + (K m k_p) s^2 + (k_z V + K A^2) s + K k_z k_p}
\] (11)

4. Conclusion

Within this article the structure of the control system on functional and organ level was proposed. In order to modernize the device it was necessary to design in detail the electro-hydraulic section of the stress equipment on organ level. To be able to control the device, it was necessary to derive mathematical models of individual components of the control system. These are expressed in form of image transfer, from which has then been compiled the resulting block diagram and the resulting image transfer of the regulated system.

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References