



Phantom cosmology without Big Rip singularity

Artyom V. Astashenok^a, Shin'ichi Nojiri^{b,c,*}, Sergei D. Odintsov^{b,d,e}, Artyom V. Yurov^a

^a *Baltic Federal University of I. Kant, Department of Theoretical Physics, 236041, 14, Nevsky st., Kaliningrad, Russia*

^b *Department of Physics, Nagoya University, Nagoya 464-8602, Japan*

^c *Kobayashi–Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya 464-8602, Japan*

^d *Institució Catalana de Recerca i Estudis Avançats (ICREA) and Institut de Ciències de l'Espai (IEEC-CSIC), Campus UAB, Facultat de Ciències, Torre C5-Par-2a pl, E-08193 Bellaterra (Barcelona), Spain*

^e *Tomsk State Pedagogical University, Tomsk, Russia*

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ABSTRACT

We construct phantom energy models with the equation of state parameter w which is less than -1 , $w < -1$, but finite-time future singularity does not occur. Such models can be divided into two classes: (i) energy density increases with time (“phantom energy” without “Big Rip” singularity) and (ii) energy density tends to constant value with time (“cosmological constant” with asymptotically de Sitter evolution). The disintegration of bound structure is confirmed in Little Rip cosmology. Surprisingly, we find that such disintegration (on example of Sun–Earth system) may occur even in asymptotically de Sitter phantom universe consistent with observational data. We also demonstrate that non-singular phantom models admit wormhole solutions as well as possibility of Big Trip via wormholes.

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1. Introduction

The discovery of accelerated expansion of the universe [1,2] led to a number of new ideas/solutions in cosmology. Recent observations of supernovae are consistent with the universe made up 71.3% of dark energy and only 27.4% of a combination of dark matter and baryonic matters [3]. Dark energy proposed to explain the cosmic acceleration should have the strong negative pressure (acting repulsively) in order to explain the observed accelerating expansion of the universe (for recent reviews, see [4–9]). The equation of state parameter w_D for dark energy is negative:

$$w_D = p_D / \rho_D < 0, \quad (1)$$

where ρ_D is the dark energy density and p_D is the pressure. We omit subscript D further for simplicity.

According to the latest cosmological data available, the uncertainties are still too large to discriminate among the three cases $w < -1$, $w = -1$, and $w > -1$: $w = -1.04^{+0.09}_{-0.10}$ [10,11]. If $w < -1$, the violation of all four energy conditions occurs. The corresponding phantom field, which is unstable as quantum field theory [12] but could be stable in classical cosmology, may be naturally described by a scalar field with the negative kinetic term. Such Lagrangians appear in some models of supergravity [13], in the

gravity theories with higher derivatives [14] and in string field theory [15].

The additional interest to the models with the phantom fields is caused by their prediction of a so-called Big Rip singularity [16–21]. Theoretically, the scale factor of the universe becomes infinite at a finite time in the future which was dubbed Big Rip singularity. There were proposed several scenarios to cure the Big Rip singularity: (i) To consider phantom acceleration as transient phenomenon. This is possible for a number of scalar potentials. (ii) To account for quantum effects which may delay/stop the singularity occurrence [22]. (iii) To modify the gravitation itself in such a way that it appears to be observationally-friendly from one side but it cures singularity (for review, see [23]). (iv) To couple dark energy with dark matter in the special way [24] or to use special (artificial) form for dark energy equation of state [25]. Note that for quintessence dark energy, other (milder) finite-time singularities may occur. The corresponding classification of such quintessence-related finite-time singularities is given in Ref. [26]. For instance, type II (sudden) singularity [27] or type III singularity [26] occurs with finite scale factor but infinite energy and/or pressure. Such quintessence-related finite-time singularities occur for instance, for the models [28–32] and were also called the “big freeze” singularities [33,34].

The closer examination shows that the condition $w < -1$ is not sufficient for a singularity occurrence. First of all, a transient phantom cosmology is quite possible. Moreover, one can construct such models that w asymptotically tends to -1 and the energy density increases with time or remains constant but there is no

* Corresponding author at: Department of Physics, Nagoya University, Nagoya 464-8602, Japan.

E-mail address: nojiri@phys.nagoya-u.ac.jp (S. Nojiri).

finite-time future singularity [26,27,35–38]. Of course, most evident case is when Hubble rate tends to constant (cosmological constant or asymptotically de Sitter space), which may also correspond to the pseudo-rip [39]. Very interesting situation is related with Little Rip cosmology [38] where Hubble rate tends to infinity in the infinite future (for further investigation, see [39–41]). The key point is that if w approaches -1 sufficiently rapidly, then it is possible to have a model in which the time required for singularity is infinite, i.e., the singularity effectively does not occur. Nevertheless, it may be shown that even in this case the disintegration of bound structures takes place in the way similar to Big Rip.

The aim of this Letter is to develop the method of constructing the phantom models without finite-time singularity. In Section 2, a general approach to this problem is developed. The examples of singular dark energy models are given there. Section 3 is devoted to the construction of the scalar Little Rip dark energy models. In Section 4, the transient phantom era which ends up at asymptotically de Sitter universe is investigated. The corresponding non-singular scalar phantom models are constructed. It is demonstrated that such models are compatible with latest data from Supernova Cosmology Project. We show that the dissolution of bound structures is possible in such asymptotically de Sitter universe for special choice of theory parameters. This gives the observationally-consistent pseudo-rip cosmology scenario. The influence of possible interaction between phantom energy and dark matter on non-singular cosmological evolution is investigated in Section 5. In Section 6, the possibilities of wormhole solutions and so-called “Big Trip” wormhole scenario in constructed cosmological models are considered. Some summary and outlook are given in Section 7.

2. Scalar dark energy models with future singularity

We start from the FRW equation and the conservation law for spatially flat universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3}, \quad \dot{\rho} = -3\left(\frac{\dot{a}}{a}\right)(\rho + p), \quad (2)$$

where ρ and p are the total energy density and pressure, a is the scale factor, $\dot{} = d/dt$, and we use the natural system units in which $8\pi G = c = 1$.

We will examine the future evolution of our universe from the point at which the pressure and the density are dominated by the dark energy. For the pressure of dark energy, one can choose the general expression

$$p = -\rho - f(\rho), \quad (3)$$

where $f(\rho)$ is a function of the energy density. The case $f(\rho) > 0$ corresponds to $w < -1$. From (2), one can obtain the following relation between the time coordinate t and $f(\rho)$:

$$t = \frac{2}{\sqrt{3}} \int_{x_0}^x \frac{dx}{f(x)}, \quad x \equiv \sqrt{\rho}. \quad (4)$$

If the integral (4) converges at $\rho \rightarrow \infty$, we have a singularity: energy density becomes infinite at a finite future $t = t_f$ in the future. The expression for scale factor

$$a = a_0 \exp\left(\frac{2}{3} \int_{x_0}^x \frac{x dx}{f(x)}\right) \quad (5)$$

indicates that there are two possibilities:

- (i) scale factor diverges at a finite time (“Big Rip”),
- (ii) scale factor reaches finite value and singularity ($\rho \rightarrow \infty$) occurs. It is type III singularity in the notations of Ref. [26].

The simple choice of f corresponding to first scenario is

$$f(x) = \beta x^\alpha, \quad 1 < \alpha \leq 2, \quad (6)$$

where β is a positive constant. The scale factor can be written as

$$a = \begin{cases} A \exp\{(B - Ct)^{-\gamma}\}, & \alpha \neq 2, \quad \gamma = \frac{2-\alpha}{\alpha-1}, \\ (D - Et)^{-\delta}, & \alpha = 2, \quad \delta = 2/3\beta, \end{cases} \quad (7)$$

where A, B, C, D , and E are positive constants. The case $\alpha = 2$ corresponds to a simplest model of phantom energy with parameter $w = -1 - \beta = \text{const}$.

If $\alpha > 2$, one has the second possibility: the energy density grows so rapidly with time that scale factor does not reach the infinite value.

Equivalent description in terms of scalar theory can be derived using the equations:

$$\rho = -\dot{\phi}^2/2 + V(\phi), \quad p = -\dot{\phi}^2/2 - V(\phi), \quad (8)$$

where ϕ is a scalar field with potential $V(\phi)$. For the scalar field and the potential, one can derive following expressions:

$$\phi(x) = \phi_0 \pm \frac{2}{\sqrt{3}} \int_{x_0}^x \frac{dx}{\sqrt{f(x)}}, \quad (9)$$

$$V(x) = x^2 + f(x)/2. \quad (10)$$

Combining Eqs. (9) and (10), we find the potential as a function of scalar field. For simplicity we choose the sign “+” in Eq. (9), hereafter. The choice (6) yields for potential (if $\alpha \neq 2$)

$$V(\phi) = F^{1-\frac{2}{\alpha}} (\phi - \phi_0)^{\frac{2}{1-\alpha}} + \frac{\beta}{2} F^{\frac{\alpha}{1-\alpha}} (\phi - \phi_0)^{\frac{\alpha}{1-\alpha}}, \quad (11)$$

$$F = \sqrt{3\beta}(1 - \alpha/2)/2.$$

For $\alpha = 2$, as expected, we have an exponential potential:

$$V(\phi) = (1 + \beta/2)x_0^2 \exp\{\sqrt{3\beta}(\phi - \phi_0)\}. \quad (12)$$

The key difference between (i) and (ii) for this model is that the potential of scalar field has a pole in a case of (ii)-singularity. Note that another type of singularity occurs if $f(x) \rightarrow \infty$ at $x = x_f < \infty$, i.e., the pressure of dark energy becomes infinite at finite energy density. The second derivative of scale factor diverges.

It is interesting however to investigate the phantom energy models without finite-time future singularities with the help of similar technique.

3. Scalar Little Rip cosmology

Let us now consider the models which provide an evolution for the universe intermediate between de Sitter evolution and phantom era with the Big Rip. These models were described in detail in [38,39,41]. The energy density grows with time but not rapidly enough for the occurrence of the Big Rip singularity. According to terminology in [38] we have a so-called “Little Rip”: eventually a dissolution of bound structures at some point in the future occurs.

The method of constructing such models is very simple (compare with [41]). In principle one can choose arbitrary monotonic function $g(x)$ well defined in domain $x > x_0$ and satisfying condition $g(x) \rightarrow \infty$ at $t \rightarrow \infty$. Then one can assume that

$$f(x) = \frac{1}{g'(x)}. \tag{13}$$

From Eqs. (4) and (5) we have

$$t = \frac{2}{\sqrt{3}}(g(x) - g(x_0)),$$

$$a = a_0 \exp \left\{ \frac{2}{3}(xg(x) - x_0g(x_0)) - \frac{2}{3} \int_{x_0}^x g(x) dx \right\}. \tag{14}$$

For the scalar field ϕ and potential $V(\phi)$, the following expressions can be written

$$\phi(x) = \phi_0 + \frac{2}{\sqrt{3}} \int_{x_0}^x \sqrt{g'(x)} dx, \quad V(x) = x^2 + \frac{1}{2g'(x)}. \tag{15}$$

The exponential growth of density with time ($g(x) = \ln(x)/\beta$, $\beta > 0$) corresponds to the scalar potential

$$V(\phi) = \frac{9\beta^2}{256}\phi^4 + \frac{3\beta^2}{8}\phi^2, \quad \phi = \phi_0 \exp\left(\frac{3^{1/2}\beta}{2}t\right) \tag{16}$$

(for simplicity, we put $\phi_0 = 4\sqrt{x_0/3\beta}$). The scale factor a grows with time in accordance with the double exponential law

$$a = a_0 \exp \left\{ \frac{2x_0}{3\beta} \left(\exp\left(\frac{3^{1/2}\beta t}{2}\right) - 1 \right) \right\}. \tag{17}$$

As shown in [38], this model can be matched with the latest data from Supernova Cosmological Project. The best-fit value for β is $3.46 \times 10^{-3} \text{ Gyr}^{-1}$. Thus, we presented the example of the scalar Little Rip cosmology where the future singularity does not effectively occurs.

However, it should be noted that the Little Rip produces the disintegration of bound structures just as in the case of Big Rip. The condition of disintegration can be derived in the following. The acceleration of the universe leads to an inertial force on a mass m as seen by a gravitational source separated by a comoving distance l

$$F_{\text{in}} = ml \frac{\ddot{a}}{a} = ml \frac{4\pi G}{3} (2\rho(a) + \rho'(a)a). \tag{18}$$

The structure disintegrates when the inertial force (18), dominated by dark energy, becomes equal to the force bounding the structure. It is convenient to define dimensionless parameter [39]

$$\bar{F}_{\text{in}} = \frac{2\rho(a) + \rho'(a)a}{\rho_0} \tag{19}$$

(ρ_0 is a dark energy density at the present time). The simple calculations allow to derive the following expression for \bar{F}_{in} as a function of time

$$\bar{F}_{\text{in}} = 2 \exp(3^{1/2}\beta t) + \frac{3\beta}{\rho_0^{1/2}} \exp\left(\frac{3^{1/2}\beta t}{2}\right). \tag{20}$$

The system of Sun and Earth, for example, disintegrates when \bar{F}_{in} reaches $\sim 10^{23}$. Therefore, the time required for this event is around $8.5 \times 10^3 \text{ Gyr}$.

4. Phantom models with asymptotically de Sitter evolution

Another interesting class of models arises if integral in Eq. (4) diverges at some finite $x = x_f < \infty$. The time required for energy density to reach $\rho = x_f^2$ is infinite, i.e., the expansion of the universe asymptotically approaches the exponential regime, which

corresponds to the pseudo-rip in [39]. The energy density tends to the constant value (“cosmological constant”) although the parameter w is always less than -1 .

For example, let us assume that

$$f(x) = A(1 - x/x_f)^\alpha, \tag{21}$$

where A and α are positive constants and we assume $\alpha \geq 1$. In this case, the integral (4) diverges at $x = x_f$. For the case that $\alpha \neq 1, 2$, algebraic calculations allow us to get the following representation for scale factor:

$$a(t) = \tilde{a}_0 \exp(x_f t / \sqrt{3}) \exp(g_\alpha(t)),$$

$$g_\alpha(t) = \frac{2x_f^2}{3A(2-\alpha)} \left(\frac{A\sqrt{3}(\alpha-1)t}{2x_f} + \left(1 - \frac{x_0}{x_f}\right)^{1-\alpha} \right)^{1+\frac{1}{1-\alpha}}. \tag{22}$$

For $1 < \alpha < 2$ one can easily see that $g(t) \rightarrow 0$ for $t \rightarrow \infty$. If $\alpha > 2$ $|g(t)| \ll x_f t / \sqrt{3}$ at $t \rightarrow \infty$. Therefore the dependence (21) asymptotically tends to de Sitter solution with vacuum energy density $\rho_\Lambda = x_f^2$.

When $t \rightarrow \infty$, the value of scalar field

$$\phi = \phi_0 + \frac{2x_f}{\sqrt{3}A} \frac{1}{1-\alpha/2} \left\{ \left(1 - \frac{x_0}{x_f}\right)^{1-\alpha/2} - \left(\frac{\sqrt{3}(1-\alpha)At}{x_f} + \left(1 - \frac{x_0}{x_f}\right)^{1-\alpha}\right)^{\frac{1-\alpha/2}{1-\alpha}} \right\} \tag{23}$$

tends to constant for $1 < \alpha < 2$ and to $\phi \rightarrow \pm\infty$ for $\alpha > 2$. As described in the second section, the scalar potential may be found as

$$V(\phi) = x_f^2 \left\{ 1 - \left(\left(1 - \frac{x_0}{x_f}\right)^{1-\alpha/2} - \frac{\sqrt{3}A(1-\alpha/2)(\phi - \phi_0)}{2x_f} \right)^{\frac{1}{1-\alpha/2}} \right\}^2 + \frac{A}{2} \left\{ \left(1 - \frac{x_0}{x_f}\right)^{1-\alpha/2} - \frac{\sqrt{3}A(1-\alpha/2)(\phi - \phi_0)}{2x_f} \right\}^{\frac{\alpha}{1-\alpha/2}}. \tag{24}$$

The cases $\alpha = 1, 2$ are more interesting. For $\alpha = 1$, the scale factor behaves as

$$a(t) = a_0 \exp(x_f t / \sqrt{3}) \exp(g_1(t)),$$

$$g_1(t) = \frac{2x_f^2}{3A} \left(1 - \frac{x_0}{x_f}\right) (\exp(-\sqrt{3}At/2x_f) - 1), \tag{25}$$

and the scalar field

$$\phi(t) = \phi_0 + \frac{2x_f}{\sqrt{3}A} \left(1 - \frac{x_0}{x_f}\right)^{1/2} (1 - \exp(-\sqrt{3}At/4x_f)), \tag{26}$$

tends asymptotically to maximum (if $x_f^2 > A/4$) or minimum (if $x_f^2 < A/4$) of corresponding potential

$$V(\phi) = x_f^2 + \lambda(\phi - \phi^*)^4 - \mu^2(\phi - \phi^*)^2,$$

$$\phi^* = \phi_0 + \frac{2x_f}{\sqrt{3}A} \left(1 - \frac{x_0}{x_f}\right)^{1/2},$$

$$\lambda = \frac{9A^2}{16x_f^2}, \quad \mu^2 = \frac{3A^2}{8x_f^2} \left(\frac{4x_f^2}{A} - 1\right). \tag{27}$$

The choice $\alpha = 2$ leads to the exponential potential:

$$a(t) = a_0 \exp(x_f t / \sqrt{3}) \exp(g_2(t)),$$

$$g_2(t) = \frac{2x_f^2}{3A} \ln\left(\frac{\sqrt{3}A(x_f - x_0)}{x_f^2} t + 1\right), \quad (28)$$

$$\phi = \phi_0 + \frac{2x_f}{\sqrt{3}A} \ln\left(1 + \frac{\sqrt{3}A}{x_f^2} t\right), \quad (29)$$

$$V = x_f^2 - 2x_f \exp(-S) + \left(1 + \frac{A}{2x_f^2}\right) \exp(-2S),$$

$$S = \sqrt{3}A(\phi - \phi_0) / 2x_f. \quad (30)$$

The appearance of exponential potentials may indicate to some connection with string theory. Eqs. (29) and (30) show that in the infinite future, the scalar field also goes to infinity and the scalar field climbs up the potential to constant $V \rightarrow x_f^2$.

The above example is a good theoretical illustration of the new phantom energy models mimicking vacuum energy. The dark energy with such a behavior can be realized if the function $f(x)$ is equal to zero at $x = x_f$ and the integral (4) diverges at $x = x_f$. In the vicinity of this point, the arbitrary function f satisfying these conditions can be expanded as

$$f(x) = (x - x_f)^\alpha + O((x - x_f)^\alpha), \quad \alpha \geq 1.$$

Thus for $t \rightarrow \infty$, one can expect that Eqs. (23), (26), and (29) for $\phi(t)$ and (24), (27), and (30) for $V(\phi)$ will be satisfied. The case $\alpha = 1$ corresponds to the most rapid growth of the energy density with time in which any singularity does not occur.

Let us consider another model:

$$f(x) = A \cos^2\left(\frac{\pi x}{2x_f}\right). \quad (31)$$

In the vicinity $x = x_f$, we find $f(x) \approx \frac{A\pi^2}{4x_f^2} (1 - x/x_f)^2$ and we can conclude that the potential of scalar field looks like that in (30) at $t \rightarrow \infty$. Indeed, this is correct. For such a model, one gets

$$V(\phi) = x_f^2 \left\{ \frac{2}{\pi} \arctan \exp(S) \right\} + \frac{2A \exp(S)}{(1 + \exp(S))^2},$$

$$S = \frac{\sqrt{3}A\pi}{x_f} (\phi - \phi_0). \quad (32)$$

The interesting question is: could such models in principle describe latest supernova data from the Supernova Cosmology Project? The analysis shows that it is possible. Moreover the construction of such models is trivial. For example let us choose

$$f(x) = \beta x^{1/2} (1 - (x/x_f)^{3/2}), \quad (33)$$

and assume that dark energy density varies from 0 to $\rho_f = x_f^2$. Let $t = t_0$ and $\rho = \rho_0$ at current universe. Eq. (5) allows to write the following relation between the dark energy density ρ and the redshift $z = a_0/a - 1$:

$$\rho(z) = \rho_f (1 - (1+z)^\gamma (1-\Delta))^{4/3},$$

$$\gamma = 3\beta \rho_f^{-3/4} / 2, \quad \Delta = \left(\frac{\rho_0}{\rho_f}\right)^{3/4}. \quad (34)$$

The equation of state parameter w_0 is

$$w_0 = -1 - \frac{2\gamma}{3} \frac{1-\Delta}{\Delta}. \quad (35)$$

For the scale factor, we have the parametric expression

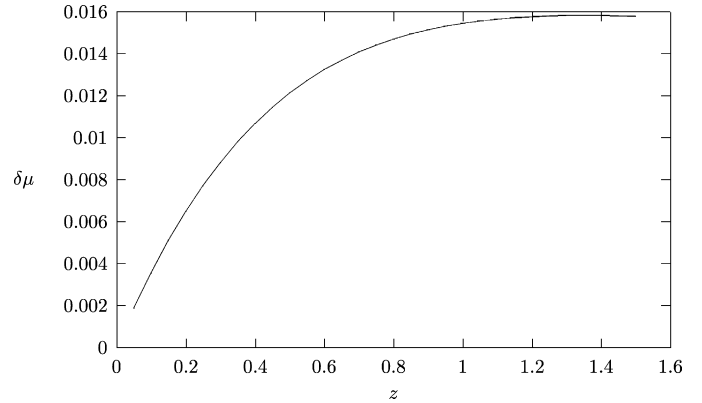


Fig. 1. The difference between modulus extracted from the toy dark energy model (33) and from the standard Λ CDM-model.

$$a(v) = \frac{a_0}{(1 - v^{3/2})^{2/3\gamma}}, \quad (36)$$

$$t(v) = t_0 + \frac{2\sqrt{3}}{\gamma x_f} \left(\frac{1}{6} \ln(v^2 + v + 1) + \frac{1}{\sqrt{3}} \arctan \frac{2v + 1}{\sqrt{3}} - \frac{\pi}{6\sqrt{3}} - \frac{1}{3} \ln(1 - v) \right). \quad (37)$$

The parameter v varies from 0 (at $t = t_0$) to 1 (at $t \rightarrow \infty$). The last term in Eq. (37) dominates at $v \rightarrow 1$. In this case, with a good accuracy one can write

$$a = \left(\frac{2}{3}\right)^{2/3\gamma} a_0 \exp\left(\frac{x_f(t - t_0)}{\sqrt{3}}\right). \quad (38)$$

For the dark and baryonic matter densities,

$$\rho_m = \rho_{m0} (1 + z)^3. \quad (39)$$

Therefore, the dependence of luminosity distance D_L from redshift z for this model is given by

$$D_L = \frac{c}{H_0} (1 + z) \int_0^z (\Omega_m (1 + z)^3 + \Omega_D h(z))^{-1/2} dz,$$

$$h(z) = \Delta^{-4/3} (1 - (1 + z)^\gamma (1 - \Delta))^{4/3}. \quad (40)$$

In Eq. (40), Ω_m and Ω_D express the fractions of matters and dark energy in the total energy budget correspondingly. For the standard Λ CDM cosmology, as well-known, we find

$$D_L^{SC} = \frac{c}{H_0} (1 + z) \int_0^z (\Omega_m (1 + z)^3 + \Omega_\Lambda)^{-1/2} dz. \quad (41)$$

If γ and Δ are very close to zero and 1 correspondingly (β is small and $\rho \rightarrow \rho_f$), then $h(z)$ is close to 1. Therefore the functions (40) and (41) are essentially indistinguishable (especially in the observable range $0 < z < 1.5$). The equation of state parameter is $w \approx -1$. Another choice of parameters

$$\Delta = 0.5, \quad \gamma = 0.075$$

corresponds to current equation of state parameter $w_0 = -1.05$. The difference $\delta\mu = 5 \lg(D/D^{SC})$ (μ is a distance modulus) for $0 < z < 1.5$ is depicted in Fig. 1 and does not exceed 0.016 (for $\Omega_m = 0.28$, $\Omega_\Lambda = \Omega_D = 0.72$). Taking into account that the errors in definition of SNe modulus are ~ 0.075 – 0.5 , we conclude that our model fits these data with excellent precision.

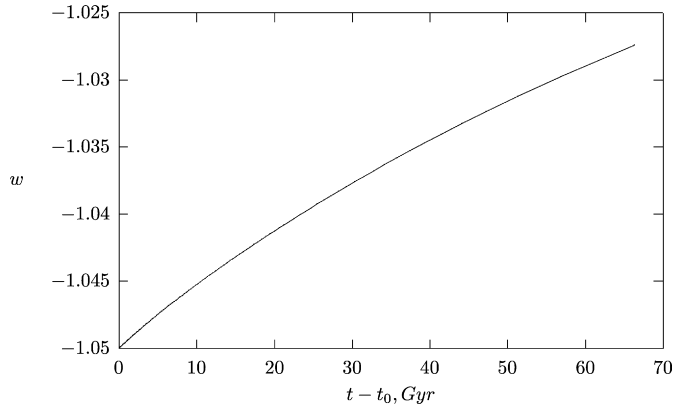


Fig. 2. The time dependence of w for the model (33).

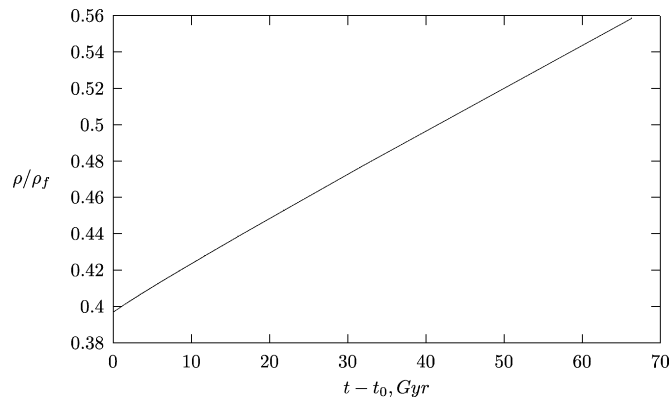


Fig. 3. The time dependence of the dark energy density for the model (33).

The equation of state parameter w and phantom energy density very slowly increase with time (Figs. 2 and 3).

Thus, we presented non-singular phantom dark energy which evolves to asymptotically de Sitter space and satisfies the observational bounds.

Let us compare the model (33) with observational data in more detail. One defines deceleration parameter q_0 , jerk parameter j_0 as follows

$$q_0 = -\frac{1}{aH^2} \frac{d^2 a}{dt^2} \Big|_{t=t_0} = -\frac{1}{H^2} \left\{ \frac{1}{2} \frac{d(H^2)}{dN} + H^2 \right\} \Big|_{N=0},$$

$$j_0 = \left\{ \frac{1}{aH^3} \frac{d^3 a}{dt^3} \Big|_{t=0} = \frac{1}{2H^2} \frac{d^2(H^2)}{dN^2} + \frac{3}{2H^2} \frac{d(H^2)}{dN} + 1 \right\} \Big|_{N=0}.$$
(42)

Here N is the e-foldings defined by

$$N \equiv -\ln(1+z).$$
(43)

For the current universe $t = t_0$, we have $N = 0$. Since

$$H^2 = \frac{1}{3}(\rho + \rho_m) = \frac{1}{3}(\rho_f(1 - e^{-\gamma N}(1 - \Delta))^{4/3} + \rho_{m0}e^{-3N}),$$
(44)

one gets

$$q_0 = \frac{2}{3}\gamma(\Delta^{-1} - 1)\Omega_\Lambda + \frac{3}{2}\Omega_m - 1,$$

$$j_0 = \left(\frac{2}{9}\gamma^2(\Delta^{-1} - 1)^2 + \left(-\frac{2}{3}\gamma^2 + 2\gamma \right) (\Delta^{-1} - 1) \right) \Omega_\Lambda + 1.$$
(45)

Since $\Delta = 0.5$, $\gamma = 0.075$, $\Omega_m = 0.28$, and $\Omega_\Lambda = 0.72$, we find

$$q_0 = -0.54, \quad j_0 = 1.11.$$
(46)

In case of Λ CDM model, which corresponds to $\Delta = 0$, we have $q_0 = -0.58$ and $j_0 = 1$, which is not so different from the values (46).

Just as it was done in the previous section, we can estimate the possibility of the disintegration of bound system in our model (33). It is easy to derive the following parametric representation for the dimensionless inertial force \bar{F}_{in} in (19) as follows:

$$\bar{F}_{in}(u) = 2\Delta^{-4/3} (1 - (1+u)^\gamma (1-\Delta))^{1/3} \times \left\{ 1 + \left(\frac{2}{3}\gamma - 1 \right) (1+u)^\gamma (1-\Delta) \right\},$$
(47)

$$t - t_0 = \frac{1}{H_0} \int_0^0 du (1+u)^{-1} (\Omega_m(1+u)^3 + \Omega_D h(u))^{-1/2}.$$
(48)

The variable $u = a_0/a - 1$ varies from 0 (at present time) to -1 (when $t \rightarrow \infty$). The function $h(u)$ coincides with $h(z)$ in Eq. (40) by changing $z \rightarrow u$. The inertial force asymptotically tends to $2\Delta^{-4/3}$. Therefore, the disintegration of the system of Sun and Earth can occur if $\Delta \leq \Delta_{min} = 10^{-17}$. The parameter γ at $\Delta = \Delta_{min}$ may vary from 0 (when $w_0 = -1$) to 2×10^{-18} (when $w_0 = -1.14$ – the lower bound from observations). The analysis shows that for any $\Delta < \Delta_{min}$ and $0 < \gamma < 10^{-1}\Delta$, our model also describes the SNe data. Therefore, the current observations in principle do not contradict with the possible disintegration of bound system in the models with an asymptotically de Sitter expansion. Note that such models were dubbed pseudo-rip in Ref. [39].

One remark is in order. The asymptotically de Sitter expansion of the universe can occur not only in the models with the phantom energy but also for the quintessence dark energy ($-1 < w < 0$). For the quintessence, the function $f(x)$ in (3) is negative and the energy density decreases with time. Eqs. (4), (5), and (10) do not change, for Eq. (9), it is sufficient to replace $f(x) \rightarrow -f(x)$. One can consider our model (21) and assume that $\alpha = 1$ and $x_0 > x_f$. The energy density asymptotically tends to $\rho \rightarrow x_f^2$ but the equation of state parameter to $w > -1$. Therefore one can conclude that the point $x = x_f$ is attractor in the case $f(x) = A(1 - x/x_f)$.

5. Coupled phantom models

Realistic cosmological scenarios should take into account that the dark energy does not have a single component of universe energy. It is quite appealing to include the possible interaction between dark energy and dark matter (for recent discussion, see [42–44]). We shall see that the addition of such a coupling may lead to interesting effects in the non-singular phantom cosmology. The cosmological solutions with interacting phantom energy exhibit much richer behavior than those considered above.

It is customary to assume that phantom dark energy and dark matter interact through a coupling term Q as

$$\dot{\rho}_m + 3H\rho_m = -Q,$$
(49)

$$\dot{\rho}_D + 3H(\rho_D + p_D) = Q.$$
(50)

Let us introduce variables x and y

$$\rho_D = x^2, \quad \rho_m = y^2.$$

For convenience, we assume that

$$Q = 3Hq(x, y),$$
(51)

where $q(x, y)$ is an arbitrary function of variables x and y . Then the Hubble rate H is given by

$$H = \left(\frac{x^2 + y^2}{3} \right)^{1/2}, \tag{52}$$

and Eqs. (49) and (50) can be rewritten as

$$\dot{y} = - \frac{3^{1/2}(x^2 + y^2)^{1/2}}{2y} (y^2 + q(x, y)), \tag{53}$$

$$\dot{x} = \frac{3^{1/2}(x^2 + y^2)^{1/2}}{2x} (f(x) + q(x, y)). \tag{54}$$

We can obtain critical points of the system (53), (54) satisfying following conditions, $\dot{x}_f = 0$ and $\dot{y}_f = 0$. Note that these critical points must satisfy the requirement $x_f \geq 0, y_f \geq 0$. Unfortunately, there are no well-motivated direct observational or theoretical bounds on the type of interaction. Usually, simple interactions $q \sim \rho_m, q \sim \rho_D$, and $q \sim (\rho_D + \rho_m)$ are extensively investigated.

For simplicity, we restrict ourselves to the cases $q(x, y) \equiv g(x)$, i.e., the intensity of interaction depends only on the phantom energy density and the rate of the Hubble rate H . Let us consider two examples.

1. The simplest phantom energy model with $f(x) = \beta x^2$ and $w = -1 - \beta$. Without interaction, one gets the Big Rip singularity. One can assume that

$$q(x, y) = -\alpha x^4, \quad \alpha > 0,$$

i.e., interaction leads to transformation of phantom energy into dark matter. On the other hand, the density of dark energy grows with time. Eventually the dynamical equilibrium between these processes is established. The equilibrium phantom energy density is

$$\rho_D^{eq} = \frac{\beta}{\alpha}.$$

The density of dark matter otherwise decreases as universe expands but the dark matter born out from the phantom energy. The density of the dark matter asymptotically tends to

$$\rho_m^{eq} = \frac{\beta^2}{\alpha}.$$

Then the Hubble rate tends to

$$H \rightarrow \left(\frac{\beta + \beta^2}{3\alpha} \right)^{1/2}, \quad t \rightarrow \infty,$$

i.e., we have expansion according to the de Sitter law with “effective” cosmological constant

$$\Lambda^{eff} = \frac{\beta}{\alpha}(\beta + 1).$$

In other words, coupling with dark matter changes the qualitative behavior of phantom dark energy making it to be non-singular, in the way similar to the one described in Ref. [24].

2. The “switch” from the Little Rip to the asymptotically de Sitter expansion. As an example, let us choose

$$f(x) = \beta x, \quad q(x) = -\alpha x^2, \quad \alpha > 0.$$

The interpretation of interaction is the same as in the previous model. Instead of the Little Rip expansion, the quasi-de Sitter expansion occurs with the effective cosmological constant:

$$\Lambda^{eff} = \frac{\beta}{\alpha^2}(\alpha + 1).$$

Thus, we demonstrated that coupling of phantom dark energy with dark matter may help in the transition from singular accelerating expansion to the non-singular one. Note that stability of such cosmologies may be investigated in the same way as in Ref. [45] where it is shown that the Little Rip often may be stable if we compare it with de Sitter universe.

6. “Big Trip” in phantom cosmology without singularities

One of the very interesting points in phantom cosmology is connected with wormholes. First of all, the existence of the static, spherically symmetric wormhole solutions of the gravitational field equations in the absence of ghost (or phantom) degrees of freedom is impossible, as it was shown in [46]. We discuss here wormholes application in phantom cosmology under discussion.

It was shown in [47,48] that as one goes towards the Big Rip, there would occur the process of the fast wormhole swelling taking the size of the wormhole throat to infinity during the finite time. The reason of such a striking behavior is the phantom energy accretion onto wormhole. This accretion induces an increase of the wormhole throat radius so quick that the wormhole would engulf the entire universe before this reached the Big Rip. Such a result has been dubbed as “Big Trip” and was later criticized in Ref. [49]. The rejoinder was contained in [50]. The issue remains open up to now and it is not proposed to deal at length with this discussion. All we need here is to consider the formal possibility of the Big Trip in phantom models without Big Rip. As we shall see, the Big Trip in phantom cosmology is a common occurrence, that often happens even if we consider models with asymptotic de Sitter evolution.

The equation which describes evolution of the throat radius of a Morris–Thorne wormhole $b = b(t)$ due to dark energy accretion has the form

$$\dot{b} = -2\pi^2 D b^2 (1 + w) \rho, \tag{55}$$

where D is positive dimensionless constant and ρ is the energy density of the dark energy fluid. This equation is immediate consequence of the result for the dynamics of the mass of a black hole due to fluid accretion [51]. In this section we consider four examples of phantom models without finite-time singularity.

1. The first and second models were obtained in [38,41]. Let put the Hubble rate as $H = H_0 e^{\lambda t}$ with positive constants H_0 and λ . As a result we have the scale factor, the density and the pressure in the form

$$a(t) = a_0 \exp \left[\frac{H_0}{\lambda} (e^{\lambda t} - 1) \right],$$

$$\rho(t) = H_0^2 e^{2\lambda t}, \quad p(t) = -\frac{H_0}{3} e^{\lambda t} [2\lambda + 3H_0 e^{\lambda t}], \tag{56}$$

so

$$w(t) = -1 - \frac{2\lambda}{3H_0} e^{-\lambda t},$$

and at $t \rightarrow \infty$ one gets the asymptotic de Sitter universe.

Substituting (56) into (55), one obtains

$$b(t) = \frac{3}{4\pi^2 D H_0 (e^{\lambda t_{BT}} - e^{\lambda t})}. \tag{57}$$

Thus we have the Big Trip at

$$t = t_{BT} = \frac{1}{\lambda} \log \left(b_0 + \frac{3}{4\pi^2 D H_0} \right),$$

where b_0 is the throat radius of the wormhole at $t = 0$: $b = b(t)$.

2. Let $H = H_0 - H_1 e^{-\lambda t}$. This is the most clear example of universe with de Sitter asymptotic behavior, if $H_0 > 0$ and $\lambda > 0$. After integration of FRW equations we have

$$w(t) = -1 - \frac{2\lambda H_1 e^{-\lambda t}}{2(H_0 - H_1 e^{-\lambda t})^2},$$

and $w(t) \rightarrow -1$ at $t \rightarrow +\infty$. It is interesting to note that this solution contains so-called w -singularity [52], to be more precise, some generalization of w -singularity which was obtained in [53]. In fact, at

$$t = t_w = -\frac{1}{\lambda} \log \frac{H_0}{H_1},$$

$w(t_w) = -\infty$ although $\rho(t_w) = 0$ and $p(t_w) = -2\lambda H_0/3 \neq \infty$. Sure, the w -singularity occurs if both H_0 and H_1 are positive constants. If $H_1 < 0$ then $w(t)$ is always finite.

After calculations we have the throat radius of the wormhole in the form:

$$b(t) = \frac{3b_0}{3 - 4\pi^2 D H_1 b_0 (1 - e^{-\lambda t})}. \quad (58)$$

For negative values of H_1 we have Big Trip for any values of positive parameters b_0 and D but have no w -singularity. If $H_1 > 0$ then Big Trip takes place only for wormholes with

$$b_0 > \frac{3}{4\pi^2 D H_1},$$

at

$$t = t_{BT} = -\frac{1}{\lambda} \log \left(1 - \frac{3}{4\pi^2 D H_1 b_0} \right).$$

For positive H_1 we have w -singularity and

$$b(t_w) = \frac{3b_0}{3 + 4\pi^2 D b_0 (H_0 - H_1)}.$$

If

$$H_1 > H_0 + \frac{3}{4\pi^2 b_0},$$

then Big Trip takes place after w -singularity, otherwise – before. In any case, it is clear that w -singularity does not influence the evolution of wormhole at all, since it is not real physical singularity.

3. Now let consider the solution (7) with $\alpha \neq 2$. Since we are interested in solutions without Big Rip one should take $\gamma = -g < 0$; this is possible if $\alpha < 1$ ($g > 1$) and $\alpha > 2$ ($0 < g < 1$). We have phantom model $w < -1$ for the $\alpha < 1$, and $w > -1$ for the $\alpha > 2$. It is interesting to note that if $g > 1$ ($\alpha < 1$, $w < -1$) then for $t \rightarrow t_s = B/C$ we have $\rho \rightarrow 0$. If $1 < g < 2$ ($\alpha < 0$, $w < -1$) then at $t \rightarrow t_s$, $p \rightarrow -\infty$, $\rho \rightarrow 0$. This is type II singularity [26]. If $g > 2$ ($0 < \alpha < 1$, $w < -1$) then $\rho(t_s) = p(t_s) = 0$, $w(t_s) = -\infty$. This is exact w -singularity for the phantom case. For $g = 2$ ($\alpha = 0$) we have generalized w -singularity.

The throat radius of the wormhole is

$$b(t) = \frac{3b_0}{3 - 4\pi^2 D g C b_0 (B^{g-1} - (B - Ct)^{g-1})},$$

so the Big Trip takes place if and only if $g > 1$, i.e., $w < -1$ which is the case for the $\alpha < 1$, as expected.

4. At last we consider the solution (22) with $g < 0$. Here we have the generalized w -singularity ($\rho(t_w) = 0$, $p(t_w) = -A/3$), at

$$t_w = \frac{2\sqrt{3}(|g|+1)x_f}{3A} \left[1 - \left(1 - \frac{x_0}{x_f} \right)^{-1/(|g|+1)} \right],$$

and the Big Trip at

$$t_{BT} = \frac{2x_f(|g|+1)}{A\sqrt{3}} \left(\left(1 - \frac{x_0 - \delta}{x_f} \right)^{-1/(|g|+1)} - \left(1 - \frac{x_0}{x_f} \right)^{-1/(|g|+1)} \right),$$

with $\delta = 3\sqrt{3}/(4\pi D b_0) > 0$.

Thus, smooth exit from the phantom inflationary phase can still be tentatively recovered by considering a Big Trip scenario where the primordial phantom universe would travel in time towards a future universe (filled with, for example, usual radiation, see [47]). Such “exit from inflation” is possible in phantom models both with and without the future Big Rip singularity.

7. Conclusion

In summary, the relatively simple method for constructing phantom energy models without finite-time future singularity is developed. The dark energy models without future singularity are attractive from the physical viewpoint because the occurrence of finite-time singularity may lead to some inconsistencies. The equivalent description of the Little Rip cosmology where singularity effectively disappears, via fluid or scalar-tensor theory is presented. Phantom models with asymptotically de Sitter evolution are described. It is demonstrated that asymptotically de Sitter expansion can be realized in the class of exponential or power-law scalar potentials. Generalization for phantom models coupled with dark matter is also discussed. It is interesting to note that disintegration of bound structure (on the example of the system of Sun and Earth) in some asymptotically de Sitter phantom universe may occur for observationally acceptable choice of parameters.

We have shown that current data make it essentially impossible to determine whether or not the universe will end in a future singularity. The above scalar dark energy models represent natural alternative for Λ CDM model, which also leads to non-singular cosmology. Nevertheless, even for the non-singular asymptotically de Sitter universe, the possibility of dramatic rip which may lead to the disappearance of bound structures in the universe remains to be possible.

It is confirmed that phantom models without the Big Rip may lead to wormholes solutions. It is demonstrated that the possible Big Trip in the phantom cosmology can happen even if we consider the models with asymptotic de Sitter evolution.

The presence of numbers of free possible parameters (for instance, the choice of equation of state (3)) gives enough space for fine-tuning the models which can be useful for fitting with observational data. Hence, the described method may be very useful for confronting of theoretical models with coming observational data.

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