Decentralized robust model reference adaptive control for interconnected time-delay systems

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Abstract

In this paper, the problem of decentralized robust model reference control for a class of interconnected time-delay systems is investigated. The interconnections with time-varying time delays considered are high order and the gains are not known. A class of decentralized adaptive feedback controllers are proposed, which can render the resulting closed-loop error system uniformly ultimately bounded stable. A numerical example is given to show the feasibility and effectiveness of the proposed design techniques.

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1. Introduction

Various engineering systems, such as electrical networks, turbojet engines, microwave oscillators, nuclear reactors, and hydraulic systems, have the characteristics of time delay. Due to the effect of time delay, these systems may possess instability, the control performance of these systems are hardly assured. So far, the stability analysis and robust control for dynamic time-delay systems attracted a number of researchers over the past years, see for example, [1–8,10–24] and references therein.

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On the other hand, in the real world many practical systems are found to be large-scale systems which are composed of a set of interconnected subsystems, such as power systems, digital communication networks, economic systems and urban traffic networks. Robust control for large-scale time-delay systems have been one of the focused study topics in the past years, and a lot of achievements have been made, see for example, [6,10,14,19–23]. In [10], the problem of robust control for a class of interconnected systems with bounded uncertainties was considered. The same system was further discussed by using the decentralized sliding mode control method in [6]. The problem of stabilization of large-scale stochastic systems with time delay was studied in [22], while stabilization of a class of time-varying large-scale systems subject to multiple time-varying delays in the interconnections was investigated in [14]. In the work [23], the robust control problem was investigated by using a linear function as a bound for the uncertain interconnections and the controller was designed based on the bounds. The adaptive control problem of a class of interconnected time-delay systems without knowledge of bounds of uncertain interconnections was considered in [21].

In another research front line, model reference approach has been extensively studied and widely used in control problem and its applications. However, to the best of the authors’ knowledge, very few attempts have been made to tackle time-delay systems by using model reference method. Oucheriah [13] firstly investigated the problem and designed the controller for a class of uncertain time-delay systems. Model reference adaptive control for interconnected systems with time delays are considered in [2], but the obtained controllers are dependent on the delays and the interconnections need to be known precisely.

In this paper we consider a class of nonlinear interconnected time-delay systems. The uncertain interconnections are bounded by high-order nonlinear functions and the gains are unknown. The model reference adaptive control problem is studied. Decentralized adaptive feedback controller, which is independent of the time delays and can render the closed-loop error system uniformly ultimately bounded stable is designed. Numerical simulation is presented to show the potential of the proposed techniques.

2. System formulation and preliminaries

The interconnected system considered in this paper includes \( N \) subsystems with the \( i \)th subsystem described by

\[
S_i : \dot{x}_i = A_i x_i + B_i u_i + \sum_{j=1}^{N} H_{ij}(x_j, x_j(t - d_{ij}(t)), t),
\]

where \( x_i \in \mathbb{R}^{n_i} \) and \( u_i \in \mathbb{R}^{m_i} \) represent the state and control vectors, respectively, of the subsystem. \( A_i \) and \( B_i \) are constant matrices with proper dimensions. \( H_{ij}(x_j, x_j(t - d_{ij}(t)), t) \) are uncertain nonlinear interconnections, which indicate the interconnections among the current states and the delayed states of systems \( S_i \) and \( S_j \), while \( d_{ij}(t) \) are bounded time-varying delays, are differentiable and satisfy

\[
0 \leq d_{ij}(t) \leq d_{ij} < \infty, \quad \dot{d}_{ij}(t) \leq d_{ij}^b < 1,
\]

where \( d_{ij} \) are the delays and \( d_{ij}^b \) are the upper bounds of the delays.
where \(d_{ij}\) and \(d_{ij}^*\) are positive scalars, and initial conditions are given as follows:

\[
\begin{align*}
\dot{x}_i(t) &= \Omega_i(t), \quad t \in [t_0 - d_{ij}, t_0], \quad i = 1, 2, \ldots, N,
\end{align*}
\]

where \(\Omega_i(t)\) are continuous functions. For the purpose of model reference, the local reference model of the \(i\)th subsystem is given by

\[
\dot{x}_{mi}(t) = A_{mi}x_{mi} + B_{mi}r_i(t),
\]

where \(x_{mi} \in \mathbb{R}^{n_i}\) is the state vector, \(r_i(t)\) is the known piecewise continuous and bounded reference input to the \(i\)th reference model. \(A_{mi}\) and \(B_{mi}\) are known matrices.

Further from (1) and (3), we obtain the following error system:

\[
\dot{e}_i = A_{mi}e_i + (A_i - A_{mi})x_i + B_iu_i + \sum_{j=1}^{N} H_{ij}(x_j, x_j(t - d_{ij}(t)), t) - B_{mi}r_i(t),
\]

where \(e_i = x_i - x_{mi}\), we assume the initial value is \(e_i(t) = \psi_i(t)\) for \(t \in [t_0 - d_{ij}, t_0]\).

In the following, some standard assumptions are imposed on system (4).

**Assumption 1.** There exist matrices \(K_i\) and positive matrices \(P_i\) satisfying the following inequality for \(i = 1, 2, \ldots, N\)

\[
(A_{mi} + B_{i}K_i)^T P_i + P_i(A_{mi} + B_{i}K_i) = -Q_i,
\]

where \(Q_i (i = 1, \ldots, N)\) are positive matrices.

**Assumption 2.** The following conditions are satisfied:

\[
\begin{align*}
H_{ij}(x_j, x_j(t - d_{ij}(t)), t) &= B_{i}\tilde{H}_{ij}(x_j, x_j(t - d_{ij}(t)), t), \\
(A_i - A_{mi}) &= B_{i}N_i, \\
B_{mi} &= B_{i}M_i,
\end{align*}
\]

where \(\tilde{H}_{ij}(\cdot)\) are proper vector function, \(M_i\) and \(N_i\) are constant matrices.

**Assumption 3.** The interconnections satisfy the following inequalities:

\[
\sum_{j=1}^{N} \|\tilde{H}_{ij}(x_j, x_j(t - d_{ij}(t)), t)\| \leq \sum_{j=1}^{N} \sum_{s=1}^{P_{ij}} z_{ij} \|x_j\|^s + \sum_{j=1}^{N} \sum_{l=1}^{Q_{ij}} \beta_{ijl} \|x_j(t - d_{ij}(t))\|^l,
\]

where \(p_{ij}\) and \(q_{ij}\) are known positive scalars, \(z_{ij}\) and \(\beta_{ijl}\) are unknown scalars.

**Assumption 4.** The states of model reference system (3) are bounded.

**Remark 1.** It should be noted that Assumption 1 is to guarantee that the pair \(\{A_{mi}, B_i\}\) can be stabilized. If \(\{A_{mi}, B_i\}\) is completely controllable, Assumption 1 will always hold. Assumption 2 is the so-called matching condition which has been widely used in robust control and filtering problems (see for example, [7,11,12,19,20,17]). Unlike in the existing literature investigating the control problem of interconnected
time-delay systems, we assume that the interconnected terms are bounded by high-order functions and the gains are unknown in Assumption 3. Therefore, the results obtained in this paper will be applicable to a large class of interconnected time-delay systems. Assumption 4 is to assure that the underlying model reference system is bounded stable.

For interconnected time-delay system (1) satisfying above assumptions, we will propose a class of decentralized adaptive feedback controllers to achieve the model reference’s objective.

**Definition.** The solution \( e(t, t_0, \psi) \) of interconnected system (4) is said to be uniformly ultimately bounded with respect to the bound \( \psi \) if for each \( \delta > 0 \) there exists \( T = T(\psi, \delta) > 0 \) independent of \( t_0 \) such that \( \| e(t, t_0, \psi) \| \leq \psi \) for all \( t \geq t_0 + T \) when \( \| x_{t_0} \| < \delta \).

3. Main results

From Assumption 3, we further obtain the following inequalities with the help of Assumption 4:

\[
\sum_{j=1}^{N} \| \tilde{H}_{ij}(x_j, x_j(t - d_{ij}(t)), t) \| \\
\leq \sum_{j=1}^{N} \sum_{s=1}^{p_{ij}} \alpha_{ijs} \| x_j \|^s + \sum_{j=1}^{N} \sum_{l=1}^{q_{ij}} \beta_{ijl} \| x_j(t - d_{ij}(t)) \|^l \\
= \sum_{j=1}^{N} \sum_{s=1}^{p_{ij}} \alpha_{ijs} \| x_{mj} \| + e_j \| ^s + \sum_{j=1}^{N} \sum_{l=1}^{q_{ij}} \beta_{ijl} \| x_{mj}(t - d_{ij}(t)) + e_j(t - d_{ij}(t)) \|^l \\
\leq \sum_{j=1}^{N} \sum_{s=1}^{p_{ij}} \tilde{\alpha}_{ijs} \| e_j \|^s + \sum_{j=1}^{N} \sum_{l=1}^{q_{ij}} \tilde{\beta}_{ijl} \| e_j(t - d_{ij}(t)) \|^l + \delta_i \\
= \sum_{j=1}^{N} \tilde{\alpha}_{ij}^T U_{ij}(\| e_j \|) + \sum_{j=1}^{N} \tilde{\beta}_{ij}^T W_{ij}(\| e_j(t - d_{ij}(t)) \|) + \delta_i, \tag{7}
\]

where \( \tilde{\alpha}_{ijs}, \tilde{\beta}_{ijl} \) and \( \delta_i \) are unknown positive scalars, and

\[
\tilde{\alpha}_{ij} = (\tilde{\alpha}_{i1j}, \tilde{\alpha}_{i2j}, \ldots, \tilde{\alpha}_{ip_ij})^T, \quad \tilde{\beta}_{ij} = (\tilde{\beta}_{i1j}, \tilde{\beta}_{i2j}, \ldots, \tilde{\beta}_{iq_ij})^T, \\
U_{ij}(\cdot) = (\| e_j \|, \| e_j \|^2, \ldots, \| e_j \|^{p_{ij}})^T, \\
W_{ij}(\cdot) = (\| e_j(t - d_{ij}(t)) \|, \| e_j(t - d_{ij}(t)) \|^2, \ldots, \| e_j(t - d_{ij}(t)) \|^{q_{ij}})^T.
\]

Since the states \( x_{mi} \) of reference model system are bounded, there always exist positive scalars \( \tilde{\alpha}_{ijs}, \tilde{\beta}_{ijl} \) and \( \delta_i \) such that inequality (7) holds.

Now, we are ready to present our main result in this paper.
Theorem 1. For system (1), the following adaptive feedback controller

\[ u_i = u_{i1} + u_{i2} + u_{i3}, \]  

where \( u_{i1} = -N_i x_i + M_i r_i + K_i e_i, \)

\[ u_{i2} = -\theta_i(t) B_i^T \frac{\partial V_i(e_i)}{\partial e_i}, \]

\[ u_{i3} = -\vartheta_i(t) B_i^T \frac{\partial V_i(e_i)}{\partial e_i} \frac{\|B_i^T \partial V_i^T(e_i) / \partial e_i\|}{\|B_i^T \partial V_i^T(e_i) / \partial e_i\|} \]  

in which \( \theta_i(t) \) and \( \vartheta_i(t) \) are adaptive parameters with adaptive laws

\[ \dot{\theta}_i = \frac{1}{2} \Gamma_i \left\| B_i^T \frac{\partial V_i(e_i)}{\partial e_i} \right\|^2 - \frac{1}{2} \Gamma_i \sigma_1 \theta_i, \]

\[ \dot{\vartheta}_i = \frac{1}{2} \Phi_i \left\| B_i^T \frac{\partial V_i(e_i)}{\partial e_i} \right\|^2 - \frac{1}{2} \Phi_i \sigma_2 \vartheta_i, \]  

where \( \Gamma_i, \Phi_i, \sigma_1 \) and \( \sigma_2 \) are positive scalars, and

\[ V_i(e_i) = \sum_{k=1}^{h_i} \frac{1}{k} (e_i^T P_i e_i)^k, \quad h_i = \max\{p_{ji}, q_{ji}\} (j \in [1, N]), \]  

\( K_i \) and \( P_i \) are matrices satisfying (5), will render the closed-loop error system uniformly ultimately bounded stable.

Proof. Define the following Lyapunov function candidate:

\[ \tilde{V}(e, \theta, t) = \sum_{i=1}^{N} \tilde{V}_i(e, \theta, t) = \sum_{i=1}^{N} \left\{ V_i(e_i) + \Gamma_i^{-1} (\theta_i - \tilde{\theta}_i)^2 + \Phi_i^{-1} (\vartheta_i - \tilde{\vartheta}_i)^2 + \sum_{j=1}^{N} v_{ij} \int_{t-d_{ij}(t)}^{t} \|e_j(\xi)\|^{2k} d\xi \right\}, \]  

where \( v_{ij} \) are sufficiently small positive scalars, and \( \tilde{\theta}_i \) and \( \tilde{\vartheta}_i \) are also positive scalars defined in (15) (below). In [9], similar Lyapunov function is employed to investigate the control problem of nonlinear systems free of time delays. In this paper, we will prove the stability of closed-loop system based on the function.
By taking the time derivative of \( \hat{V}(\cdot) \) along the trajectories of the closed-loop system, we have

\[
\hat{V}(e, \theta, t) = \sum_{i=1}^{N} \hat{V}_i(e, \theta, t) \\
\leq \sum_{i=1}^{N} \sum_{j=1}^{h_i} \left( \epsilon_i^T P_i e_i \right)^{k-1} \epsilon_i^T (A_{mi} + B_i K_i) \epsilon_i \\
+ \sum_{i=1}^{N} \left( \frac{\partial V_i(e_i)}{\partial e_i} B_i \sum_{j=1}^{N} \hat{H}_{ij}(x_j, x_j(t - d_{ij}(t)), t) \right) \\
+ \sum_{i=1}^{N} \sum_{j=1}^{N} v_{ij}\left(\|e_j\|^2 - (1 - d_{ij}^*)\|e_j(t - d_{ij}(t))\|^2\right)^{2k} \\
+ \sum_{i=1}^{N} \left( 2r_i^{-1}(\theta_i - \bar{\theta}_i)\dot{\theta}_i - 2\Phi_i^{-1}(\delta_i - \bar{\theta}_i)\dot{\delta}_i + \frac{\partial V_i(e_i)}{\partial e_i} B_i u_{ij} \right) \\
- \sum_{i=1}^{N} \theta_i \left\| B_i^T \frac{\partial V_i(e_i)}{\partial e_i} \right\|^2.
\] (13)

Also, the following inequality holds

\[
\frac{\partial V_i(e_i)}{\partial e_i} B_i \sum_{j=1}^{N} \hat{H}_{ij}(x_j, x_j(t - d_{ij}(t)), t) \\
\leq \sum_{j=1}^{N} \bar{a}_{ij} U_{ij}(\|e_j\|) \left\| B_i^T \frac{\partial V_i(e_i)}{\partial e_i} \right\| + \delta_i \left\| B_i^T \frac{\partial V_i(e_i)}{\partial e_i} \right\| \\
+ \sum_{j=1}^{N} \bar{b}_{ij} W_{ij}(\|e_j(t - d_{ij}(t))\|) \left\| B_i^T \frac{\partial V_i(e_i)}{\partial e_i} \right\| \\
\leq \sum_{j=1}^{N} \left( \frac{\|\bar{a}_{ij}\|^2}{4\epsilon_i j} \left\| B_i^T \frac{\partial V_i(e_i)}{\partial e_i} \right\|^2 + \epsilon_{ij} U_{ij}(\|e_j\|)\| \right)^2 \right) \\
+ \sum_{j=1}^{N} \left( \frac{\|\bar{b}_{ij}\|^2}{4v_{ij}(1 - d_{ij}^*)} \left\| B_i^T \frac{\partial V_i(e_i)}{\partial e_i} \right\|^2 + \epsilon_{ij} W_{ij}(\|e_j(t - d_{ij}(t))\|)\| \right)^2 \right) \\
+ \delta_i \left\| B_i^T \frac{\partial V_i(e_i)}{\partial e_i} \right\|^2.
\] (14)
Let

\[ \tilde{\theta}_i = \sum_{j=1}^{N} \left( \frac{\|x_{ij}\|^2}{4\epsilon_{ij}} + \frac{\|\beta_{ij}\|^2}{4(1 - d_{ij}^*)v_{ij}} \right). \]  

(15)

Then, we have

\[ \frac{\partial V_i(e_i)}{\partial e_i} B_i \sum_{j=1}^{N} \tilde{H}_i(x_j, x_j(t - d_{ij}(t)), t) \leq \tilde{\theta}_i \left\| B_i^T \frac{\partial V_i(e_i)^T}{\partial e_i} \right\|^2 + \delta_i \left\| B_i^T \frac{\partial V_i(e_i)^T}{\partial e_i} \right\| \]

\[ + \sum_{j=1}^{N} \left( \sum_{k=1} {q_{ij}} (1 - d_{ij}^*)v_{ij} \|e_j(t - d_{ij}(t))\|^2 + \sum_{k=1} {p_{ij}} \|e_j\|^2 \right). \]  

(16)

We know

\[ \frac{\partial V_i(e_i)}{\partial e_i} B_i u_i = -\dot{\vartheta}_i(t) \left\| B_i^T \frac{\partial V_i(e_i)^T}{\partial e_i} \right\|. \]  

(17)

Substituting (16) and (17) into (13), we further have

\[ \dot{\tilde{V}} (x, 0, t) = \sum_{i=1}^{N} \tilde{V}_i(e_i, 0, t) \]

\[ \leq \sum_{i=1}^{N} \left[ h_i \sum_{k=1}^{h_i} (e_i^T P_i e_i)^{k-1} (-e_i^T Q_i e_i) + \sum_{j=1}^{N} \left( \sum_{k=1} {q_{ij}} v_{ij} \|e_j\|^2 + \sum_{k=1} {p_{ij}} \|e_j\|^2 \right) \right] \]

\[ + \sum_{i=1}^{N} (2\Phi_i^{-1}(\tilde{\theta}_i - \tilde{\vartheta}_i)\dot{\theta}_i - 2\Phi_i^{-1}(\dot{\delta}_i - \dot{\theta}_i)\dot{\theta}_i) - \sum_{i=1}^{N} (\dot{\theta}_i - \tilde{\theta}_i) \left\| B_i^T \frac{\partial V_i^T(e_i)}{\partial e_i} \right\|^2 \]

\[ + \sum_{i=1}^{N} (\dot{\delta}_i - \dot{\theta}_i(t)) \left\| B_i^T \frac{\partial V_i^T(e_i)}{\partial e_i} \right\|. \]

By applying (10), one has

\[ \dot{\tilde{V}} (x, 0, t) \leq \sum_{i=1}^{N} \left\{ - \sum_{k=1}^{h_i} (e_i^T P_i e_i)^{k-1} (e_i^T Q_i e_i) + (\tilde{\delta}_i - \tilde{\theta}_i)(-\sigma_{1i}\tilde{\theta}_i) + (\delta_i - \dot{\theta}_i(t))(\sigma_{2i}\tilde{\theta}_i) \right\} \]

\[ + \sum_{j=1}^{N} \left( \sum_{k=1} {q_{ij}} v_{ij} \|e_j\|^2 + \sum_{k=1} {p_{ij}} \|e_j\|^2 \right) \].

If we choose parameters

\[ v_j = \max_i \{v_{ij}\}, \quad \epsilon_j = \max_i \{\epsilon_{ij}\} \quad \text{for} \quad i \in [1, N], \]  

(18)
the following inequalities hold:

\[
\sum_{i=1}^{N} \left\{ -\sum_{k=1}^{h_i} (e_i^T P_i e_i)^{k-1} (e_i^T Q_i e_i) + \sum_{j=1}^{N} \left( \sum_{k=1}^{q_{ij}} v_{ij} \| e_j \|^{2k} + \sum_{k=1}^{p_{ij}} \epsilon_{ij} \| e_j \|^{2k} \right) \right\} \\
\leq \sum_{i=1}^{N} \left\{ -\sum_{k=1}^{h_i} (e_i^T P_i e_i)^{k-1} (e_i^T Q_i e_i) + \sum_{j=1}^{N} \left( \sum_{k=1}^{h_j} v_{ij} \| e_j \|^{2k} + \sum_{k=1}^{h_j} \epsilon_{ij} \| e_j \|^{2k} \right) \right\} \\
= \sum_{i=1}^{N} \sum_{k=1}^{h_i} \left\{ -(e_i^T P_i e_i)^{k-1} (e_i^T Q_i e_i) + N \| v_i \| e_i \|^{2k} + N \| \epsilon_i \| e_i \|^{2k} \right\},
\]

where \( v_j = 0 \) (\( q_{ij} < j \leq h_j \)) and \( \epsilon_j = 0 \) (\( p_{ij} < j \leq h_j \)). Considering the following equality:

\[
(\theta_i - \bar{\theta}_i)(-\sigma_{1i} \theta_i) + (\sigma_{2i} \bar{\theta}_i) = -\sigma_{1i} \left( \theta_i - 2^{-1} \bar{\theta}_i \right)^2 + \frac{1}{4} \sigma_{1i} \bar{\theta}_i^2 \\
- \sigma_{2i} \left( \sigma_{2i} - 2^{-1} \bar{\theta}_i \right)^2 + \frac{1}{4} \sigma_{2i} \bar{\theta}_i^2,
\]

we further have

\[
\dot{\hat{V}}(e, \theta, t) \leq \sum_{i=1}^{N} \left( \sum_{k=1}^{h_i} \left\{-\lambda_{\min}(P_i)^{k-1} \lambda_{\min}(Q_i) \| e_i \|^{2k} + N v_i \| e_i \|^{2k} + N \| \epsilon_i \| e_i \|^{2k} + \frac{1}{4} \sigma_{1i} \bar{\theta}_i^2 + \frac{1}{4} \sigma_{2i} \bar{\theta}_i^2 \right\}\right).
\]

(19)

For \( P_i \) and \( Q_i \) are positive matrices, parameters \( v_{ij} \) and \( \epsilon_{ij} \) can be selected to be small enough to render that the following inequality holds:

\[
-\lambda_{\min}(P_i)^{k-1} \lambda_{\min}(Q_i) + N v_i + N \| \epsilon_i \| e_i \|^{2k} - \Pi_i < 0,
\]

(20)

where \( \Pi_i \) are positive scalars. Furthermore, one has

\[
\dot{\hat{V}}(e, \theta, t) \leq \sum_{i=1}^{N} \left\{-h_i \Pi_i \| e_i \|^{2k} + \frac{1}{4} \sigma_{2i} \bar{\theta}_i^2 + \frac{1}{4} \sigma_{1i} \bar{\theta}_i^2 \right\}.
\]

(21)

Based on Lyapunov stability theory, the proposed decentralized state feedback controller (8)–(10) will guarantee the closed-loop error system is uniformly ultimately bounded stable. \( \square \)

**Remark 2.** For the controller \( u_{i3} \) in (9), we can choose the following candidate to avoid zero appearing in the denominator:

\[
u_{i3} = \frac{-\sigma_i(t) B_i^T \partial_i V_i^T(e_i) / \delta_i e_i}{\| B_i^T \partial_i V_i^T(e_i) / \delta_i e_i \| + \alpha e^{-rt}},
\]
where $\varepsilon$ and $r$ are positive scalars. With the controller above, it is easy for us to obtain the closed-loop system is uniformly ultimately bounded stable.

**Remark 3.** For inequality (21), it is easy for us to obtain that the stable bounds of error $e$ can be rendered sufficiently small by reducing the values of parameters $\sigma_{1i}$ and $\sigma_{2i}$.

**Remark 4.** In this section we have investigated the control problem for interconnected time-delay systems with the uncertainties bounded by high-order polynomials. With the gains unknown, we employed adaptive control idea and designed the controllers. Specially, if the uncertainties are bounded by first-order polynomials, that is: $p_{ij} = q_{ij} = 1$, the model reference adaptive control problem is considered in [2]. In [2] the time delay interconnections are needed to be precisely known and the controllers designed are dependent of the time delays. In this paper, the conditions imposed on the interconnected systems are looser and the controllers constructed are independent of the time delays. Therefore, the results obtained in this part are expected to solve the decentralized model reference control problem for a larger class of interconnected time-delay systems.

### 4. Numerical example

In this section, we give a numerical example to verify the validity of the controller designed in previous section. Consider the following interconnected time-delay system:

**Subsystem I:**

$$
\dot{x}_1 = \begin{pmatrix} -1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u + \begin{pmatrix} 0 \\ \delta_{11}x_{11} + \delta_{12}x_{22} + \delta_{13}x_{12}x_{21}(t - 0.5) + \delta_{14}x_{11}^2(t - 0.25(1 + \sin(t))) \end{pmatrix}.
$$

**Subsystem II:**

$$
\dot{x}_2 = \begin{pmatrix} -1 & 0 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} x_{21} \\ x_{22} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u + \begin{pmatrix} 0 \\ \delta_{21}x_{21} + \delta_{22}x_{12}^2 + \delta_{24}x_{12}^2(t - 1) + \delta_{23}x_{12}x_{22}(t - 0.5(1 + \cos(t))) \end{pmatrix},
$$

where the parameters $\delta_{ij}$ are unknown parameters. Based on the theorem proposed in this paper, we will design the decentralized adaptive feedback controller. The desired reference models are selected as

$$
\dot{x}_{m1} = \begin{pmatrix} -1 & 0 \\ -6 & -5 \end{pmatrix} \begin{pmatrix} x_{m11} \\ x_{m12} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r_1(t),
$$

$$
\dot{x}_{m2} = \begin{pmatrix} -1 & 0 \\ -4 & -5 \end{pmatrix} \begin{pmatrix} x_{m21} \\ x_{m22} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r_2(t),
$$

$$
r_1(t) = 100 \sin(t), \quad r_2(t) = 100 \cos(t).
$$
Therefore, we obtain the following controller from Theorem 1:

\[
u_1 = -(9x_{11} + 6x_{12}) - 20\theta_1(t)(e_{12} + (e_{11}^2 + e_{12}^2)e_{12}) + r_1(t) - 10e_{12}
\]

\[\frac{2\dot{\theta}_1(t)(e_{12} + (e_{11}^2 + e_{12}^2)e_{12})}{\|e_{12} + (e_{11}^2 + e_{12}^2)e_{12}\|},\]

\[
u_2 = -(x_{21} + 8x_{22}) - 20\theta_2(t)(e_{22} + (e_{21}^2 + e_{22}^2)e_{22}) + r_2(t)
\]

\[-10e_{22} - \frac{2\dot{\theta}_2(t)(e_{22} + (e_{21}^2 + e_{22}^2)e_{22})}{\|e_{22} + (e_{21}^2 + e_{22}^2)e_{22}\|}\]

and the adaptive laws

\[\dot{\theta}_1 = \|e_{12} + (e_{11}^2 + e_{12}^2)e_{12}\|^2 - 0.01\theta_1, \quad \dot{\theta}_2 = \|e_{22} + (e_{21}^2 + e_{22}^2)e_{22}\|^2 - 0.01\theta_2,\]

\[\dot{\varphi}_1 = 2\|e_{12} + (e_{11}^2 + e_{12}^2)e_{12}\| - 0.01\varphi_1, \quad \dot{\varphi}_2 = 2\|e_{22} + (e_{21}^2 + e_{22}^2)e_{22}\| - 0.01\varphi_2.\]

We choose \(\delta_{ij} = 1\) and the initial conditions are

\[x_{11} = 8, \quad x_{12} = 4, \quad x_{21} = -4, \quad x_{22} = -8, \quad x_{m11} = 1, \quad x_{m12} = 3, \quad x_{m21} = -1, \quad x_{m22} = -2.\]

The simulation results are shown in Figs. 1–4, from which we can see that the decentralized feedback controller can render the states of the controlled system quickly track the states of model reference system. Specially, we consider that the signals \(r_1(t)\) and \(r_2(t)\) are chosen as \(r_1(t) = r_2(t) = 0.\) It is easy to get
that the reference model is asymptotically stable. With the designed controller, the states response of the closed loop system are shown in Fig. 5, from which we can see that the closed-loop system is uniformly ultimately bounded stable.
5. Conclusion

In this paper, model reference adaptive control problem for a class of large-scale time-delay systems is investigated. The decentralized feedback controllers and corresponding adaptive laws are designed. Based
on Lyapunov stability theory, we prove the resulting closed-loop error system is uniformly ultimately bounded stable. A numerical example is given to verify the feasibility and validity of the main results. Unlike the existing literature, in this paper the uncertain interconnections with time-varying time delays are bounded by high-order nonlinear functions and the gains need not to be known. Therefore, the results obtained are expected to apply to a large class of interconnected systems.

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