Modelling and Forecasting the Residential Electricity Consumption in Brazil with Pegels Exponential Smoothing Techniques

P.M. Maçaira\textsuperscript{a*}, R.C. Souza\textsuperscript{b}, F.L. Cyrino Oliveira\textsuperscript{a}

\textsuperscript{a}Pontifical Catholic University of Rio de Janeiro, Industrial Engineering Department - PUC-Rio, Rio de Janeiro, RJ 22453-900, Brazil
\textsuperscript{b}Pontifical Catholic University of Rio de Janeiro, Electrical Engineering Department - PUC-Rio, Rio de Janeiro, RJ 22453-900, Brazil

Abstract

The importance of the residential class in the consumption of electricity in the Brazilian Electric System (BES) can be recognized by its quantitative size, as it, in 2013, concentrates 27% of the total consumption and 85% among all consumers. Also, in this class are the main public policies such as subsidies for consumer units inhabited by low-income families, labelling and increased energy efficiency of appliances used in the home and others. This paper aims to model and forecast the Brazilian residential energy consumption, up to 2050, with Pegels exponential smoothing techniques. In addition to the forecasts with the best model in sample, an optimization procedure of the model’s hyper parameters is carried out in order to adjust the projections provided by the Energy Research Company (ERC). The results obtained show that it is possible to predict satisfactorily the electricity consumption for the proposed horizon with minimum error in sample. And the exercise of optimization proved to be important for providing level and trend equations for the official expectations regarding the residential electricity consumption in Brazil.

Keywords: Brazilian residential electricity sector; Exponential Smoothing; Pegels.

1. Introduction

The Brazilian Electric System (BES) is divided in four basic classes of consumption, namely: industrial, residential, commercial and others. The residential class has its importance, in the total consumption of electricity, recognized by its quantitative size, as it, in 2013, concentrates 27% of the total consumption and 85% among all...
consumers [1]. Also, in this class are the main public policies such as subsidies for consumer units inhabited by low-income families, labelling and increased energy efficiency of appliances used in the home and others [2].

Due to this importance, it is essential have accuracy forecasts of the residential consumption to a good future planning of the sector. The Energy Research Company (ERC for short) is in charge of publishing two official reports: “The Ten Year Energy Planning” (PDE) and “The National Energy Planning” (PNE) which contain, among other things, the forecast for longer lead times of the Brazilian electricity consumption. In the present formulation these horizons are 2023 [1] and 2050 [3]. Both studies provide forecasts regarding economics and energy demand, together with the desired growth of the matrix of generation and fuel. For the PDE the latest version has annual estimates for the years between 2014 and 2023 inclusive, being called PDE 2014-2023. The most recent PNE forecasts up to 2050 (PNE 2050) and publish the expected consumption for the years 2014, 2020, 2030, 2040 and 2050.

The methods used by ERC to generate the official projections of consumption are not disclosed clearly, so this paper aims to applying exponential smoothing methods to fill this gap and provide level and trend equations for the residential consumption forecasts. In addition to the forecasts with the best model in sample, the methods Standard and Damped Pegels are applied to perform an optimization procedure of the model’s hyper parameters in order to adjust the forecasted values provided by the Energy Research Company for the residential sector.

This paper is organized as follows: in section Methodology is described the theoretical frameworks and motivations for applying the exponential smoothing models with mathematical details relevant to the understanding of the reader as the modeling of the problem. In section Descriptive Analysis it is presented the residential energy consumption series of Brazil, and conduct analyzes of the results expected by the PDE 2014-2023 and PNE 2050. Section Results detailed, compares and evaluates the results obtained through errors measures and graphical views that facilitate the understanding. Finally, in the last section are drawn some conclusions and directions for further researches on this topic.

2. Methodology

In this section a review of exponential smoothing methods will be carried out, since to find the model that returns the smallest error in-sample , an extensive search is required.

For series that present only a level pattern, the exponential smoothing method most indicated among all is the Simple Exponential Smoothing (SES). The concept behind SES is attach larger weights to more recent observations, and the forecasts are calculated using weighted averages where the weights decrease exponentially as observations come from further in the past. The mathematical representation of SES method for a time series \( y_t, \ldots, y_T \) can be seen next.

\[
\begin{align*}
    l_t &= \hat{y}_{t+1|t} = \alpha y_t + (1 - \alpha)l_{t-1} \quad (1) \\
    \hat{y}_{T+h|t} &= l_T 
\end{align*}
\]

where \( l_t \) is the estimate of the level at time \( t \), \( y_t \) is the observed time series at time \( t \), \( \hat{y}_t \) is the forecasted value at time \( t \) and \( \alpha \) is the smoothing parameter of the level, \( 0 \leq \alpha \leq 1 \).

To fit and forecast series with only level and trend pattern, Charles C. Holt [4] developed the Holt's Linear Trend method, that now involves a forecast equation and two smoothing equations, one for the level and one for the trend.

\[
\begin{align*}
    l_t &= \alpha y_t + (1 - \alpha)(l_{t-1} + b_{t-1}) \quad (3) \\
    b_t &= \gamma (l_t - l_{t-1}) + (1 - \gamma)b_{t-1} \quad (4)
\end{align*}
\]
\[ \hat{y}_{T+h|T} = l_T + h b_T \] (5)

where \( b_t \) denotes an estimate of the trend (slope) of the series at time \( t \) and \( \gamma \) is the smoothing parameter of the trend, \( 0 \leq \gamma \leq 1 \).

A variation from Holt's method is known as Exponential Trend method or Standard Pegels, developed by Pegels [5], is achieved by allowing the level and the slope to be multiplied rather than added.

\[ l_t = \alpha y_t + (1 - \alpha)(l_{t-1} b_{t-1}) \] (6)

\[ b_t = \gamma \frac{l_t}{l_{t-1}} + (1 - \gamma) b_{t-1} \] (7)

\[ \hat{y}_{T+h|T} = l_T b_T^h \] (8)

The predictions provided by Holt's method exhibit a constant trend independently of the number of steps ahead being even more extreme when these projections are generated by the Standard Pegels method, thus increasing the chance of overestimating future values. Motivated by this information, Gardner and Mckenzie [6] introduced a parameter that dampens the trend to a flat line sometime in the future being known as Additive Damped Trend method. Thus, the equations of this method have, in addition to the terms of smoothing (\( \alpha \) e \( \gamma \)), a damping parameter \( \phi \).

\[ l_t = \alpha y_t + (1 - \alpha)(l_{t-1} + \phi b_{t-1}) \] (9)

\[ b_t = \gamma (l_t - l_{t-1}) + (1 - \gamma) \phi b_{t-1} \] (10)

\[ \hat{y}_{T+h|T} = l_T + b_T \sum_{i=1}^{h} \phi^i \] (11)

If \( 0 < \phi < 1 \) the trend is damped, if \( \phi = 1 \), the equations become identical to the Holt's Linear Trend method. Tashman and Kruk [7] show that there may be value in allocating \( \phi > 1 \), if applied in series with a strong tendency, especially with exponential trend. However, Taylor [8] showed that if the process is exponential, then a suitable method would be one that actually model such trend, as the method Standard Pegels. So Taylor introduced a damping parameter to the Exponential Trend method resulting to a Multiplicative Damped Trend method or Damped Pegels. The respective equations are shown below.

\[ l_t = \alpha y_t + (1 - \alpha)(l_{t-1} b_{t-1}^\phi) \] (12)

\[ b_t = \gamma (l_t - l_{t-1}) + (1 - \gamma) b_{t-1}^\phi \] (13)

\[ \hat{y}_{T+h|T} = l_T + b_T \sum_{i=1}^{h} \phi^i \] (14)

If \( 0 < \phi < 1 \), the multiplicative trend is damped, if \( \phi = 1 \), the Damped Pegels method becomes identical to the Standard Pegels, but if \( \phi > 1 \), the forecast function has an inflationary multiplicative term that grows overtime.

It is expected that this new technique, introduced by Taylor, produces even more conservative forecasts than the Additive Damped Trend method when compared to Holt's linear.
3. Descriptive Analysis

In this paper will be analyze the residential electricity consumption series in Brazil. The historical data is annual, between 1995 and 2013, have been obtained from the ERC site, measured in TWh. In addition to the consumption series it will be also analyzed, the projections provided by PDE 2014-2023 and PNE 2050.

From Fig. 1, which presents the history of residential electricity consumption in Brazil, an approximately linear growth trend is noticed and a structural break in 2001. This break was a result of the record energy supply crisis, which culminated in a compulsory energy rationing plan.

![Energy Consumption in Brazil](image)

Following in Fig. 2, the projections profiles provided by ERC through the PDE 2014-2023 and PNE 2050 reports are presented. The main difference found when comparing the PDE and the PNE is that for the second survey, the projections made for the year 2020 (164.73 TWh) are more pessimistic than those performed by the PDE (168.37 TWh). This may have occurred due to the fact that the year 2013 was bellow ERC expectations while for the PNE, the year 2013 is given as done.
In Table 1 and Table 2 are the values published by the ERC. According to the PDE 2014-2023 the average annual growth expected for the residential sector is 6.7 TWh by 2023, while the PNE 2050 expects an annual growth of 6 TWh between 2014 and 2050.

Table 1. PDE 2014-2023 expected consumption.

<table>
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<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PDE 2014-2023</td>
<td>129,98</td>
<td>135,79</td>
<td>142,08</td>
<td>148,39</td>
<td>154,88</td>
<td>161,54</td>
<td>168,37</td>
<td>175,38</td>
<td>182,57</td>
<td>189,93</td>
</tr>
</tbody>
</table>

Table 2. PNE 2050 expected consumption.

<table>
<thead>
<tr>
<th>Survey</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>PNE 2050</td>
<td>164,73</td>
<td>232,37</td>
<td>297,45</td>
<td>345,20</td>
</tr>
</tbody>
</table>

4. Results

From now on, it will be shown all the results obtained by applying the methodologies described in the Methodology section. As in the previous section it was noted that the residential consumption series has a growth in its history, so it will be applied the methods: Holt's linear trend method, Standard Pegels and Damped Pegels to adjust the best model in-sample and then generate projections up to 2050. For best model understand like the choice of the collection of parameters that among all produced the lowest error between adjusted and predicted values in history.
Following, the hyper parameters of the techniques Standard and Damped Pegels were optimized to find the equations that obtain better performance in: (1) forecast jointly the expected values of the PDE 2014-2023; and (2) forecast all the years of the PNE.

To adjust the optimal hyper parameters it was used the Excel Solver add-in and to forecast until 2050 with the respective equations, the authors has developed a code in R allowing the use of any value for the parameter $\varphi$ in the Damped Pegels method. This code was needed since the ets function did not allow any value of $\varphi$, becoming a problem to adjust some values of the ERC researches.

In Table 3 is shown the values of the hyper parameters and MAPE in-sample by adjust the consumption series with Holt’s linear trend method, Standard and Damped Pegels. It is possible to conclude that all the techniques were good in fitting the historical since the MAPE’s measures are very low, highlighting Pegels techniques that decreased the error measure from almost for 4% with Holt to approximately 2.6%.

Table 3. Hyper parameters and MAPE with Holt, Standard and Damped methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\varphi$</th>
<th>MAPE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holt’s linear trend</td>
<td>1.000</td>
<td>0.329</td>
<td>-</td>
<td>3.916</td>
</tr>
<tr>
<td>Standard Pegels</td>
<td>1.000</td>
<td>0.001</td>
<td>-</td>
<td>2.643</td>
</tr>
<tr>
<td>Damped Pegels</td>
<td>1.000</td>
<td>0.150</td>
<td>0.999</td>
<td>2.642</td>
</tr>
</tbody>
</table>

The next step, after fitting, is forecast up to 2050, with the graphics results presented in Fig. 3. Is also present in this graph the ERC projections, and comparing the values predicted by the exponential smoothing models with the expected by PDE and PNE is noticed that Holt drew near the expected, but the Standard and Damped methods overestimated both surveys.

Fig. 3. Forecasts with best model in-sample up to 2050 and comparison with ERC projections

To optimize the hyper parameters for the two cases mentioned above, it was needed an intense computing power, so for this exercise only the hyper parameters of the Pegels' techniques were adjusted. The optimized
hyper parameters, MAPE in-sample and Average Error founded with Standard Pegels techniques are in Table 4, where the last column is the average error relative to PDE and the PNE simultaneously.

Table 4. Optimized hyper parameters and MAPE in sample with Standard Pegels.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>MAPE in-sample (%)</th>
<th>Average Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDE 2014-2023</td>
<td>1.00</td>
<td>0.04</td>
<td>2.79</td>
<td>9.29</td>
</tr>
<tr>
<td>PNE 2050</td>
<td>0.02</td>
<td>0.59</td>
<td>15.37</td>
<td>3.88</td>
</tr>
</tbody>
</table>

Note that the since $\alpha=1$ for the PDE study, for this model the smoothing level is estimated as the observed value. The equations that provide the best adjust to PNE 2050 do not adjust well to the series historical because the MAPE in-sample is 15.37%, but are efficient to adjust the forecasts of both surveys since the Average Error is 3.88%.

The results with hyper parameters optimized via Damped Pegels are in Table 5. See that for both cases $\alpha=1$ and for PDE study $\gamma=1$, meaning that the smoothing trend at time $t$ is the smoothing trend at time $t-1$ damped by $\varphi$. Both MAPE's measures founds for this case are satisfactorily, proving that both model were efficient in fitting the historical. Another highlight is that the Average Error by applying the parameters fitted for the PNE 2050 is very low, showing that this is an excellent model to fit both ERC researches, providing a level and trend equations.

Table 5. Optimized hyper parameters and MAPE in sample with Damped Pegels.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\varphi$</th>
<th>MAPE in-sample (%)</th>
<th>Average Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDE 2014-2023</td>
<td>1.00</td>
<td>0.00</td>
<td>0.99</td>
<td>2.68</td>
<td>6.43</td>
</tr>
<tr>
<td>PNE 2050</td>
<td>1.00</td>
<td>0.34</td>
<td>0.97</td>
<td>3.11</td>
<td>0.73</td>
</tr>
</tbody>
</table>

To finalize the results found, see in Fig. 4 the graphical forecasting with each one of the models applying both Standard and Damped Pegels. And reinforcing what was analysed with the error measurements, the optimized model equations with Damped Pegels for PNE 2050 adhere very well the projections published by ERC, as can be seen by the pink line.

![Forecasts with optimized hyper parameters](image)

Fig. 4. Forecasts with optimized hyper parameters up to 2050 and comparison with ERC projections.
5. Conclusions

This paper aimed to fit and forecast the residential consumption of electricity in Brazil through exponential smoothing methods. In particular with Pegels’ techniques, called Standard and Damped, hyper parameters optimizations were performed to adjust the official projections expected by Energy Research Company (ERC), through the “The Ten Year Energy Planning” (PDE) and “The National Energy Planning” (PNE).

In a first moment were applied Holt's linear trend method, Standard Pegels and Damped Pegels to fit the best model in-sample and then generate projections up to 2050. For best model understand the one that produce the lowest error in sample. For this exercise all the techniques shown to be well adjusted to the historical data, with a larger error only in 2001, explained by the effect of rationing. When performing the prediction with these models until 2050 and compare with the ERC researches, Holt’s linear trend method proved to be a good approximation of the results of PDE 2014-2023 and PNE 2050 while Standard and Damped Pegels overestimate both studies.

The second exercise comprised in optimized the hyper parameters of the techniques Standard and Damped Pegels to find the equations that obtain better performance in: (1) forecast jointly the expected values of the PDE 2014-2023; and (2) forecast all the years of the PNE. The great value of this exercise is model the official expectations for the residential energy consumption in Brazil, with time series methods, since the approaches used by ERC to generate the official projections are not disclosed clearly. With this exercise is also possible to know the equations of level and trend for each one of the projections of PDE and PNE. After a large computational effort it was possible to see that the model with Damped Pegels that adjust the PNE 2050 is the best model to approximate both researches (PDE and PNE) simultaneously, concluding then that the goal was achieved.

As an extension of this work, the authors intend to applied the same methodology to others energy sectors of Brazil and also for the total Brazil. Also, for future research the authors intend to use others techniques to fit the official projections.

Acknowledgements

The authors would like to thank the colleagues from PUC-Rio for their valuable comments and suggestions, which improved the paper and the R&D program of the Brazilian Electricity Regulatory Agency (ANEEL) for the financial support (R&D project PD-7625-0003/2014). Finally, the authors would like to thank CNPq and CAPES, which are governmental agencies that have also given support to this research.

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