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Two-loop fermionic electroweak corrections to the Z-boson width and production rate

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ABSTRACT

Improved predictions for the Z-boson decay width and the hadronic Z-peak cross-section within the Standard Model are presented, based on a complete calculation of electroweak two-loop corrections with closed fermion loops. Compared to previous partial results, the predictions for the Z width and hadronic cross-section shift by about 0.6 MeV and 0.004 nb, respectively. Compact parametrization formulae are provided, which approximate the full results to better than 4 ppm.

The recent discovery of the Higgs boson [1] has been a tremendous success of the Standard Model (SM). Remarkably, the observed mass of the Higgs boson, M_H, agrees very well with the value that has been predicted many years earlier from electroweak (EW) precision observables, which are quantities that have been measured with very high accuracy at lower energies, and that can be calculated comparably precisely within the SM. Besides predicting the Higgs mass, global fits to all available electroweak precision observables are crucial for testing the SM at the quantum level and constraining new physics (see Refs. [2–4] for recent examples).

On the theory side, these precision tests rely on calculations of radiative corrections for the relevant observables. At the current level of precision, two-loop EW and higher-order QCD corrections are numerically important [5,6]. Complete two-loop contributions are known for the prediction of the mass of the W boson, M_W [5,7–9], and the leptonic effective weak mixing angle, $\sin^2\theta^\ell_\mathrm{eff}$ [6,7,10,11], which is related to the ratio of vector and axial-vector couplings of the Z boson to leptons. Furthermore, the leading three- and four-loop corrections to these observables in the limit of large values of the top-quark mass, m_t, have been obtained. These are EW contributions of order $\mathcal{O}(\alpha^3 m_\mathrm{t}^6)$ [12] and mixed EW/QCD terms of $\mathcal{O}(\alpha\alpha_\mathrm{s}^2)$ [13], $\mathcal{O}(\alpha^2\alpha_\mathrm{s} m_\mathrm{t}^4)$ [12], and $\mathcal{O}(\alpha\alpha_\mathrm{s}^3 m_\mathrm{t}^2)$ [14]. Similarly precise results are available for the effective weak mixing angle of quarks [11,15] and the ratio of the Z-boson partial widths into $b\bar{b}$ and all hadronic final states, $R_b \equiv \Gamma_{Z\to b\bar{b}}/\Gamma_{Z\to\mathrm{had.}}$ [16], except that the so-called *bosonic* EW two-loop corrections stemming from diagrams without closed fermion loops are not known for these quantities. However, detailed analyses and experience from the calculation of M_W and $\sin^2\theta^\ell_\mathrm{eff}$ indicates that these are small. Most of the published results have been implemented into the public code ZFITTER [17] and several private packages [4,18].

However, for the decay width and production cross-section of the Z boson, so far only approximate results for the EW two-loop corrections have been calculated in a large-m_t expansion, including the next-to-leading order $\mathcal{O}(\alpha^2 m_\mathrm{t}^2)$ [19] for leptonic final states and quarks of the first two generations, while for the $Z\to b\bar{b}$ partial width merely the leading $\mathcal{O}(\alpha^2 m_\mathrm{t}^4)$ coefficient is known [20]. It was estimated that the missing EW two-loop corrections lead to an uncertainty of several MeV for the prediction of the Z width [21], which is comparable to the experimental error, but has not been properly accounted for in the global SM fits. In this Letter, this gap is filled by presenting the complete *fermionic* EW two-loop corrections (*i.e.* from diagrams with one or two closed fermion loops) for the Z-boson width and production rate. With these new results, the theoretical uncertainty from missing higher-order contributions in electroweak fits will be significantly reduced.

The width of the Z boson is defined through the imaginary part of the complex pole $s_0 \equiv \overline{M}_\mathrm{Z}^2 - i\overline{M}_\mathrm{Z}\overline{\Gamma}_\mathrm{Z}$ of the propagator

$$\left[s - \overline{M}_\mathrm{Z}^2 + \Sigma_\mathrm{Z}(s)\right]^{-1}, \tag{1}$$

where $\Sigma_\mathrm{Z}(s)$ is the transverse part of the Z self-energy, resulting in

$$\overline{\Gamma}_\mathrm{Z} = \frac{1}{\overline{M}_\mathrm{Z}}\,\mathrm{Im}\,\Sigma_\mathrm{Z}(s_0). \tag{2}$$

This definition is consistent and gauge-invariant too all orders [22]. Expanding Eq. (2) up to next-to-next-to-leading order in the electroweak coupling and using the optical theorem leads to (see Ref. [23] for details)

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$$\overline{\Gamma}_Z = \sum_f \overline{\Gamma}_f, \quad \overline{\Gamma}_f = \frac{N_c \overline{M}_Z}{12\pi}\left[\mathcal{R}_V^f F_V^f + \mathcal{R}_A^f F_A^f\right]_{s=\overline{M}_Z^2}, \tag{3}$$

$$\begin{aligned} F_V^f = {} & v_{f(0)}^2\left[1 - \operatorname{Re}\Sigma'_{Z(1)} - \operatorname{Re}\Sigma'_{Z(2)} + \left(\operatorname{Re}\Sigma'_{Z(1)}\right)^2\right] \\ & + 2\operatorname{Re}(v_{f(0)} v_{f(1)})\left[1 - \operatorname{Re}\Sigma'_{Z(1)}\right] + 2\operatorname{Re}(v_{f(0)} v_{f(2)}) \\ & + |v_{f(1)}|^2 - \frac{1}{2}\overline{M}_Z \overline{\Gamma}_Z v_{f(0)}^2 \operatorname{Im}\Sigma''_{Z(1)}, \end{aligned} \tag{4}$$

where v_f is the effective vector $Zf\bar{f}$ coupling, which includes EW vertex corrections and Z–γ mixing contributions, and F_A^f is defined similarly in terms of the axial-vector coupling a_f. The subscripts in brackets indicate the loop order and Σ'_Z is the derivative of Σ_Z. The radiator functions $\mathcal{R}_{V,A}$ take into account final-state QED and QCD radiation and are known up to $\mathcal{O}(\alpha_s^4)$ in the limit of massless quarks and $\mathcal{O}(\alpha_s^3)$ for the kinematic mass corrections [24].

Note that the mass and width defined through the complex pole of (1) differ from the experimentally reported values, M_Z and Γ_Z, since the latter have been obtained using a Breit–Wigner formula with a running width. The two are related via

$$\overline{M}_Z = M_Z \Big/ \sqrt{1 + \Gamma_Z^2/M_Z^2}, \qquad \overline{\Gamma}_Z = \Gamma_Z \Big/ \sqrt{1 + \Gamma_Z^2/M_Z^2}, \tag{5}$$

amounting to $\overline{M}_Z \approx M_Z - 34$ MeV and $\overline{\Gamma}_Z \approx \Gamma_Z - 0.9$ MeV.

Now let us turn to the Z-boson cross-section. After subtracting contributions from photon exchange and box diagrams, the amplitude for $e^+e^- \to f\bar{f}$ can be written as an expansion about the complex pole,

$$\mathcal{A}_Z(s) = \frac{R}{s - s_0} + S + (s - s_0)S' + \cdots. \tag{6}$$

Instead of the total cross-section, it is customary to use the hadronic peak cross-section defined as

$$\sigma_{\text{had}}^0 = \frac{1}{64\pi^2 M_Z^2} \sum_{f=u,d,c,s,b} \int d\Omega \left|\mathcal{A}_Z(M_Z^2)\right|^2. \tag{7}$$

Starting from (6), an explicit calculation yields

$$\sigma_{\text{had}}^0 = \sum_{f=u,d,c,s,b} \frac{12\pi}{\overline{M}_Z^2} \frac{\overline{\Gamma}_e \overline{\Gamma}_f}{\overline{\Gamma}_Z^2}(1 + \delta X), \tag{8}$$

where δX occurs first at two-loop level [25] and is given by[1]

$$\delta X_{(2)} = -\left(\operatorname{Im}\Sigma'_{Z(1)}\right)^2 - 2\overline{\Gamma}_Z \overline{M}_Z \operatorname{Im}\Sigma''_{Z(1)}. \tag{9}$$

The calculation of the $\mathcal{O}(\alpha^2)$ corrections to $\overline{\Gamma}_Z$ and σ_{had}^0 was carried out as follows: Diagrams for the form factors $v_{f(n)}$ and $a_{f(n)}$ were generated with FeynArts 3.3 [26]. For the renormalization the on-shell scheme has been used, as described in Ref. [5]. Two-loop self-energy integrals and vertex integrals with sub-loop self-energy bubbles have been evaluated with the method illustrated in Section 3.2 of Ref. [11], while for vertex diagrams with sub-loop triangles the technique of Ref. [27] has been employed. As a cross-check, the results for $\sin^2\theta_{\text{eff}}^\ell$ [6], $\sin^2\theta_{\text{eff}}^b$ [15], and R_b [16] have been reproduced using the effective couplings $v_{f(n)}$ and $a_{f(n)}$, and very good agreement within theory uncertainties has been obtained.

For the presentation of numerical results, the one- and two-loop EW corrections have been combined with virtual loop corrections of order $\mathcal{O}(\alpha\alpha_s)$ [7], which have been re-computed for this work, and partial higher-order corrections proportional to $\alpha_t \equiv \alpha m_t^2$, of order $\mathcal{O}(\alpha_t\alpha_s^2)$ [13], $\mathcal{O}(\alpha_t^2\alpha_s)$, $\mathcal{O}(\alpha_t^3)$ [12], and $\mathcal{O}(\alpha_t\alpha_s^3)$ [14]. Final-state QED and QCD radiation has been included via the radiator functions $\mathcal{R}_{V,A}$ as described after Eq. (4). However, the factorization between EW corrections ($F_{V,A}$) and final-state radiation ($\mathcal{R}_{V,A}$) in Eq. (3) is not exact, but additional non-factorizable contributions appear first at $\mathcal{O}(\alpha\alpha_s)$ [28,29]. All fermion masses except for m_t have been neglected in the EW two-loop corrections, but a finite b quark mass has been retained in the $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$ contributions, and non-zero bottom, charm and tau masses are included in the radiators $\mathcal{R}_{V,A}$.

Table 1
Input parameters used for Tables 2 and 3, from Refs. [3,30]. Here $\Delta\alpha$ is the shift in the electromagnetic coupling due to loop corrections from leptons [31] and the five light quark flavors, $\Delta\alpha = \Delta\alpha_{\text{lept}}(M_Z) + \Delta\alpha_{\text{had}}^{(5)}(M_Z)$.

Parameter	Value	Parameter	Value
M_Z	91.1876 GeV	$m_b^{\overline{MS}}$	4.20 GeV
Γ_Z	2.4952 GeV	$m_c^{\overline{MS}}$	1.275 GeV
M_W	80.385 GeV	m_τ	1.777 GeV
Γ_W	2.085 GeV	$\Delta\alpha$	0.05900
M_H	125.7 GeV	$\alpha_s(M_Z)$	0.1184
m_t	173.2 GeV	G_μ	1.16638×10^{-5} GeV^{-2}

Table 2
Loop contributions to Γ_Z and σ_{had}^0 with fixed M_W as input parameter. Here N_f and N_f^2 refer to corrections with one and two closed fermion loops, respectively, and $\alpha_t = \alpha m_t^2$. In all rows the radiator functions $\mathcal{R}_{V,A}$ with known contributions through $\mathcal{O}(\alpha_s^4)$, $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$ are included.

	Γ_Z [MeV]	σ_{had}^0 [pb]
$\mathcal{O}(\alpha)$	60.26	−48.85
$\mathcal{O}(\alpha\alpha_s)$	9.11	3.14
$\mathcal{O}(\alpha_t\alpha_s^2, \alpha_t\alpha_s^3, \alpha_t^2\alpha_s, \alpha_t^3)$	1.20	0.48
$\mathcal{O}(N_f^2\alpha^2)$	5.13	−1.03
$\mathcal{O}(N_f\alpha^2)$	3.04	9.07

The final combined result is evaluated as a perturbative expansion in α and α_s, rather than the Fermi constant G_μ. Using the input parameters in Table 1, except G_μ, the size of various loop contributions is shown in Table 2.

However, it is common practice not to use the W mass, M_W, as an input parameter, but instead to determine M_W from G_μ. Using the results from Refs. [5,8,9] for the computation of M_W, the numbers in Table 3 are obtained. For comparison, also shown are the corresponding results based on the approximation of the EW two-loop corrections for large values of m_t [19,20].[2] The difference illustrates the impact of the full fermionic two-loop corrections beyond the large-m_t approximation, which can be seen to be of moderate size, about 0.6 MeV for Γ_Z and about 0.004 nb for σ_{had}^0.

The new results, including all corrections described above and the currently most precise result for M_W [9], can be accurately described by the simple parametrization formula

$$\begin{aligned} X = {} & X_0 + c_1 L_H + c_2\Delta_t + c_3\Delta_{\alpha_s} + c_4\Delta_{\alpha_s}^2 \\ & + c_5\Delta_{\alpha_s}\Delta_t + c_6\Delta_\alpha + c_7\Delta_Z, \\ L_H = {} & \log\frac{M_H}{125.7\text{ GeV}}, \qquad \Delta_t = \left(\frac{m_t}{173.2\text{ GeV}}\right)^2 - 1, \\ \Delta_{\alpha_s} = {} & \frac{\alpha_s}{0.1184} - 1, \qquad \Delta_\alpha = \frac{\Delta\alpha}{0.059} - 1, \\ \Delta_Z = {} & \frac{M_Z}{91.1876\text{ GeV}} - 1. \end{aligned} \tag{10}$$

[1] Note that Eq. (9) differs from the expression shown in Ref. [25] since the non-resonant terms in Eq. (6) were not included there. See Ref. [23] for details.

[2] These numbers have been kindly supplied by S. Mishima based on the work in Ref. [4].

Table 3
Results for Γ_Z and σ^0_{had}, where in contrast to Table 2 M_W has been calculated from G_μ using the same order of perturbation theory as indicated in each line. In all cases, the complete radiator functions $\mathcal{R}_{V,A}$ are included. The last two lines show the comparison between the result based on the full fermionic two-loop EW corrections and the large-m_t approximation [4,19,20]. For consistency of the comparison, the small $\mathcal{O}(\alpha_t\alpha_s^3)$ contribution has been removed in the next-to-last line, since this part is also missing in the last line, but the three-loop corrections or order $\mathcal{O}(\alpha_t\alpha_s^2, \alpha_t^2\alpha_s, \alpha_t^3)$ are included in both.

	Γ_Z [GeV]	σ^0_{had} [nb]
Born $+\mathcal{O}(\alpha)$	2.49769	41.4687
$+\mathcal{O}(\alpha\alpha_s)$	2.49648	41.4758
$+\mathcal{O}(\alpha_t\alpha_s^2, \alpha_t\alpha_s^3, \alpha_t^2\alpha_s, \alpha_t^3)$	2.49559	41.4770
$+\mathcal{O}(N_f^2\alpha^2, N_f\alpha^2)$	2.49423	41.4884
$-\mathcal{O}(\alpha_t\alpha_s^3)$	2.49430	41.4882
Large-m_t exp. for EW 2-loop	2.49485	41.4840

The coefficients are given by

$$X = \Gamma_Z\,[\text{MeV}]:\quad X_0 = 2494.24,\quad c_1 = -2.0,\quad c_2 = 19.7,\quad c_3 = 58.60,\quad c_4 = -4.0,\quad c_5 = 8.0,\quad c_6 = -55.9,\quad c_7 = 9267; \tag{11}$$

$$X = \sigma^0_{had}\,[\text{pb}]:\quad X_0 = 41488.4,\quad c_1 = 3.0,\quad c_2 = 60.9,\quad c_3 = -579.4,\quad c_4 = 38.1,\quad c_5 = 7.3,\quad c_6 = 85.4,\quad c_7 = -86\,027. \tag{12}$$

This formula approximates the full results to better than 0.01 MeV and 0.1 pb, respectively, for the input parameters within the ranges $M_H = 125.7 \pm 2.5$ GeV, $m_t = 173.2 \pm 2.0$ GeV, $\alpha_s = 0.1184 \pm 0.0050$, $\Delta\alpha = 0.0590 \pm 0.0005$ and $M_Z = 91.1876 \pm 0.0042$ GeV.

In summary, the complete fermionic two-loop electroweak corrections to the Z-boson decay width and the cross-section for $e^+e^- \to$ hadrons have been computed within the Standard Model. Compared to known previous calculations, the new contributions lead to shifts of 0.6 MeV in Γ_Z and 0.004 nb in σ^0_{had}, which are smaller then, but of comparable order of magnitude as the current experimental uncertainties (2.3 MeV and 0.037 nb [30]). Therefore, the new results are important for robust predictions of these quantities, while the remaining theory uncertainty is estimated to be relatively small. It mainly stems from the missing bosonic $\mathcal{O}(\alpha^2)$ contributions and $\mathcal{O}(\alpha^2\alpha_s)$, $\mathcal{O}(\alpha^3)$ and $\mathcal{O}(\alpha\alpha_s^2)$ corrections beyond the large-m_t approximation, leading to the estimates $\delta\Gamma_Z \approx 0.5$ MeV and $\delta\sigma^0_{had} \approx 0.006$ nb [23].

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