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Addendum

Addendum to: “Dislocations in second strain gradient elasticity” [Int. J. Solids Struct. 43 (2006) 1787–1817]

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In the present note we want to give some remarks on the general case of anisotropic second strain gradient elasticity given by Lazar et al. (2006) and especially on the symmetries of the material tensors entering in the generalized stress–strain relationships. In Lazar et al. (2006), the strain energy density W has the following form

$$W = \frac{1}{2}C_{ijkl}E_{ij}E_{kl} + \frac{1}{2}C_{ijklmn}E_{ij,k}E_{lm,n} + \frac{1}{2}C_{ijklmnpq}E_{ij,kl}E_{mn,pq} + D_{ijklm}E_{ij}E_{kl,m} + D_{ijklmn}E_{ij}E_{kl,mn} + D_{ijklmnp}E_{ij,k}E_{lm,np}, \tag{1}$$

where the last three contributions are cross terms. From Eq. (1) we obtain the following constitutive equations for the second (Cauchy-like), third (double), fourth (triple) order stress tensors

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}E_{kl} + D_{ijklm}E_{kl,m} + D_{ijklmn}E_{kl,mn}, \\ \tau_{ijk} &= D_{lmijk}E_{lm} + C_{ijklmn}E_{lm,n} + D_{ijklmnp}E_{lm,np}, \\ \tau_{ijkl} &= D_{mnijkl}E_{mn} + D_{mnpijkl}E_{mn,p} + C_{ijklmnpq}E_{mn,pq}, \end{aligned} \tag{2}$$

where $\sigma_{ij} = \partial W/\partial E_{ij}, \tau_{ijk} = \partial W/\partial E_{ij,k}$ and $\tau_{ijkl} = \partial W/\partial E_{ij,kl}$. Here $C_{ijkl}, C_{ijklmn}, C_{ijklmnpq}, D_{ijklm}, D_{ijklmn}$ and $D_{ijklmnp}$ are the material tensors with the constitutive coefficients, which Lazar et al. (2006) assumed, for simplicity, to possess additional symmetry properties than those implied by their definitions (see also Agiasofitou and Lazar (2009)). In general, the material tensors in anisotropic second strain gradient elasticity possess only the following symmetries:

$$\begin{aligned} &C_{(ij)(kl)} \equiv C_{(kl)(ij)} \equiv C_{(ij)|(kl)}, \\ &C_{(ij)k(lm)n} = C_{(lm)n(ij)k} \equiv C_{(ij)k|(lm)n}, \\ &C_{(ij)(kl)(mn)(pq)} = C_{(mn)(pq)(ij)(kl)} \equiv C_{(ij)(kl)|(mn)(pq)}. \end{aligned} \tag{3}$$

The material tensors $D_{(ij)(kl)m}, D_{(ij)(kl)(mn)}$ and $D_{(ij)k(lm)(np)}$ do not possess additional symmetries. The additional symmetries for $C_{(ij)k(lm)n}$ and $C_{(ij)(kl)(mn)(pq)}$ given in (2.27) by Lazar et al. (2006) were obtained by substituting Eq. (2) into the stress equilibrium condition

$$\mathring{\sigma}_{ij,j} = 0, \qquad \mathring{\sigma}_{ij} = \sigma_{ij} - \tau_{ijk,k} + \tau_{ijkl,kl}. \tag{4}$$

The terminology adopted by Lazar et al. (2006) for $\mathring{\sigma}_{ij}$ was ‘total stress’ whereas the terminology ‘Cauchy stress’ was adopted for the stress σ_{ij} with the dimension of force/(length)^2, while the terminology ‘double stress’ and ‘triple stress’ was adopted for the hyperstress τ_{ijk} which has the dimension of force/length and τ_{ijkl} which has the dimension of force. Alternatively, the self-equilibrating stress $\mathring{\sigma}_{ij}$ could be identified with the Cauchy stress (e.g. Aifantis (2010) and references quoted therein) and then, Eq. (4.b) would play the role of a ‘constitutive equation’ for $\mathring{\sigma}_{ij}$.

In order to obtain the additional symmetries for $D_{(ij)(kl)m}$ and $D_{(ij)k(lm)(np)}$ given in (2.27) by Lazar et al. (2006), we assumed that the crossing terms of the tensors of odd rank cancel each other out in Eq. (4). Moreover, the symmetry for the material tensor $D_{(ij)(kl)(mn)}$ which is of even rank was assumed for simplicity. Of course, from the group theoretical point of view, the material tensors possess only the symmetries given above, i.e. those listed in Eq. (3).

If we substitute the constitutive relations given by Eq. (2) into the equilibrium condition given by Eq. (4), we find for the general anisotropic case the result:

$$\begin{aligned} &C_{(ij)|(kl)}E_{kl,j} - C_{(ij)k|(lm)n}E_{lm,jkn} + C_{(ij)(kl)|(mn)(pq)}E_{mn,jklpq} \\ &\quad + (D_{(ij)(kl)m} - D_{(kl)(ij)m})E_{kl,jm} + (D_{(ij)(kl)(mn)} + D_{(kl)(ij)(mn)})E_{kl,jmn} \\ &\quad - (D_{(ij)k(lm)(np)} - D_{(lm)k(ij)(np)})E_{lm,jknp} = 0. \end{aligned} \tag{5}$$

In addition, the stress tensor $\mathring{\sigma}_{ij}$, for the general case of anisotropic second strain gradient elasticity, reads

$$\begin{aligned} \mathring{\sigma}_{ij} &= C_{(ij)|(kl)}E_{kl} - C_{(ij)k|(lm)n}E_{lm,kn} + C_{(ij)(kl)|(mn)(pq)}E_{mn,klpq} \\ &\quad + (D_{(ij)(kl)m} - D_{(kl)(ij)m})E_{kl,m} + (D_{(ij)(kl)(mn)} + D_{(kl)(ij)(mn)})E_{kl,mn} \\ &\quad - (D_{(ij)k(lm)(np)} - D_{(lm)k(ij)(np)})E_{lm,knp}. \end{aligned} \tag{6}$$

Lazar et al. (2006) considered, in Eqs. (2.23)–(2.27), the case of anisotropic second strain gradient elasticity. Afterwards they specialized to the isotropic case. All the solutions derived by Lazar et al. (2006) are valid for the isotropic case.

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