Numerical modeling of large deformation and nonlinear frictional contact of excavation boundary of deep soft rock tunnel

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Abstract: Roadways excavated in soft rocks at great depth are difficult to be maintained due to large deformation of surrounding rocks, which greatly influences the safety and efficiency of deep resources exploitation. During the excavation process of a deep soft rock tunnel, the rock wall may be compacted due to large deformation. In this paper, the technique to address this problem by a two-dimensional (2D) finite element software, large deformation engineering analyses software (LDEAS 1.0), is provided. By using the Lagrange multiplier method, the kinematic constraint of non-penetrating condition and static constraint of Coulomb friction are introduced to the governing equations in the form of incremental displacement. The numerical example demonstrates the efficiency of this technology. Deformations of a transportation tunnel in inclined soft rock strata at the depth of 1 000 m in Qishan coal mine and a tunnel excavated to three different depths are analyzed by two models, i.e. the additive decomposition model and polar decomposition model. It can be found that the deformation of the transportation tunnel is asymmetrical due to the inclination of rock strata. For extremely soft rock, large deformation can converge only for the additive decomposition model. The deformation of surrounding rocks increases with the increase in the tunnel depth for both models. At the same depth, the deformation calculated by the additive decomposition model is smaller than that by the polar decomposition model.

Key words: deep soft rock tunnel; large deformation; contact problem; Lagrange multiplier method

1 Introduction

After excavation of a soft rock tunnel at great depth, the deformation of surrounding rocks, such as roof subsidence, floor heaving and sidewall shrinkage, is generally large due to the mechanical properties of soft rocks and high in-situ stresses. For large deformation analyses of such a deep soft rock engineering, three types of nonlinear behaviors, i.e. material nonlinear components, geometrical nonlinearity, and contact nonlinearity are involved. Therefore, in order to perform reasonable large deformation analyses for deep soft rock tunnels, besides reliable constitutive models to describe the mechanical responses of soft rock masses, the nonlinear kinematic theories for determining strain and rotation appropriately, and the efficient algorithms for modeling the dynamic contact of excavation boundary are necessary.

It is known that the linear small deformation theory is only suitable for small displacement field. For large deformation analyses, there are two nonlinear models, i.e. polar decomposition model and additive decomposition model. The polar decomposition model is based on the classical nonlinear large deformation theory, which uses the Green strain tensor for strain definition and rotation tensor defined by Finger-Truesdell’s polar decomposition theorem [1, 2]. The additive decomposition model uses the nonlinear large deformation theory based on S-R decomposition theorem proposed by Chen [3, 4], where deformation gradient is decomposed into a positive definite strain tensor and an orthogonal rotation tensor by using a comoving coordinate system. The strain tensors defined by the two nonlinear theories can overcome the limitations of the Cauchy linear strain tensor, which is an incorrect strain measurement for finite rotation.

The main drawback of the classical nonlinear large deformation theory is that two different tensile tensors are related to the same rotation and therefore the rotation is incompatible with the strain.
Nearly all the existing commercial finite element softwares only provide the polar decomposition model for large deformation analysis. In order to improve this constrain, He et al. [5] started to develop a finite element software in 2005 for large deformation analysis of soft rock engineering at great depth, i.e. LDEAS. Two work groups, Institute of Geotechnical Engineering, China University of Mining and Technology (Beijing) and FEGEN Software Co. Ltd., worked together for this purpose. Finite element program generator (FEPG) developed by Liang in 1990 was used for generating the finite element codes. The main idea of FEPG is that for any kind of problem, it can automatically generate a complete source code based on partial different equation (PDE) and finite element algorithm expression, which can save 90% of programming time and guarantee the accuracy and consistency of programs.

The LDEAS has the following algorithms to enable it to perform large deformation analysis for soft rock engineering at great depth: (1) two models for large deformation analysis, i.e. the polar decomposition model and the additive decomposition model; (2) sixteen calculation programs, including total or incremental polar or additive decomposition analysis model for elastic or plastic materials under plane stress or plane strain; (3) element types of rocks, joints and supports, such as bolts, cables, beams and trusses; and (4) simulation of excavation and construction processes.

During the excavation process of a deep soft rock tunnel, the surrounding rocks may become compacted due to large deformation. Such a dynamic contact problem can be solved by the Lagrange multiplier method or the penalty function method generally [6, 7]. To satisfy the inequalities of static constraint on contract boundaries precisely and to introduce the frictional contact condition, the Lagrange multiplier method is adopted in the LDEAS 1.0.

In this paper, the focus is put on the capability of the software in modeling the contact and large deformation of soft rock engineering at great depth. At first, the basic incremental equations of the two models without contact for large deformation analysis are established. Then, the technique to solve the contact problem based on the Lagrange multiplier method is given. Finally, the large deformation of tunnels excavated at great depth is analyzed by the two models. The numerical example demonstrates the efficiency of the technology.

2 Basic equations of the two models for large deformation analysis

2.1 The additive decomposition model

The additive decomposition theorem states that any invertible linear differential transformation \( F \) has a unique additive decomposition [3, 4]:
\[
F = S + R, \quad F_j^i = S_j^i + R_j^i
\]
(1)
\[
S_j^i = \frac{1}{2} [(u_i^j + (u_i^j)^T)] - (1 - \cos \Theta)L_i^j L_j^k
\]
(2)
\[
R_i^j = \delta_i^j + L_i^j \sin \Theta + (1 - \cos \Theta) L_i^j L_j^k
\]
(3)
\[
\Theta = \pm \arcsin(-\omega_i^j / 2)^{1/2}
\]
(4)
\[
L_i^j = \omega_i^j / \sin \Theta
\]
(5)
where \( S \) is a symmetrical and positive definite sub-transformation, representing a strain tensor; \( R \) is an orthogonal sub-transformation, representing a local mean rotation tensor; \( \Theta \) is the mean rotation angle; and \( L_i^j \) is the unit vector in rotation axis direction.

When large deformation occurs in a body, its configuration may change drastically. Therefore, incremental analysis and updated coordinate method should be used, in which stress and strain are defined in real-time deformable state. For a material point in the deformed body at time \( t \), its updated comoving coordinates can be identified by \( 'x' \) \((i = 1, 2, 3)\), and its strain tensor and Euler stress tensor can be denoted by \( 'S' \) and \( '\sigma' \), respectively. At time \( t + \Delta t \), the position of the material point changes to \( i + \Delta x' \) \((i = 1, 2, 3)\). The displacement increment is \( \Delta u' = i + \Delta x' - 'x' \), and the strain tensor increment and Euler stress tensor increment are \( \Delta S_j^i = i + \Delta S_j^i - 'S' \) and \( \Delta \sigma_j^i = i + \Delta \sigma_j^i - '\sigma' \), respectively.

The covariant derivative of the displacement increment with respect to the updated comoving coordinates is defined as
\[
\Delta u_j^i = \partial \Delta u^j / \partial 'x_j^i + \Delta u_k \Gamma_k^j
\]
(6)
where \( \Gamma_k^j \) is the Christoffel symbol in the updated comoving coordinate system.

The kinematic additive decomposition of the covariant derivative of the displacement increment is given by
\[
\Delta u_j^i = \Delta S_j^i + \Delta R_j^i
\]
(7)
\[ \Delta S'_j = \frac{1}{2} [\Delta u'_j, + (\Delta u'_j \mid x)^T] \]
\[ \Delta R'_j = \frac{1}{2} [\Delta u'_j, - (\Delta u'_j \mid x)^T] \]

where \( \Delta S'_j \) is the strain increment, and \( \Delta R'_j \) is the local rotation increment.

Equilibrium equation is

\[ {^i v} \sigma_j = D'_j, \Delta S'_j \]
\[ \Delta \sigma_j = \Delta \sigma'_j - \sigma'_j \Delta S'_j + \sigma'_j \Delta S'_j \]

where \( \Delta \sigma'_j \) is the objective increment of the Euler stress, and \( D'_j \) is the tangential stiffness matrix of the material.

Boundary conditions for displacement and external force are

\[ {^i v} u' = {^i v} u \]
\[ \sigma'_j = {^i v} T_i \]

where \( {^i v} T_i \) is the surface force, and \( {^i v} u' \) is the total displacement at time \( t + \Delta t \), \( {^i v} u' = u' + \Delta u' \).

Then, the variational formula with respect to the updated comoving configure [8] can be obtained:

\[ \int_{\Omega} D'_j D'_j \delta u' \mid d \Omega = {^i v} W - \int_{\Omega} f \delta u' \mid d \Omega \]
\[ {^i v} W = \int_{\Omega} f \delta u' \mid d \Omega + \int_{\Omega} {^i v} T_i \delta u' \mid d \Omega \]

where \( {^i v} W \) is the external virtual work.

### 2.2 The polar decomposition model

The polar decomposition theorem [9] is a fundamental step in the development of kinematic description of continuous body motion. It allows locally (at a point) decomposing any deformation gradient \( F \) into a pure deformation motion plus a pure rotation motion and vice versa, i.e. the right and left polar decomposition can be respectively expressed as

\[ F = RU = V \bar{R} \quad F_j = \bar{R}_j U'_j = V'_j \bar{R}_j \]

where \( \bar{R} \) is the second-order orthogonal rotation tensor; \( U \) and \( V \) are the second-order symmetric right and left tensile tensors, respectively, and they can be determined as follows:

\[ U = (F^T F)^{1/2}, \quad V = (FF^T)^{1/2} \]

The relations among the Green strain tensor \( E \) and the Almansi strain tensor \( \varepsilon \) and the right and left tensile tensors are

\[ E = \frac{1}{2} (U^2 - I), \quad \varepsilon = \frac{1}{2} (I - V^2) \]

where \( I \) is the second-order unit tensor.

The weak variational formula in the updated Lagrange formulation [6, 8] can be given by

\[ \int_{\Omega} D_{12} \delta \varepsilon_{ij} \delta \varepsilon_{jk} d \Omega = \frac{1}{2} \frac{\partial \Delta u}{\partial \varepsilon^{ij}} + \frac{\partial \Delta u}{\partial \varepsilon^{ji}} \]

where \( \Delta \varepsilon_{ij} \) is the linear Cauchy strain increment, and \( \Delta \varepsilon_{ij} \) is the nonlinear Green strain increment. Both of them can be measured by the configuration at time \( t \).

### 3 The Lagrange multiplier method for frictional contact problems

For a deformable body, two material points that contact each other should not penetrate, which gives kinematic constraints for relative movement in each pair of contact points:

\[ A^T \Delta u + g = 0 \]

where \( g \) is the allowable displacement vector; for each pair of contact points, \( A_t = [I, -I] \). Therefore, \( A^T \Delta u = \Delta u - \Delta u_c \).

By introducing the Lagrange multiplier, the finite element equations for contact problems with large deformation can be formulated by solving the following functional equation:

\[ J(\Delta u, \lambda) = \frac{1}{2} \Delta u^T K \Delta u - u^T \Delta Q + \lambda^T (A^T \Delta u + g) \]

The conditions that the functional equation (Eq.(22)) takes stationary values are

\[ \frac{\partial J(\Delta u, \lambda)}{\partial \Delta u} = K \Delta u - \Delta Q + A \lambda = 0 \quad \text{(23a)} \]
\[ \frac{\partial J(\Delta u, \lambda)}{\partial \lambda} = A^T \Delta u + g = 0 \quad \text{(23b)} \]

According to Eq.(23a), the displacement can be related to the Lagrange multiplier by

\[ \Delta u = K^{-1} (\Delta Q - A \lambda) \quad \text{(24)} \]

Substituting Eq.(24) into Eq.(23b), the Lagrange multiplier \( \lambda \), which represents the contact force at contact points, can be obtained:

\[ A^T K^{-1} A \lambda = A^T K^{-1} \Delta Q + g \quad \text{(25)} \]
Based on the Coulomb friction law, static constraints for the contact force can be expressed in local coordinate system of contact points as

$$\lambda_n^l \geq 0, \ |\lambda_t^l| \leq \mu \lambda_n^l$$  \hspace{1cm} (26)

where $\lambda_n^l$ and $\lambda_t^l$ are the normal and tangential components of contact force, respectively; and $\mu$ is the friction coefficient of material. The Lagrange multiplier in local coordinate system, $\lambda^l$, can be related to the Lagrange multiplier in a global coordinate system by the transition matrix $T$ of the two coordinate systems:

$$\lambda^l = T \lambda, \ \lambda = T^T \lambda^l$$  \hspace{1cm} (27)

In general, the contact position and contact state in a deformable body are unknown previously, which are related to the nonlinear deformation of body and nonlinear mechanical response of material. Therefore, for large deformation analysis of a body with dynamic frictional contact boundary, the technique of trial and check is needed to solve the incremental displacement and contact state [6].

When the Lagrange multiplier method is applied to the incremental analysis of large deformation of deep soft rock engineering, the excavation boundary is taken as a possible contact boundary. At the beginning of every incremental calculation step, contact points will be searched according to the displacement obtained from the previous step. Then, the Gauss-Seidel iterative method is used to solve the Lagrange multiplier (contact force) and justify it to satisfy the inequalities of static constraint on contact boundaries (Eq.(26)). Finally, the incremental displacement in this step is solved according to Eq.(24).

### 4 Numerical examples

#### 4.1 Verification of contact algorithm

The performance of the software in modeling dynamic frictional contact on excavation boundary of soft rock engineering at great depth is verified by comparing the analysis results of large deformation with and without contact algorithm for a transportation tunnel in Liuhai coal mine. For simplicity, the constitutive model used in the analysis is assumed to be elastic.

The cross-section of the transportation tunnel is U-shaped. The radius of the arch is 1.91 m, and the width and height of the straight wall are 3.82 and 1.86 m, respectively. The calculation zone is 30 m wide and 30 m high. The excavation process of the tunnel is simulated in five steps from top to bottom. Material parameters of rock masses and material definition are shown in Table 1 and Fig.1, respectively. The weight of the upper strata is 12 MPa. The Young’s moduli of the straight wall and the floor (rock strata No.3–5 and No.8–10) are decreased to consider the softening mechanism of soft rock due to exposure to the air.

#### Table 1 Material parameters of rock masses in the transportation tunnel.

<table>
<thead>
<tr>
<th>Rock (material No.)</th>
<th>Unit weight (kN/m³)</th>
<th>Young’s modulus (GPa)</th>
<th>Poisson’s ratio</th>
<th>Friction coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium sandstone (1)</td>
<td>16.62</td>
<td>10</td>
<td>0.18</td>
<td>0.531</td>
</tr>
<tr>
<td>Fine-grained sandstone (2, 6, 7)</td>
<td>28.57</td>
<td>10</td>
<td>0.18</td>
<td>0.466</td>
</tr>
<tr>
<td>Siltstone (3, 8, 9, 10)</td>
<td>12.8</td>
<td>0.2</td>
<td>0.25</td>
<td>0.487</td>
</tr>
<tr>
<td>Mudstone (4, 5)</td>
<td>22.94</td>
<td>0.05</td>
<td>0.25</td>
<td>0.700</td>
</tr>
</tbody>
</table>

![Fig.1 Material definition of the transport tunnel.](image1)

The boundary conditions are given as: (1) the bottom is fixed in horizontal and vertical directions, (2) two sides are fixed in horizontal direction, and (3) vertical stress is applied on the top of the model.

Using the finite element meshes shown in Fig.2, the deformed meshes in the polar decomposition model with and without contact algorithm are shown in Fig.3. It can be observed that the non-penetrating condition of the floor and the sidewall on excavation boundary is guaranteed with this contact algorithm.

![Fig.2 Finite element meshes of the transportation tunnel.](image2)
4.2 Asymmetrical large deformation of a crossheading in Qishan coal mine

Asymmetrical deformation of a crossheading in inclined soft rock strata at the depth of 1000 m in Qishan coal mine is analyzed by the two methods.

The cross-section of the crossheading is also U-shaped. The radius of the arch is 2.1 m, and the width and height of the straight wall are 4.2 and 1.4 m, respectively. The calculation zone is 30 m in width and 30 m in height. The excavation process of the crossheading is simulated in three steps. The strata incline at 25° towards horizontal direction. Material parameters of rock masses, material definition and finite element meshes are shown in Table 2, Figs. 4(a) and (b), respectively. The overburden stress of the upper strata is 20 MPa. The boundary conditions are the same as those of the first example.

**Table 2** Material parameters of rock masses in the crossheading.

<table>
<thead>
<tr>
<th>Rock (material No.)</th>
<th>Unit weight (kN/m³)</th>
<th>Young’s modulus (GPa)</th>
<th>Poisson’s ratio</th>
<th>Friction coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy shale (1, 2)</td>
<td>26.5</td>
<td>4</td>
<td>0.2</td>
<td>0.839</td>
</tr>
<tr>
<td>Siltstone (3, 4)</td>
<td>25</td>
<td>2</td>
<td>0.3</td>
<td>0.700</td>
</tr>
<tr>
<td>Fractured siltstone (5–7)</td>
<td>20</td>
<td>0.5 (case a), 0.03 (case b)</td>
<td>0.34</td>
<td>0.577</td>
</tr>
</tbody>
</table>

At early stages, the crossheading was supported by bolts and shotcrete with wire meshes. U29 steel profile was used for repair. However, the deformation of the crossheading was not controlled. During the 75-day observation period, the average roof subsidence, sidewall shrinkage and roof-to-floor convergence were 60, 116 and 150 cm, respectively. The asymmetrical deformation observed was remarkable due to rock strata inclination, as shown in Fig. 5. To consider the softening mechanism of rocks due to exposure to the air, two values of Young’s modulus of the straight wall and the floor (rock strata No. 5–7), 500 and 30 MPa, noted as cases a and b (Table 2), respectively, are analyzed by the two models. In case a, the displacements obtained by the additive and polar large deformation analyses are convergent; while in case b, only that obtained by the additive large deformation analysis is convergent. The displacement vectors and deformation of surrounding rocks in the crossheading are shown in Figs. 6 and 7 and Table 3.

X-ray diffraction experiment finds that the contents of clay mineral in roof and floor strata are 56.2% and 79.3%, respectively. The contents of high-expansion minerals, such as illite-smectite, are 51% and 54%, respectively.
Fig. 7 Displacement vectors of the crossheading in case b by the additive decomposition analysis.

Table 3 Deformations of the crossheading.

<table>
<thead>
<tr>
<th>Case</th>
<th>Method</th>
<th>Roof subsidence (m)</th>
<th>Floor heaving (m)</th>
<th>Sidewall shrinkage (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Polar decomposition</td>
<td>0.47</td>
<td>0.204</td>
<td>0.637</td>
</tr>
<tr>
<td></td>
<td>Additive decomposition</td>
<td>0.107</td>
<td>0.176</td>
<td>0.596</td>
</tr>
<tr>
<td>b</td>
<td>Additive decomposition</td>
<td>0.226</td>
<td>0.527</td>
<td>1.673</td>
</tr>
</tbody>
</table>

It can be found that the deformation of the crossheading is asymmetrical due to the inclination of rock strata. Roof subsidence, floor heaving and sidewall shrinkage increase with the decrease in the Young’s modulus of surrounding rocks. For the same Young’s modulus, the deformation calculated by the polar decomposition model is smaller than that by the additive decomposition model. For extremely soft rock, only the large deformation analyzed by the additive decomposition model converges.

4.3 Large deformation of a coal tunnel excavated to different depths

Large deformations of a coal tunnel at −800 m level in Jiahe mine with different in-situ stresses are analyzed by the two decomposition models. For simplicity, the constitutive model used here is also elastic.

The cross-section of the coal tunnel is quadrilateral. Its width is 4.1 m, and heights of the left and right sidewalls are 2.8 and 2 m, respectively. The excavation process of the tunnel is simulated by one step. Material definition and finite element meshes of the tunnel are shown in Figs. 8(a) and (b), respectively. Material parameters of rock masses are listed in Table 4.

X-ray diffraction experiment shows that the contents of clay mineral in roof and floor are 50% and 52%, respectively. The content of high-expansion minerals, such as montmorillonite and illite-smectite, is 30%. Microcracks develop fully in the tunnel.

The tunnel was initially supported by bolts and shotcrete with wire meshes. During the 75-day observation period, the average roof subsidence, sidewall shrinkage and floor heaving were 42, 56 and 75 cm, respectively. The length of the gateway that has large sidewall convergence is about 40% of the total length. The amount of displacement is beyond the allowable value. The observed deformation of the tunnel is shown in Fig. 9. Three design depths of the tunnel were 421, 423 and 425 m, respectively. The calculated parameters of rock masses are listed in Table 4.

![Fig. 8 The calculation zone of the coal tunnel.](image)

![Fig. 9 Large deformation of the coal tunnel.](image)
tunnel, 500, 1,000 and 1,500 m, are studied. According to Brown and Hoek [10], the vertical in-situ stress (σ_v) and the ratio of the horizontal in-situ stress to the vertical one (k) in China can be estimated by

\[
\sigma_v = 0.027h \\
\frac{100}{h} + 0.3 \leq k \leq \frac{1}{500} + 0.5
\]

The in-situ stresses at the three depths are obtained according to Eq.(28), which are listed in Table 5. The measured in-situ stresses show that the value of k varies between 0.5 and 1.

**Table 5** In-situ stresses measured at the design depths.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Vertical stress (MPa)</th>
<th>Horizontal stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>13.5</td>
<td>6.75</td>
</tr>
<tr>
<td>1,000</td>
<td>27</td>
<td>13.5</td>
</tr>
<tr>
<td>1,500</td>
<td>40.5</td>
<td>20.25</td>
</tr>
</tbody>
</table>

The boundary conditions are given as follows. The bottom is fixed in the horizontal and vertical directions. The horizontal stress acts on the two sides, and the vertical stress is loaded at the top.

The displacements of surrounding rocks at different depths obtained by the two models are shown in Figs.10 and 11, respectively. The curves of sidewall shrinkage, floor heaving and roof subsidence vs. the tunnel depth are plotted in Fig.12.

For both models, the deformation of surrounding rocks increases with the increase in tunnel depth. It is noted that the floor heaving changes fastest, and the sidewall shrinkage changes smallest. At the same depth, the deformation calculated by the additive decomposition model is relatively smaller than that by the polar decomposition model.
the polar decomposition model.

5 Conclusions

The paper presents the technology for solving the contact problem with LDEAS 1.0, a 2D finite element software for large deformation analysis of soft rock engineering at great depth. By using the Lagrange multiplier method, the constraints of non-penetrating condition and the Coulomb friction are introduced into the basic equations in the form of incremental displacement. The deformation of a transportation tunnel in Qishan coal mine after excavation was analyzed using the software. It is demonstrated that the software can successfully eliminate the unreasonable penetration between the floor and sidewalls due to great floor heaving, and thus can model the dynamic contact and large deformation of soft rock tunnel reasonably.

Asymmetrical deformation of a crossheading in inclined soft rock strata at the depth of 1 000 m in Qishan coal mine was analyzed by the two methods. It is found that the deformation of the crossheading is asymmetrical due to the inclination of rock strata. For extremely soft rock, only the large deformation analyzed by the additive decomposition model converges.

Deformation of a coal tunnel at −800 m level in Jiahe mine was analyzed at three different depths by the two models. The results show that the deformation of surrounding rocks increases with the increase in tunnel depth for both models. It is noticeable that the floor heaving changes fastest, and the sidewall shrinkage changes smallest. At the same depth, the deformation calculated by the additive decomposition model is relatively smaller than that by the polar decomposition model.

References