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## Constraint Effects using a Stress-State Dependent Cohesive Model

Nishant Kanhurkar, Faizan Md. Rashid, Anuradha Banerjee\*

*Department of Applied Mechanics, Indian Institute of Technology Madras, Chennai – 600036, India*

### Abstract

In the present work, constraint effects on growth curves of a mode-I crack are determined using a triaxiality dependent cohesive model. Plane strain elastic-plastic analysis based on the modified boundary layer formulation is performed and for modeling the fracture process, the cohesive parameters and the mechanical properties for a mild steel are taken from literature. From the analysis, the resistance curves for a range of constraint parameter are obtained. A discussion is developed on the effectiveness of the triaxiality dependent model in capturing the well-known effect of constraint and also on the effect of the two model parameters on the resistance curves.

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**Keywords:** Triaxiality dependent cohesive zone model; T-stress; Fracture curve

### 1. Introduction

Characterization of the effects of constraint on the resistance of a material to ductile fracture has been of significant interest to ensure safe operation and life assessment of structures. One of the primary reason is that it allows transfer of fracture properties obtained from simple laboratory tests to different geometries and loading configurations.

In modelling of the ductile fracture process, the phenomenological approach of cohesive zone concept finds wide application. It assumes the fracture process to be localized in a thin layer, the constitutive behavior of which is represented by an assumed traction-separation law:  $T_n = \sigma_{max} f(\delta_n)$  with the cohesive strength,  $\sigma_{max}$ , and cohesive energy, the area under the traction-separation curve, as the two model parameters (Needleman, 1987; Tvergaard and Hutchinson, 1992, 1994; Sigmund and Brocks, 2000).

More recently, by incorporating the effect of triaxiality on the traction-separation behavior, researchers have found the cohesive energy as well as the cohesive strength to be dependent on stress-state and therefore, not material constants (Anvari et al., 2006; Banerjee and Manivasagam, 2009; Rashid and Banerjee, 2013; Scheider et al., 2011; Sigmund and Brocks, 1999).

\* Corresponding author. Tel.: +9-1442-257-4075 ; fax: +9-1442-257-4052.

E-mail address: [anuban@iitm.ac.in](mailto:anuban@iitm.ac.in)

Rashid and Banerjee (2013) take a combined experimental and computational approach to determine model parameters of the triaxiality dependent model (TCZM) by Banerjee and Manivasagam (2009) to replicate experimental data from fracture tests for a wide range of triaxiality. In the present study, the effectiveness of the TCZM in reproducing the well-known effects of constraint is tested. Further, a discussion on the effect of the model parameters on the nature of crack growth is developed.

## 2. Problem formulation

Resistance curve for a ductile material in case of small scale yielding is expressed in terms of crack length and critical energy release rate as

$$K = K_R(\Delta a) \quad (1)$$

where,  $K_R = \sqrt{E\Gamma_R(1-\nu^2)}$  such that  $\Gamma_R(\Delta a)$  is the critical energy release rate,  $E$  and  $\nu$  are the elastic constants of the material. In the present study, the elastic-plastic properties of mild steel (IS-2062) used for the current analysis were obtained from the uniaxial tension test of round tensile bars Rashid and Banerjee (2013).

Region very near to the crack tip was modeled as a circular region having a radially focused mesh. Two dimensional 4-noded plane strain element with reduced integration technique available in Abaqus v.10 library was used to model the region. The finite element mesh and boundary condition at a particular instant of time are shown in Figure 1.

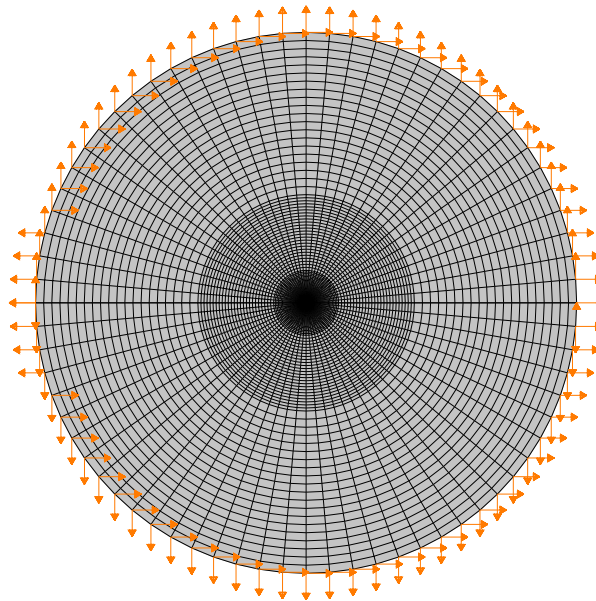


Fig. 1. Radially focused mesh around the crack tip with displacement boundary conditions on the remote outer boundary.

## 2.1. T-stress

According to the small strain linear elastic solution, the in-plane stress components near the crack-tip are of the form

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + T \delta_{1i} \delta_{1j} \quad (2)$$

where  $(r, \theta)$  are polar coordinates and  $\delta_{ij}$ , is the Kronecker delta. For mode I loading,  $K/\sqrt{2\pi r}$  is the amplitude of the singular stress field, while  $T$  is a non-singular stress term, acting parallel to the crack plane. Based on the mode I plane strain conditions, loads were applied in the form of time varying displacements on the remote outer boundary  $r_b$  of the model.

$$u_x(t) = K(t) \frac{1+\nu}{E} \sqrt{\frac{r_b}{2\pi}} \cos \frac{\theta}{2} (3-4\nu - \cos \theta) + T(t) r_b \cos \theta \left( \frac{1-\nu^2}{E} \right) \quad (3)$$

$$u_y(t) = K(t) \frac{1+\nu}{E} \sqrt{\frac{r_b}{2\pi}} \sin \frac{\theta}{2} (3-4\nu - \cos \theta) - T(t) r_b \sin \theta \left( \frac{\nu(1+\nu)}{E} \right) \quad (4)$$

In the current analysis the displacement due to  $T$ -stress was applied first. Subsequently, additional displacements were specified on the outer circular boundary according to the singular  $K$ -field solution around the crack-tip. The uniform  $T$ -stress field applied initially is below the yield limit, and the additional loading is applied by incrementally increasing the amplitude  $K$  for the displacements on the circular boundary.

Crack growth resistance curves are plotted in the form of  $K$  versus  $\Delta a$  for the following cases of  $T$ -stress:

- $T = -\frac{1}{2}\sigma_y$ ,  $K = 15000$  MPa
- $T = 0$ ,  $K = 15000$  MPa
- $T = \frac{1}{2}\sigma_y$ ,  $K = 15000$  MPa

Triaxiality dependent cohesive zone model (TCZM) formulated by Banerjee and Manivasagam (2009) was used to simulate the fracture process. Based on the work of Rashid and Banerjee (2013), the mechanical properties were  $E = 210$  GPa,  $\sigma_y = 197$  MPa and  $n = 0.228$ . The model parameters,  $C$ , a non-dimensional constant that defines an upper bound on the equivalent plastic strain required for failure at low triaxiality and  $S/C$ , another non-dimensional constant that defines the lower bound on equivalent plastic strain to failure at high triaxiality were taken as in the study and a range was defined for each parameter around these reference values for the parametric study. In order to characterize the resistance curve, two reference quantities firstly a reference stress intensity factor  $K_o$ ,  $K_o = \sqrt{E\Gamma_R(1-\nu^2)}$  such that  $\Gamma_R(\Delta a)$  is the cohesive energy i.e. the area under the curve for high biaxiality,  $\alpha = 0.8$ , representing near crack-tip stress-state. secondly, another reference length parameter  $a_o$  was defined which represents the size of the plastic zone when  $K = K_o$ , such that

$$a_o = \frac{1}{3\pi} \left( \frac{K_o}{\sigma_y} \right)^2 \quad (5)$$

The implementation of TCZM in finite element was done through user subroutine UEL. The triaxiality of the stress-state from the adjacent continuum elements was extracted using another subroutine, UVARM.

The numerical implementation of TCZM and its experimental validation has been reported by Rashid and Banerjee (2013). Further, the role of triaxiality on the mode-I resistance curves has been investigated by comparing the growth curves predicted by the stress-state dependent model and a stress-state independent cohesive model and reported in Nishant et al. (20XX).

### 3. Results and Discussion

#### 3.1. Constraint effects on resistance curves

In order to establish the ability of the triaxiality dependent cohesive model in capturing the well known effect of constraint on the resistance curves, a parametric study was performed for a range of constraint parameter,  $T$ -stress. Also, the effect of the two model parameters of the TCZM model,  $C$  and  $S/C$ , on the resistance curves was examined as presented in Figure 2 and 3. The resistance curves show the rising R-curve behavior that is typical of ductile solids. From the resistance curves developed for a range of model parameter  $C$ , with  $S/C$  held constant, as presented in Figure 2(a)-(c). at different levels of constraint, it is evident that increase in model parameter  $C$  results in increase in overall toughness of the material. Corresponding study with parameter  $C$  held constant for a range of the model parameter  $S/C$  show that with increase in parameter  $S/C$  the overall toughness decreases. The effect of loss of constraint, associated with negative  $T$ -stress, as shown in Figure 2(a) and 3(a), in increasing toughness of the material is well reproduced by the triaxiality dependent model by Banerjee and Manivasagam (2009). In contrast to the studies that simulated resistance curves using stress-state independent cohesive model (Tvergaard and Hutchinson, 1992, 1994), the resistance curves predicted by TCZM show some local instability in the crack growth process such that the growth curves have regions that correspond to slow crack growth interspersed with regions when crack has a short burst of rapid growth.

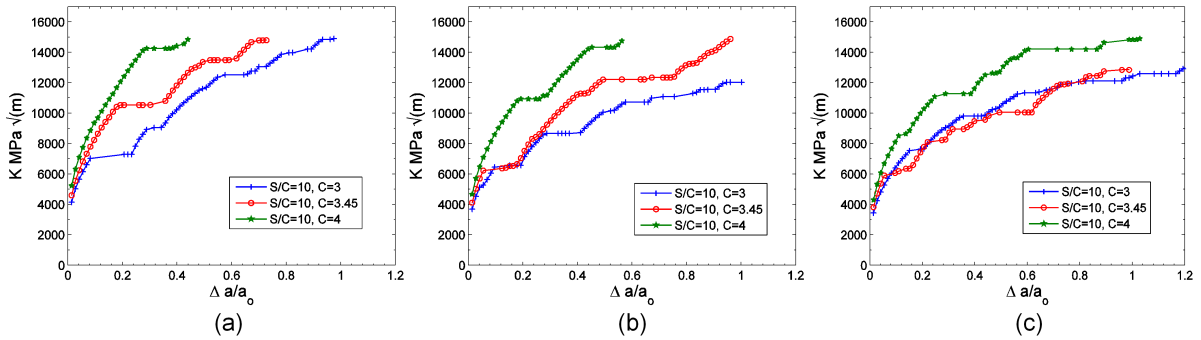


Fig. 2. Effect of model parameter  $C$  with  $S/C = 10$ , at conditions of (a) negative  $T$ -stress (b) zero  $T$ -stress (c) positive  $T$ -stress.

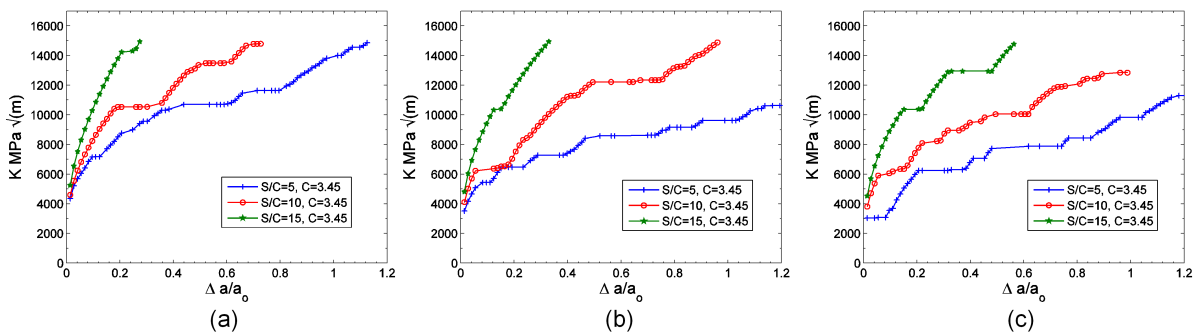


Fig. 3. Effect of model parameter  $S/C$  with  $C = 3.45$ , at conditions of (a) negative  $T$ -stress (b) zero  $T$ -stress (c) positive  $T$ -stress.

### 3.2. Positive $T$ -stress: Equivalent plastic strain

The details of the fracture process are further highlighted by comparing the plastic wake from simulations of crack growth. The equivalent plastic strain contours for simulations corresponding to negative  $T$ -stress, as in Figure 2(c) and Figure 3(c), are presented in Figures 4(a) and (b) respectively. The extent of regions with slow crack growth in the resistance curves are seen to correspond to the severity of the residual plastic strain at the crack flanks, as seen for higher  $C$  and  $S/C$  simulations. At the other end of spectrum, when the resistance curve largely constitutes regions of rapid bursts of growth, as in case of  $S/C = 5$ ,  $C = 3.45$ , the extent of crack growth is much higher and the severe localization of residual plastic strain is absent. In between these extremes, the model parameters are such that the resistance curves have distinct regions of slow and fast crack growth that is reflected in a pattern of high and low strains in the plastic wake.

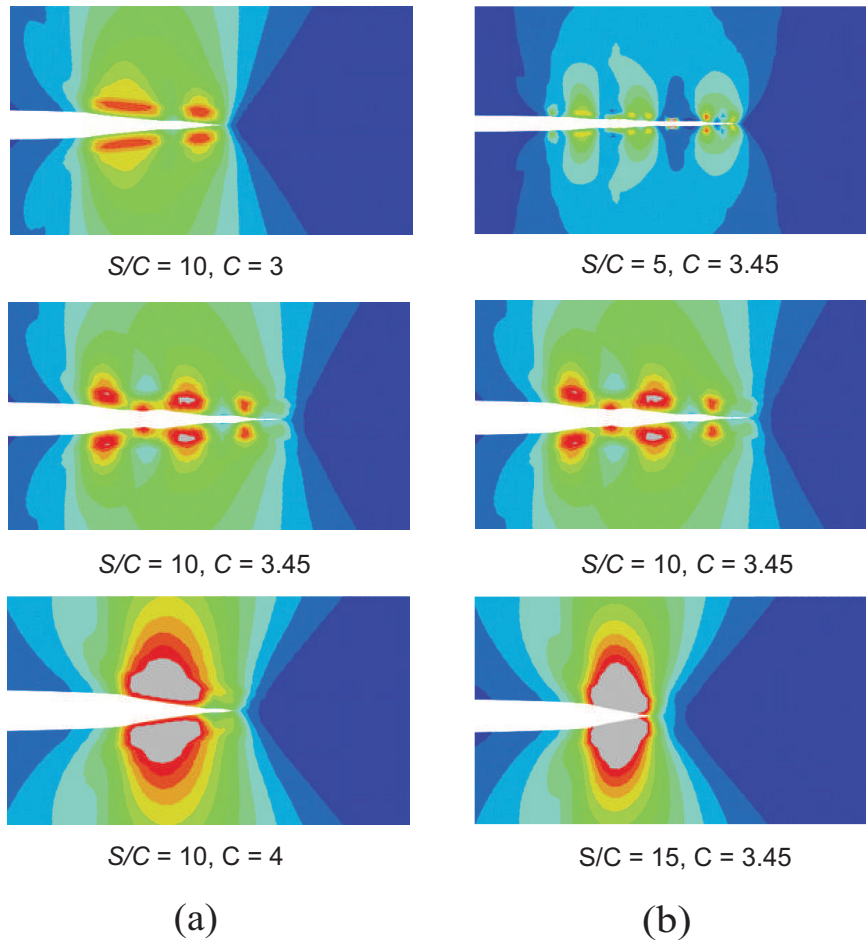


Fig. 4. Positive  $T$ -stress: contours of equivalent plastic strain

#### 4. Conclusion

Effect of constraint on plane strain, mode-I crack growth curves was simulated as per modified boundary layer formulation. Fracture process was modeled by a triaxiality dependent cohesive zone model. The model was shown to be effective in capturing the well-known effect of increase in the toughness due to loss of constraint.

The effect of model parameters,  $C$ , that defines the upperbound on plastic strain required for failure at low triaxiality and  $S/C$ , that defines a lower bound on the plastic strain sufficient for rapid damage growth at high triaxiality, is such that when cohesive energy is high the crack growth curves are smooth and have insignificant bursts of rapid crack growth.

The nature of crack growth in terms of variations in the crack growth rate are manifested in the plastic wake. Depending on the crack growth rate, whether slow or rapid, the plastic wake has presence of regions of high or low residual plastic strain respectively.

#### Acknowledgments

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