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## Description and simulation of cyclic stress-strain response during residual stress relaxation under cyclic load

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### Abstract

Compressive residual stresses are often induced by mechanical surface treatment such as shot peening for the purpose of improving fatigue life of components. Cyclic relaxation of the induced compressive residual stress, however, reduces the benefit. For component design and life management purposes it is important to be able to assess the effect of residual stresses and their relaxation on fatigue life. The problem, however, is that the phenomenon of residual stress relaxation under cyclic mechanical loads is not well understood. Furthermore, there is the technical challenge of accurately quantifying residual stress relaxation *during* component operation. The difficulties in measuring residual stress relaxation in real-time impede the consideration of tracking and assessing its effect on remaining fatigue life.

In this paper a previously proposed cyclic hardening/softening model is adapted and modified to account for residual stress relaxation. Considering residual stress as virtual *mean stress* the cyclic stress-strain response during residual stress relaxation is described and simulated using the Matlab/Simulink Program. The virtual *mean stress* has a corresponding *mean strain* which conditions result in biased stress/strain limits under cyclic load. It is shown that residual stress relaxation is associated with *mean stress shift* in stress-axis direction and *mean strain shift* in the strain axis direction. The significance of this mean strain shift is briefly discussed in terms of tracking and quantitatively determining residual stress relaxation in real-time.

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**Keywords:** residual stress; cyclic stress; cyclic plastic deformation; mean-strain shift; stress relaxation.

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### 1. Introduction:

Residual stresses are those stresses existing along a cross-section of a component without applied external force [1]. In general compressive residual stresses at the component surface are beneficial while tensile residual stresses are detrimental to fatigue resistance. Therefore compressive residual stresses are often induced by mechanical surface treatment such as [2] shot-peening, laser shock peening, low-plasticity burnishing, autofretage, and hole-expansion, for the purpose of improving fatigue life of components.

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Cyclic relaxation of the induced compressive residual stress, however, reduces the benefit and would result in lower fatigue life than predicted or expected, e.g. [3]. For purposes of components design and life management, it is important to be able to assess the effect of residual stresses and their relaxation on fatigue crack initiation and propagation. To this end it is important to be able to monitor and quantitatively measure residual stress relaxation under cyclic/fatigue loads. A technical challenge, however, is the lack of understanding of the phenomenon and how to accurately quantify residual stress relaxation *during* component operation. These difficulties impede the consideration of tracking and assessing its effect on remaining fatigue life.

In this paper a previously proposed cyclic hardening/softening model [4] is adapted and modified to describe and simulate residual stress relaxation under cyclic loads. The objective is to contribute to the quest for a better understanding of the process of residual stress relaxation and to suggest a practical method of monitoring and evaluating reduction in residual stress in real-time.

### Nomenclature

$\sigma_a$	Applied stress amplitude
$\sigma_R$	Residual stress
$\sigma$	Total stress
$\sigma_{pt}$	Tensile peak stress
$\sigma_{pc}$	Compressive peak stress
$\sigma_{RD}$	Initial residual stress,
$\sigma_{Ri}$	Instantaneous residual stress
$\Delta\sigma_R$	Change (reduction) in residual stress
$\varepsilon$	Total strain
$\varepsilon_p$	Plastic strain
$\varepsilon_{max}$	Maximum strain in a cycle
$\varepsilon_{min}$	Minimum strain
$\varepsilon_m$	Mean strain
$\varepsilon_{m0}$	Initial mean (residual) strain
$\varepsilon_{mi}$	Instantaneous mean strain
$\Delta\varepsilon_m$	Mean-strain shift
$E$	Modulus of Elasticity
$\theta_0$	Initial strain-hardening coefficient
$N$	Number of cycles
$f$	Frequency of loading
$t$	Time

## 2. Description and Simulation of residual stress-relaxation under cyclic loads

In general, stress-strain response under fatigue load exhibits hysteresis behaviour. For materials that exhibit nonlinear/strain-hardening behaviour, the cyclic stress-strain response can be described as [4]:

$$\frac{d\sigma}{d\varepsilon} = E, \text{ for } \sigma\left(\frac{d\sigma}{d\varepsilon}\right) < 0, \quad (1)$$

$$\frac{d\sigma}{d\varepsilon} = E\left(1 - \frac{\sigma}{\sigma_{pt}}\right), \text{ for } \sigma\left(\frac{d\sigma}{d\varepsilon}\right) \geq 0 \text{ and } \sigma > 0, \quad (2a)$$

$$\frac{d\sigma}{d\varepsilon} = E\left(1 - \frac{\sigma}{\sigma_{pc}}\right), \text{ for } \sigma\left(\frac{d\sigma}{d\varepsilon}\right) \geq 0 \text{ and } \sigma < 0, \quad (2b)$$

$$\frac{d\sigma_{pt}}{d\varepsilon_p} = \theta_b\left(1 - \frac{\sigma_{pc}}{\sigma_{ptc}}\right) \quad (3)$$

$$\frac{d\sigma_{pc}}{d\varepsilon_p} = \theta_b\left(1 - \frac{\sigma_{ptc}}{\sigma_{pc}}\right) \quad (4)$$

Eqns. (1)-(4) have been used and or modified to simulate stress-strain behaviour of *cyclic hardening* of copper under strain-control loading [4] and *cyclic creep* of copper in stress-control loading [5]. It is hereby extended to describe and simulate relaxation of compressive residual stress under completely-reversed cyclic loads. To this end it is considered that residual stress is a *virtual mean stress* with a corresponding *virtual mean strain*. Therefore, combination of compressive residual stress and completely-reversed cyclic stress would yield biased cyclic loading conditions such that

$$\sigma_{pc} = -(\sigma_a + \sigma_R) \quad (5a)$$

$$\sigma_{pt} = \sigma_a - \sigma_R \quad (5b)$$

$$\sigma_{pt} - \sigma_{pc} = 2\sigma_a \quad (5c)$$

In the context of eqn.(5), stress relaxation would decrease the magnitude of  $\sigma_{pc}$  but increase that of  $\sigma_{pt}$ , and would result in a vertical shift of the stress-strain curve. Also, due to the biased loading, plastic deformation in the compressive (bias) direction would not be completely recovered during tensile loading, and would also result in a horizontal shift of the cyclic stress-strain curve.

Fig.1 shows the Matlab/Simulink block model diagram of eqns.(1)-(5) and used to simulate the cyclic stress-strain response of Fig. 2, showing relaxation of compressive residual stress under imposed constant amplitude load. Note that Fig.2 does not include the initial monotonic curve but commences from the initial compressive peak stress. Fig.3 shows simulated cyclic strain as a function of time as derived from the output of the scope of Fig.1. Although the strain varies with time the strain amplitude remains more or less constant. This indicates that stress relaxation is associated with mean-strain shift. Note that the time-axis may be changed to cycle number  $N$ , by multiplying the time ( $t$ ) by frequency ( $f$ ) of loading.

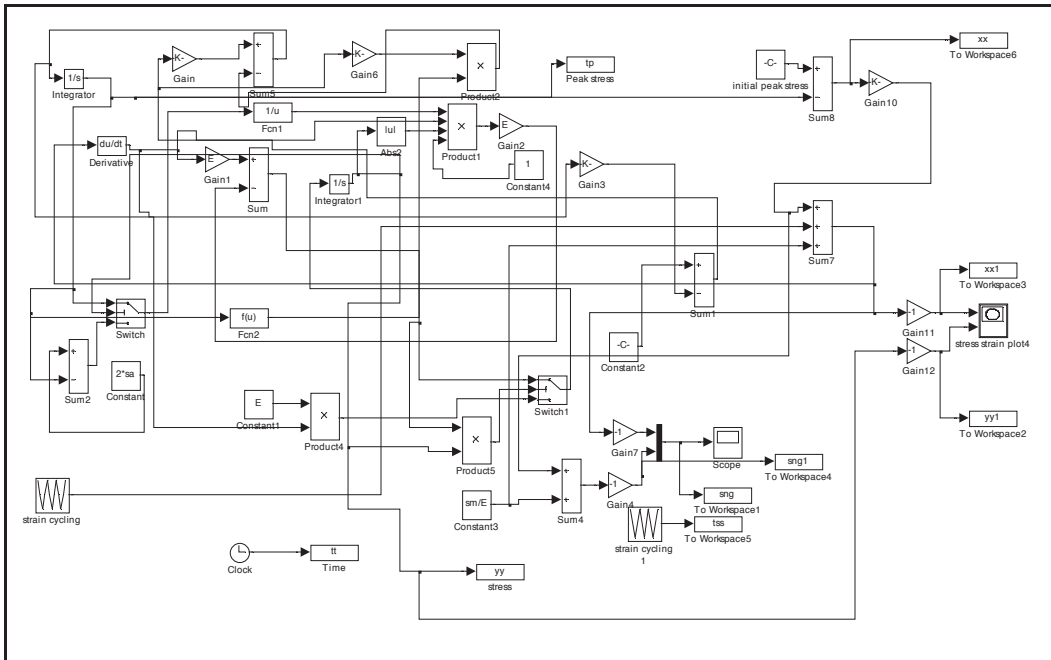


Fig.1. Matlab/Simulink Block Model for simulating residual stress relaxation under cyclic loads

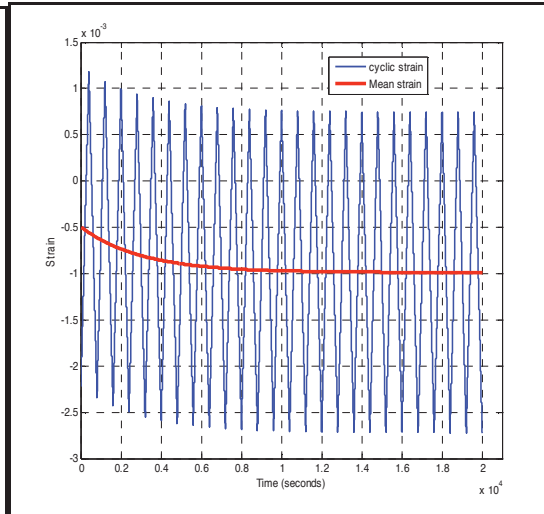
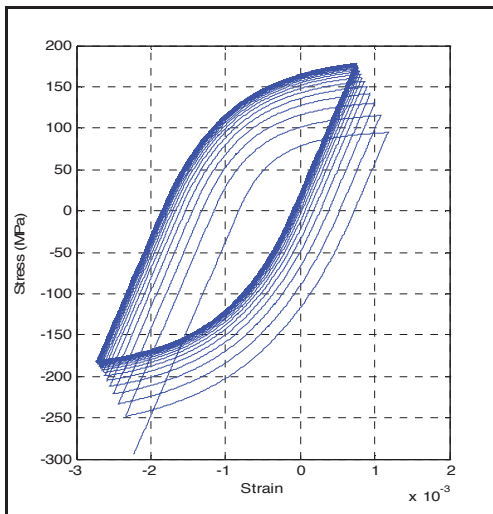
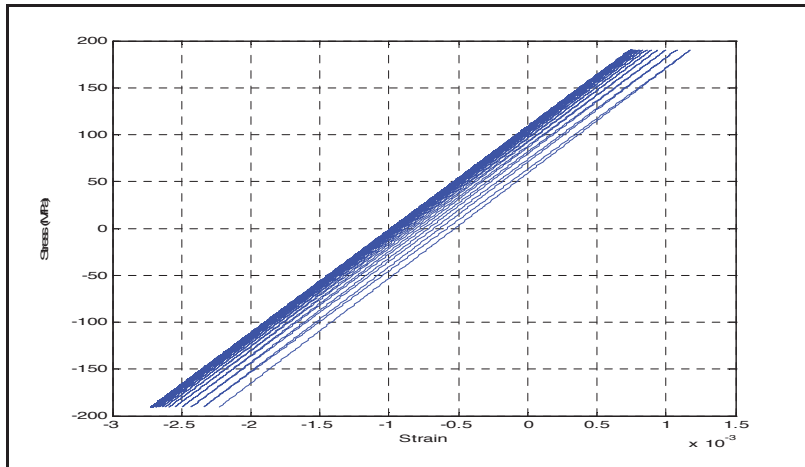


Fig. 2

Fig.3

Fig. 2 Simulated cyclic stress-strain response showing stress relaxation of compressive residual stress subjected to constant amplitude cyclic loading. Initial monotonic compressive loading curve is excluded. Fig.3 Simulated cyclic strain as a function of time indicating mean-strain shift during stress relaxation of compressive residual stress.

It is important to mention that the cyclic stress/strain characteristics of Fig.2 may not be observed experimentally from the use of strain-gauges. This is because strain gauge, like other strain sensors, cannot measure residual stress directly, but only measures changes in strain. Stress/strain output from strain-gauges would be similar to that of Fig. 4, which shows the variation of only the applied cyclic stress (without the residual stress) with increasing strain toward the biased loading direction. Rao *et al* [6]



**Fig. 4** Expected cyclic stress-strain output from strain gauge (or strain sensor) during stress relaxation of compressive residual stress under constant amplitude loading. The shift of mean strain with cycle is evidence of stress relaxation.

observed similar strain gauge output for welded specimens of tensile residual stress subjected to constant amplitude cyclic stress. In this case, however, the strain-shift was in the tensile (biased loading) direction.

### 3. Discussion

Under cyclic loading residual stress may be considered as mean-stress, resulting in biased stress/strain loading condition. Cyclic plastic deformation in the biased direction would not be completely recovered in the opposite direction, and would result in the shift of the mean strain in the direction of bias and away from zero-mean position. Also, plastic deformation would relax residual stress and, for constant stress amplitude, would result in the vertical shift of the stress-strain curves in the anti-bias direction towards the zero-mean stress position. The shift in the vertical direction is a mean-stress shift and is equivalent to reduction in residual stress. Obviously, stress relaxation (mean-stress shift) is associated with mean-strain shift which occurs as a result of stress redistribution.

Under completely-reversed constant amplitude loading, residual stress  $\sigma_R$  and its corresponding strain  $\epsilon_m$  form a mean-strain/mean-stress coordinate  $(\epsilon_m, \sigma_R)$  system on the stress-strain axes. Now let the initial coordinates with respect to the loading cycle be  $(\epsilon_{m0}, \sigma_{R0})$  and the coordinates after stress relaxation be  $(\epsilon_{mi}, \sigma_{Ri})$ . Then the shift in the vertical (stress) axis is equal to  $(\sigma_{R0} - \sigma_{Ri})$  while that of the horizontal (strain) axis is  $(\epsilon_{m0} - \epsilon_{mi})$ . Noting that both residual stress and residual strain are elastic the ratio of change in residual stress to change in mean strain must be equal to the elastic modulus of the material. That is

$$\frac{(\sigma_{R0} - \sigma_{Ri})}{(\epsilon_{m0} - \epsilon_{mi})} = \frac{\Delta\sigma_R}{\Delta\epsilon_m} = E \quad (6a)$$

$$\epsilon_m = \frac{\epsilon_{max} + \epsilon_{min}}{2} \quad (6b)$$

Eqn.(6) provides the key to monitoring and evaluating residual stress relaxation in real-time as follows:

- Carefully *attach* resistance strain-gauge to position of interest on the component prior to the application of cyclic load
- *Record* the variation in cyclic strain with time (or cycle) under service loads, as in Fig. 3. This may be done by means of X-Y plotter or computer program.
- *Extract* or derive the mean-strain versus time (or cycle) curve or relation from the cyclic strain-time (or cycle) record. The mean strain may be derived according to eqn. (6b).

- Evaluate the reduction in residual stress at given time (or cycle) as the product of the *change in mean strain* and the *elastic modulus of the material*, as indicated by (Eq. (6a)).

This approach of monitoring and evaluating residual stress relaxation is real-time, simpler, cheaper, and less time-consuming as compared to currently used methods such as the X-ray diffraction methods, e.g. [7-9] and hole-drilling techniques [10]. Furthermore, it does not require much skill in the determination of stress-relaxation.

#### 4. Conclusions

The theoretical stress-strain response during compressive residual stress relaxation under cyclic loading is described and simulated using the Matlab/Simulink Program. The following conclusions are derived:

- Under cyclic loads residual stress can be regarded as mean-stress, causing biased stress cyclic loading conditions within the material
- Cyclic plasticity occurs when the sum of cyclic stress and residual stress exceeds the local yield stress of the material. The occurrence of plastic deformation causes reduction in the residual stress which, under the constant amplitude load, causes a shift in the mean stress position
- Plastic deformation in the biased-strain direction (compressive direction) would not be completely recovered by the opposite loading direction (tensile direction) which, under constant amplitude loading, results in the shift of *mean-strain position*.
- The cyclic stress-strain behaviour during residual stress relaxation can be simulated using Matlab/Simulink Block Model of Fig.1, as shown in Fig. 2.
- By means of resistance strain-gauge attached to the component, the mean-strain shift can be recorded during component operation, from which the reduction in residual stress may be evaluated, from eqn. (6a).
- Techniques may now be developed for monitoring and evaluating stress relaxation in real-time, for purposes of predicting and managing fatigue life of critical components.

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