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# Dark matter from a gas of wormholes

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**Abstract**

The simplistic model of the classical spacetime foam is considered, which consists of static wormholes embedded in Minkowski spacetime. We explicitly demonstrate that such a foam structure leads to a topological bias of point-like sources which can equally be interpreted as the presence of a dark halo around any point source. It is shown that a non-trivial halo appears on scales where the topological structure possesses local inhomogeneity, while the homogeneous structure reduces to a constant renormalization of the intensity of sources. We also show that in general dark halos possess both (positive and negative) signs depending on scales and specific properties of the topological structure of space.


## 1. Introduction

The discrepancy between the luminous matter and the dynamic, or gravitating mass was first identified in clusters of galaxies [1]. Since then it has widely been accepted that the leading contribution to the matter density of the Universe comes from a specific non-baryonic form of matter (Dark Matter (DM), e.g., see Ref. [2]). Apart from some phenomenological properties of DM (it starts to show up in galactic halos, it is non-baryonic, it is cold at the moment of recombination, it remains to be cold in clusters and at larger scales (e.g., see Ref. [3] and references therein), but in a strange way, it turns out to be worm in galaxies[1] [4], etc.) nothing is known of its nature. Particle physics suggests various hypothetical candidates for dark matter, while we still do not observe such particles in direct laboratory experiments. Moreover, DM displays so non-trivial properties (it is worm or self-interacting in galaxies, however it was cold at the moment of recombination and it is still cold on larger (than galaxies) scales, DM fraction is practically absent in intracluster gas [6]), etc., that it is difficult to find particles capable of reconciling such observations. These facts suggest us to try, as an alternative to DM hypothesis, the possibility to interpret the observed discrepancy between luminous and gravitational masses as a violation of the law of gravity.

Possible violations of the gravity law (or modifications of general relativity (GR)) have widely been discussed, e.g., see Refs. [7,8]. The common feature of such theories is the presence of some characteristic energy scale $E_0$ (e.g., some kind of a mass of gravitons Ref. [8] or even a fundamental acceleration in the modified Newtonian dynamics [9]) which represents the threshold upon which DM-type phenomena (violations of the gravity law) start to show up. However, DM features pointed out above clearly indicate a non-linearity of DM phenomena and that there cannot be a single fundamental scale in DM physics. Again, observations demonstrate that DM halos have different properties (distributions) in different galaxies which also cannot be prescribed to a single fundamental scale. In other words, it turns out to be rather difficult to get a modification of GR which is flexible enough to reconcile all the observational DM data. Moreover, there exist fundamental theoretical arguments (e.g., the massless nature of gravitons, etc.) which make any modification to be undesirable from particle physics standpoint.

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[1] Cold DM should necessary form a cusp in centers of galaxies $\rho_{\rm DM} \sim 1/r$ [5], while observations definitely show the cored distribution $\rho_{\rm DM} \sim$ const [4]. The only way to destroy the cusp and get the cored distribution is to introduce some self-interaction in DM or to consider worm DM. Both possibilities are rejected at large scales by observing $\Delta T/T$ spectrum (e.g., see Ref. [3] and references therein).

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It turns out however, that a modification of the theory itself is not the only possibility to violate the Newton's law. The standard Newton's law can easily be modified when the topological structure of space is different from $R^3$. In the first place the non-trivial topological structure was shown to display itself by the topological bias of all physical sources (e.g., see Ref. [10] and references therein) which is equivalent to the presence of DM. Moreover, there exist the very basic theoretical arguments in favour of the presence of a non-trivial topological structure of space. Indeed, as it was first suggested by Wheeler, at Planck scales spacetime should undergo quantum topology fluctuations (the so-called spacetime foam) [11]. Such fluctuations were strong enough to form a foam-like structure of space during the quantum stage of the evolution of the Universe. There are no convincing theoretical arguments of why such a foam-like structure should decay upon the quantum stage. Moreover, the presence of a considerable portion of Dark Energy in the present Universe [3] (and in the past, on the inflationary stage) may serve as the very basic indication of a non-trivial topological structure of space.[2]

We note that inflationary stage in the past [12] should enormously stretch all physical scales and, therefore, we should expect relics of the primordial foam-like structure to survive at very large (astronomically considerable) scales. The foam-like structure, in turn, was shown to be flexible enough to account for the all the variety of DM phenomena (e.g., see Refs. [10, 13]), for parameters of the foam may arbitrary vary in space to produce the observed variety of DM halos in galaxies (e.g., the universal rotation curve for spirals constructed in Ref. [14] on the basis of the topological bias perfectly fits observations). Moreover, the topological nature of the bias means that the DM halos surrounding point-like sources appear due to the scattering on topological defects (on the foam-like structure) and if a source radiates, such a halo turns out to be luminous too [13] which seems to be the only way to explain naturally the observed absence of DM fraction in intracluster gas [6].

A general foamed Universe can be viewed as the standard Friedman model filled with a gas of wormholes [13]. However, a priori it is not clear if the presence of such a gas is sufficient to get DM phenomena. In the present Letter we consider the simplistic exact model of the spacetime foam, which consists of a static gas of wormholes embedded in the Minkowski space and demonstrate how basic DM effects can be explicitly evaluated. We note that simplistic models of the spacetime foam have been already considered in the literature (e.g., see Ref. [15] and references therein where also other topological defects were considered). However the primary interest was there focused on setting observational bounds on the foam-like structure at extremely small scales (which correspond to the energies higher than 100 MeV), while DM phenomena suggest that the characteristic scale of the spacetime foam (and respectively of wormholes) should be of the galaxy scale, e.g., of the order of a few kpc. The rigorous bounds obtained indicate that at small scales spacetime is extremely smooth up to the scales $\gtrsim 10^2 L_{\text{pl}}$ (where $L_{\text{pl}}$ is the Planck length), that was to be expected. Indeed, at those scales topology fluctuations have only virtual character and due to the renormalizability of physical field theories should not directly contribute to observable (already renormalized) effects. Topology fluctuations were strong only during the quantum stage of the evolution of the Universe, while the subsequent inflationary phase considerably increases all characteristic scales of the foam. Therefore, the only possibility to find effects of the relic foam-like structure of space is to seek for them at very large scales, rather than at very small ones.

## 2. Modification of the Newton's law in the presence of a single wormhole

In the present section we, for the sake of simplicity, consider the flat $R^3$ space. We consider first a single wormhole, which represents a couple of conjugated spheres $S_\pm$ of the radius $a$ and with a distance $d = |\vec{R}_+ - \vec{R}_-|$ between centers of spheres. The interior of the spheres is removed and surfaces are glued together. Our aim is to find the Green function $\Delta G(r, r_0) = 4\pi\delta(r - r_0)$ for such a topology.

In the absence of the wormhole the solution is well known $G_0(r) = -1/r$ which represents the standard Newton's law. In the case of a non-trivial topology of space (i.e., in the presence of the wormhole) the Newton's law violates, however, we still can use the standard Green function (the standard Newton's law), while the non-trivial topology (i.e., the proper boundary conditions) will be accounted for by the topological bias of the source [10,13] $\delta(r - r_0) \to \delta(r - r_0) + b(r, r_0)$, where $b(r, r_0) = \sum e_A \delta(r - f_A(r_0))$ describes ghost images which produce the topological corrections to the Newton's law. The equivalent description is the introduction of the topological permeability $\hat{\varepsilon}$, i.e., the modification of the equation itself $\Delta\hat{\varepsilon}G(r, r_0) = 4\pi\delta(r - r_0)$ which gives[3] $G(r, r_0) = -\hat{\varepsilon}^{-1}(1/|r - r_0|) = -1/|r - r_0| - \int b(r, r')/|r' - r_0| dV'$. In the situation when $\hat{\varepsilon} = \text{const}$ (e.g., at very large scales) the topological permeability renormalizes merely the value of a source $\Delta G(r, r_0) = (4\pi/\hat{\varepsilon})\delta(r - r_0)$ (or equivalently the value of the interaction constant $\gamma \to \gamma/\hat{\varepsilon}$ [17]).

We note that the wormhole can be equally viewed as a couple of spherical conjugated mirrors, so that while the incident signal falls on one mirror the reflected signal comes from the conjugated mirror. We point also out that gas of spherical mirrors has many common features with the gas of wormholes. In particular, in the case of a homogeneous distribution of stationary sources, statistical properties of the Newton or gravitational

[2] Recall that DE violates the energy domination condition. Save speculative theories (or pure phenomenological models), there is no matter which meets such a property. However in the presence of a non-trivial topology, vacuum polarization effects are known to give rise quite naturally to such a form of matter. By other words, up to date the only rigorous way to introduce Dark Energy is to consider the vacuum polarization effects on manifolds of a non-trivial topological structure.

[3] In the case of homogeneous and isotropic topological structure (i.e., when $b = b(|r - r_0|)$) the relation between $b$ and $\hat{\varepsilon}$ is trivial in the Fourier representation, i.e., $\varepsilon(k) = 1/(1 + b(k))$.

potential remain the same. As we shall see the difference appears only when we consider the topological permeability of space. In the case of wormholes we can distinguish two types of permeabilities[4]: one which gives $\hat{\varepsilon} < 1$ and the second part with $\hat{\varepsilon} > 1$, while mirrors possess only one type of permeability ($\hat{\varepsilon} < 1$, i.e., anti-screening). The difference appears also when we consider the propagation of signals in such a medium, which we will discuss elsewhere. However one spherical mirror gives the simplest example of a non-trivial topology and from the methodical standpoint it is more convenient to start with this case[5] which we latter on generalize to the case of a wormhole.

### 2.1. *The case of a single spherical mirror*

Consider a spherical mirror of the radius $a$ and at the position $\vec{R}$. Then points $|\vec{r} - \vec{R}| < a$ represent the non-physical region and the exact form of the topological bias and the Green function depends on the way of how we continue coordinates in the non-physical region. We need not to say that the values of the Green function in the physically admissible region $|\vec{r} - \vec{R}| \geqslant a$ do not depend on the continuation at all. However the bias and the form of the Green function in the "non-physical" region do depend on this.

First, we consider the continuation which we use in astrophysics. Recall that in astrophysics we map the physical space $\mathcal{M}$ onto $R^3$ as follows [13]. We take a point $O$ in space (the position of an observer) and issue geodesics from $O$ in every direction. Then points in $\mathcal{M}$ can be labeled by the distance from $O$ and by the direction of the corresponding geodesic. In other words, for an observer at $O$ the space will always look as $R^3$. However if we take a point $P \in \mathcal{M}$, there may exist many homotopically non-equivalent geodesics connecting $O$ and $P$. Thus, the point $P$ will have a number of images in $R^3$. Recall that the observer might determine the topology of $\mathcal{M}$ by noticing that in the observed space $R^3$ there is a fundamental domain $\mathcal{D}$ such that every radiation or gravity source in $\mathcal{D}$ has a number of copies $N$ outside $\mathcal{D}$. Then the actual manifold $\mathcal{M}$ is obtained by identifying the copies $R^3/N$.

In the case of one spherical mirror every point in the physically admissible region has only one copy in the non-physical region $|\vec{r} - \vec{R}| < a$ which corresponds to the one-to-one map (i.e., the reflection law) $\vec{r} \to \vec{f}(r) = \vec{R} + a^2(\vec{r} - \vec{R})/(\vec{r} - \vec{R})^2$. In this picture the interior of the sphere $|\vec{r} - \vec{R}| \leqslant a$ is absolutely equivalent to the outer region, i.e., $|\vec{f}(\vec{r}) - \vec{R}| \geqslant a$ and the metric within the sphere $|\vec{r} - \vec{R}| \leqslant a$ is flat and is given by $dl^2 = d\vec{f}^2(r)$. In particular, in this case the volume within the sphere is infinite, for it coincides exactly with the volume of the outer region of the sphere. However while in the outer region geodesics are straight lines in the inner region geodesics are represented by circles which go through the center of the sphere. In such a picture every source of gravity at the position $\vec{r}_0$ ($r_0 > a$) will be accompanied with the only source at the position $\vec{r}_1 = \vec{f}(r_0)$ within the sphere ($|r_1 - R| < a$).

In the present Letter we however will use the more standard way when the sphere represents merely a portion of $R^3$ with the standard flat metric ($dl^2 = d\vec{r}^2$) within it and the volume of the sphere being $V(a) = \frac{4}{3}\pi a^3$. In such a case we can use the inversion method (see the standard books, e.g., Ref. [16]). In the case of one spherical mirror the proper boundary conditions can be satisfied if we place within the sphere a couple of odd image ("ghost") sources, i.e.,

$$\delta(\vec{r} - \vec{r}_0) \to \delta(\vec{r} - \vec{r}_0) + \frac{a}{y}\delta(\vec{r} - \vec{r}_1) - \frac{a}{y}\delta(\vec{r} - \vec{R}), \tag{1}$$

where $\vec{r}_1 = \vec{R} + \frac{a^2}{y^2}\vec{y}$ and $\vec{y} = \vec{r}_0 - \vec{R}$. The negative source, at the center of the sphere, is here added to compensate the reflected source at $\vec{r}_1$. Physically, this means that the mirror does not radiate (virtual photons or gravitons) itself but only redistributes the existing radiation. In the electrodynamics this means that such a medium (gas of mirrors) possesses some polarization property which gives rise to the origin of magnetic and dielectric permeabilities Ref. [16] (see also Ref. [15]).

Thus (1) defines the topological bias in the form

$$b(r, r_0) = b^{(+)} - b^{(-)} = \frac{a}{y}\big(\delta(\vec{r} - \vec{r}_1) - \delta(\vec{r} - \vec{R})\big), \tag{2}$$

which has the property $\int b(r, r_0)\, d^3r = 0$. We see that the bias is solely defined in the non-physical region (interior of the sphere) and therefore its values depend essentially on the way of continuation discussed. When we use the astrophysical way the interior and the outer region of the sphere are simply coincide and we need not to introduce the additional negative source[6] and we will get $\int_{|r-R|<a} b(r, r_0)\, d^3r \equiv \int_{|r-R|>a} \delta(\vec{r} - \vec{r}_0)\, d^3r = 1$.

Thus, in the physically admissible region ($|\vec{r} - \vec{R}| \geqslant a$) the exact form of the Green function is given by

$$-G(r) = \frac{1}{|\vec{r} - \vec{r}_0|} + \frac{a}{y}\frac{1}{|\vec{r} - \vec{r}_1|} - \frac{a}{y}\frac{1}{|\vec{r} - \vec{R}|}, \tag{3}$$

while its form in the non-physical region ($|\vec{r} - \vec{R}| < a$) depends essentially on the continuation procedure. The standard continuation gives the same expression (3) for the non-physical region $|\vec{r} - \vec{R}| < a$, while the continuation by the astrophysical way gives $G \to G(f(r))$ (i.e., while $r$ runs the non-physical region $|\vec{r} - \vec{R}| < a$, $f(r)$ runs the region $|\vec{f}(r) - \vec{R}| > a$).

### 2.2. *The case of a wormhole*

In the case of a wormhole we have a couple of conjugated mirrors, so that while the incident signal falls on one mirror the reflected signal comes from the conjugated mirror. Thus, we have to replace the positive image source in (1) into the conjugated sphere and rotate it with some matrix $U$, which

[4] As it will be shown latter, in the case of mirrors the susceptibility of space $\chi = (\hat{\varepsilon} - 1)/4\pi$ is always negative (anti-screening $\chi < 0$), i.e., the polarizability is opposite to the external field, while for wormholes there appear two types of polarization ($\chi > 0$ and $\chi < 0$). In analogy with the magnetic susceptibility one can speak of dia- and para-susceptibilities of space.

[5] The simplest example is given by a plane mirror, but this case is trivial and we think that every reader can reconstruct such a case by himself.

[6] On the surface of the mirror the negative surface source is automatically generated, for the distance between opposite points on the sphere is infinite.

defines the gluing procedure. Moreover, every image (ghost sources, positive and negative ones) undergoes again the reflections upon the conjugated mirror and thus produces a countable set of images. Let $\vec{R}_+$, $\vec{R}_-$ be the vectors for the positions of centers of the spheres and let us define the transformations

$$\vec{r}_{\pm 1} = T_\pm \vec{r}_0 = \vec{R}_\pm + \frac{a^2}{(\vec{r}_0 - \vec{R}_\mp)^2} U^{\pm 1}(\vec{r}_0 - \vec{R}_\mp). \tag{4}$$

Applying such a transformation many times we get for the positions of extra positive images

$$\vec{r}_{\pm n} = T_\pm^n \vec{r}_0 = \vec{R}_\pm + \frac{a^2}{(T_\pm^{n-1}\vec{r}_0 - \vec{R}_\mp)^2} \times U^{\pm 1}\big(T_\pm^{n-1}\vec{r}_0 - \vec{R}_\mp\big), \tag{5}$$

which define the positive part of the topological bias in the form

$$b^{(+)}(r, r_0) = \sum_{n=0}^{\infty} b_{+n}^{(+)} \delta\big(\vec{r} - T_+^{n+1}\vec{r}_0\big) + b_{-n}^{(+)} \delta\big(\vec{r} - T_-^{n+1}\,\vec{r}_0\big), \tag{6}$$

where

$$b_{\pm n}^{(+)} = \prod_{m=0}^{n} \frac{a}{|T_\pm^m \vec{r}_0 - \vec{R}_\mp|}. \tag{7}$$

In the analogous way by means of the use of the transformation (4) and starting with sources $\frac{a}{|\vec{r}_0 - \vec{R}_\pm|}\delta(\vec{r} - \vec{R}_\pm)$ we define the negative part of the bias $b^{(-)}(r, r_0)$. We note that all images $\vec{r}_{\pm n}$ lie within the respective spheres $S_\pm$.

The above expressions solve the problem posed and the exact Green function (e.g., the gravitational potential for a point source at the position $r_0$) is given by

$$-G(r) = 1/|r - r_0| + \sum b_{\pm n}^{(+)}/|r - r_{\pm n}| - \sum b_{\pm m}^{(-)}/\big|r - r_{\pm m}^{(-)}\big|.$$

Here the first term represents the standard Newton's law, while the sums describe topological corrections, which in observations can be equally interpreted as the presence of some amount of extra (or dark) matter.

Consider now the degree of polarization of space produced by a wormhole. Since by definition the topological bias has the property $\int b(r, r_0)\, d^3r \equiv 0$ and the characteristic distance between spheres is $d = |\vec{R}_+ - \vec{R}_-|$ it can be expressed by the positive part of the bias, i.e.,

$$Q^{(+)} = \int b^{(+)}(r, r_0)\, d^3r = \sum_{n=0}^{\infty} b_{+n}^{(+)} + b_{-n}^{(+)}. \tag{8}$$

Consider the first term of this sum, i.e., $\sum b_{+n}^{(+)} = I_+$. It is convenient to extract the common multiplier $I_+ = b_{+0}^{(+)}(1 + \sum_{n=1}^{\infty} b_{+n}^{(+)}/b_{+0}^{(+)})$, where $b_{+0}^{(+)} = a/|\vec{r}_0 - \vec{R}_-|$ depends essentially on the positions of the source and the wormhole. Suppose that the wormhole obeys the condition $d \gg a$. Then in the product (7) for $m \geqslant 1$ we can use the approximation $|T_\pm^m \vec{r}_0 - \vec{R}_\mp| \approx |\vec{R}_\pm - \vec{R}_\mp| = d$ (the next terms have the order $a/d \ll 1$). In this approximation coefficients (7) take the form

$$b_{\pm n}^{(+)} \simeq b_{\pm 0}^{(+)} \left(\frac{a}{d}\right)^n \tag{9}$$

and the sum gives

$$Q^{(+)} = \big(b_{+0}^{(+)} + b_{-0}^{(+)}\big) \sum_{n=0}^{\infty} \left(\frac{a}{d}\right)^n = \left(\frac{a}{|\vec{r}_0 - \vec{R}_-|} + \frac{a}{|\vec{r}_0 - \vec{R}_+|}\right) \frac{d}{d - a}. \tag{10}$$

The factor $d/(d - a) \approx 1$ describes corrections of multiple reflections of images while the leading contribution comes from the first order images. We recall that by the construction the source lies always outside the spheres, which means that $a/|\vec{r}_0 - \vec{R}_\pm| \leqslant 1$ (the equality can be achieved only when the source comes close to one of the spheres $S_\pm$). Thus we see that the amplitude of additional sources produced by a single wormhole may reach the order $\sim 1$.

The expression (9) shows that the intensity of a ghost source which corresponds to the multiple reflection (of the order $n$) decreases as $(a/d)^n$ (recall that $a/d < 1$). Then for wormholes obeying the condition $a/d \ll 1$ it is sufficient to retain only the first order images which define the bias

$$b(r) = \frac{a}{R_-}\big[\delta(\vec{r} - \vec{r}_{+1}) - \delta(\vec{r} - \vec{R}_-)\big] + \frac{a}{R_+}\big[\delta(\vec{r} - \vec{r}_{-1}) - \delta(\vec{r} - \vec{R}_+)\big] \tag{11}$$

where we have used the coordinate system in which $r_0 = 0$ and values $\vec{r}_{\pm 1}$ are defined by (4).

## 3. Static gas of wormholes

In what follows we, for the sake of simplicity, will assume that the source is at the origin $r_0 = 0$. First, we consider some general qualitative properties of the bias which can be obtained from simple geometric consideration and which should be valid also in more general situations (e.g., when the non-trivial topology cannot be reduced to a gas of wormholes).

Indeed, the basic effect of a non-trivial topology is that it cuts some portion of the volume of the coordinate space. Therefore, the volume of the physically admissible region becomes smaller, while the density of virtual gravitons/photons (or equivalently, the density of lines of the strength of force) becomes higher. From the standard flat space standpoint this will effectively look as if the amplitude of a source renormalizes. Let $M$ be the value of the source and consider a ball of the radius $r$ around the source. Then the physical volume of the ball is $V_{\rm ph}(r) = V_{\rm coor}(r) - V_m(r)$, where the coordinate volume is $V_{\rm coor} = (4\pi/3)r^3$ and $V_m(r)$ is the volume of mirrors or wormholes which get into the ball. Therefore, the actual value of the surface which restricts the ball is given by $S_{\rm ph}(r) = \frac{d}{dr} V_{\rm ph}(r)$. Then we can use the Gauss divergency theorem to estimate the renormalization of the source. Indeed, the Gauss theorem states that

$$\int\limits_{S(R)} n\nabla G\,dS = 4\pi \int\limits_{r<R} M\delta(r)\,dV = 4\pi M,$$

where $G$ is the true Green function. Then for an isotropic distribution of wormholes it defines the normal projection of the force as $F_n(R) = n\nabla G = 4\pi M/S_{\rm ph}(R)$.

This can be rewritten as in the ordinary flat space (i.e., in terms of the standard Green function $G_0 = -1/r$ and the coordinate surface $S_{\rm coor} = 4\pi R^2$) $F_n(R) = M'(R)/R^2$, where $M'(R) = 4\pi R^2 M/S_{\rm ph}(R)$ which defines the bias in the form $M'(R)/M = 1 + 4\pi\int_0^R b(r)r^2\,dr$ or

$$b(r) = \frac{1}{r^2}\frac{d}{dr}\frac{r^2}{\frac{d}{dr}V_{\rm ph}(r)}. \qquad (12)$$

Thus, we see that a non-trivial bias $b(r)$ appears, in the fist place, due to the discrepancy in the behavior of the physical volume $V_{\rm ph}(r)$ and that of $V_{\rm coor}(r)$. At scales where the distribution of wormholes (or mirrors) crosses over to homogeneity we get $\bar{V}_{\rm ph}(R) = \varepsilon V_{\rm coor}(R) = 4/3\pi R^3\varepsilon$ with a constant $\varepsilon < 1$. This gives $\bar{b}(r) = 0$ at such scales, but defines the renormalization of the point source as $M'/M = 1/\varepsilon$.

Consider now a set of wormholes with parameters $\vec{R}_{n,\pm}$, $U_n$ and $a_n$ ($n = 1, 2, \ldots, N$). We shall assume that the gas is sufficiently rarefied (i.e., $n \ll 1$ where $n = N/V$ is the density of wormholes). Therefore, we can neglect the feedback of wormholes, i.e., images which appear due to the reflection between wormholes, and evaluate the permeability of space $\varepsilon$ (and the bias $b(r)$) in the linear approximation for the external field of the form $\phi = -1/r$. The permeability of a dense gas can then be obtained in the standard way. Indeed, if we present $\varepsilon = 1 + 4\pi\chi$, where $\chi$ is the susceptibility of space, then for a dense gas it is related to the linear susceptibility $\chi_0$ as $\chi = \chi_0/(1 - 4/3\pi\chi_0)$, e.g., see part 4 in Ref. [16].

It is convenient to distinguish in (11) the two parts of the bias $b = b_0 + b_1$, where $b_0$ resembles the bias of the spherical mirrors (2)

$$b_0(r) = \sum_{\sigma=\pm}\frac{a}{R_\sigma}\big[\delta(\vec{r} - \vec{r}_{\sigma 1}) - \delta(\vec{r} - \vec{R}_\sigma)\big], \qquad (13)$$

while the rest part is given by

$$b_1(r) = a\left(\frac{1}{R_+} - \frac{1}{R_-}\right)\big[\delta(\vec{r} - \vec{r}_{-1}) - \delta(\vec{r} - \vec{r}_{+1})\big]. \qquad (14)$$

Both parts give different contributions to $\varepsilon$ and should be considered separately.

### 3.1. The case $\varepsilon < 1$

Consider first the part of the bias (13) which coincides formally with the bias produced by a gas of spherical mirrors. In this case the topological bias is

$$b_0(r) = \sum \frac{a_n}{R_{\pm,n}}\big[\delta(\vec{r} - \vec{r}_{\pm,n}) - \delta(\vec{r} - \vec{R}_{\pm,n})\big], \qquad (15)$$

where $r^\alpha_{\pm n} = R^\alpha_{\pm,n} - a_n^2/R^2_{\mp,n}U^{\pm 1}_{n,\alpha\beta}R^\beta_{\mp,n}$, see (4). We shall assume that for all wormholes $a/R_{\pm,n} \ll 1$ and, therefore, (15) can be expanded by the small parameter $a/R_\pm$ which gives

$$b_0(r) = \nabla_\alpha \sum U^{\pm 1}_{n,\alpha\beta}R^\beta_{\mp,n}\frac{a^3}{R_{\pm,n}R^2_{\mp,n}}\delta(\vec{r} - \vec{R}_{\pm,n}), \qquad (16)$$

where $\nabla_\alpha = \partial/\partial r^\alpha$.

Let $F(R_\pm, a, U)$ be the density of wormholes with parameters $R_-$, $R_+$, $U$ and $a$, i.e.,

$$F(R_\pm, a, U) = \sum_n \delta\big(\vec{R}_- - \vec{R}^n_-\big)\delta\big(\vec{R}_+ - \vec{R}^n_+\big) \times \delta(a - a_n)\delta(U - U_n), \qquad (17)$$

which allows to rewrite (16) in the form

$$b_0(r) = \nabla_\alpha \frac{1}{r}\sum_{s=\pm}\int \frac{R^\beta_{-s}}{R^2_{-s}}\delta(\vec{r} - \vec{R}_s) \times H^s_{\alpha\beta}(\vec{R}_+, \vec{R}_-)\,d^3R_+\,d^3R_-, \qquad (18)$$

where

$$H^\pm_{\alpha\beta}(R_+, R_-) = \int a^3 U^{\pm 1}_{\alpha\beta}F(R_\pm, a, U)\,da\,dU,$$

which has the property $H^-_{\alpha\beta} = H^+_{\beta\alpha}$ (i.e., $U^{-1}_{\alpha\beta} = U_{\beta\alpha}$).

As it was pointed out above this part of the bias for wormholes resembles formally that for mirrors. To see this analogy we evaluate now the bias for a gas of mirrors. This case can be formally obtained by setting $R_+ = R_- = R$ and $U_{\alpha\beta} = \delta_{\alpha\beta}$. Then from (16) and (18) we get

$$b_0(r) = \nabla_\alpha\left(h(\vec{r})\frac{r^\alpha}{r^3}\right) = \frac{\partial h(\vec{r})}{\partial r^\alpha}\frac{\partial(-1/r)}{\partial r^\alpha} + 4\pi h(0)\delta(\vec{r}), \qquad (19)$$

where $h(R) = \int a^3 F(R, a)\,da$, $F(R, a)$ is the distribution of mirrors analogous to (17), and we used the property $\nabla^2(-1/r) = 4\pi\delta(r)$. From (19) we see that the bias $b(r)$ acquires a non-trivial dependence on the radius $r$ only due to the local inhomogeneity of the gas (i.e., the first term in (19) $\sim \partial h(\vec{r})$), while in the case of a homogeneous distribution $\bar{F}(R, a) = nf(a)$ we find $\bar{h}(r) = n\bar{a}^3$ ($n$ is the density of mirrors), the firs term in (19) disappears and, therefore, the mean bias $\bar{b}_0(r)$ reduces merely to the renormalization of the point source $M'/M = (1 + 4\pi n\bar{a}^3)$ which corresponds to the case $\varepsilon = 1/(1 + 4\pi n\bar{a}^3) < 1$.

In the case of wormholes the bias $b_0(r)$ has the same structure. Indeed, from (18) we see that $b_0(r) = \nabla_\alpha(f^\alpha(r)/r)$ with some vector $f^\alpha(r)$ defined by the integral in (18) and if we assume isotropic distribution of wormholes this vector can be proportional to the radius only, i.e., $f^\alpha(r) = r^\alpha h(r)/r^2$, with $h(r) = (\vec{r}, \vec{f}(r))$. Thus we get the same expression (19) with the function $h(r)$ defined from (18) by

$$h(r) = \int \frac{r^\alpha R^\beta}{R^2}\big[H^+_{\alpha\beta}(\vec{r}, \vec{R}) + H^+_{\beta\alpha}(\vec{R}, \vec{r})\big]\,d^3R.$$

We point out that the function $h(r)$ (together with $F$, $H_{\alpha\beta}$) has, in general, quite irregular behavior and require some averaging out. The smooth halo $\bar{b}_0(r)$ around the point source (i.e., dark matter halo, e.g., Ref. [13]) appears due to the local inhomogeneity of the function $\bar{h}(r)$ and, therefore, due to the local inhomogeneity of the topological structure, which is in agreement with (12). We also stress that the applicability of the expression (19) is restricted by sufficiently large distances, at which the number of wormholes within the radius $r$ is $N(r) = 4/3\pi r^3 n \gg 1$. At small distances the density of wormholes fluctuates strongly, e.g., sufficiently close to a source wormholes are absent which means that $h(r)$, $b_0(r) \to 0$ and the permeability $\varepsilon$ tends to the vacuum value $\varepsilon \to 1$.

### 3.2. The case $\varepsilon > 1$

Consider now the second part of the bias (14). For astrophysical implications (characteristic scales $L \gg a$) it is sufficient to consider the approximation $\vec{r}_{\pm 1} \approx \vec{R}_{\pm}$, i.e., throats (every sphere of the radius $a_n$) look like point-like objects and every ghost image is assumed to be in the center of a wormhole. Then, the topological bias is given by

$$b_1(r) = \sum_n a_n \left( \frac{1}{R_{-,n}} - \frac{1}{R_{+,n}} \right) \times \left[ \delta(\vec{r} - \vec{R}_{+,n}) - \delta(\vec{r} - \vec{R}_{-,n}) \right]. \tag{20}$$

In this approximation the bias does not depend on the matrix $U$ and the density of wormholes (17) reduces to $F(R_{\pm}, a) = \int F\, dU$. The homogeneity and isotropy of the topological structure mean that $\bar{F} = nF(|\vec{R}_- - \vec{R}_+|, a)$ where $n = N/V$ is the number density of wormholes in space. Then the mean bias can be presented as

$$\bar{b}_1(r) = 2n \int \left( \frac{1}{R} - \frac{1}{r} \right) f\left(|\vec{R} - \vec{r}|\right) d^3\vec{R}, \tag{21}$$

where $f(X) = \frac{1}{n} \int aF(X, a)\, da$ (so that $\int f(x)\, d^3x = \bar{a}$) and which in the Fourier representation $b(k) = (2\pi)^{-3/2} \int b(r) \times e^{-ikr}\, d^3r$ takes the simplest form

$$\bar{b}_1(k) = 2n \frac{4\pi(f(k) - f(0))}{k^2}. \tag{22}$$

The topological permeability is then given by $\varepsilon(k) = 1/(1 + b(k)) = 1 - b(k)/(1 + b(k))$.

Thus, for a specific distribution of wormholes $f(k)$ the relation (22) defines the mean topological polarizability of space (the mean bias $\bar{b}(k)$) in the field of the external source $\phi_{\text{ext}} = -1/r$. We recall that by the construction $d$ is here defined in the range $d = |\vec{R}_+ - \vec{R}_-| \geqslant 2a$ which means that $f(k) \to 0$ as $k > \pi/\bar{a}$, while for $k \to 0$ it gives

$$\bar{b}(k) \approx 8\pi n \left( \frac{1}{2} f''(0) + \cdots \right).$$

For sufficiently large distances $r \to \infty$ ($k \to 0$) we get $\bar{b}(k) \approx 4\pi n f''(0)$, which defines merely the renormalization of the point source $M'/M = (1 + 4\pi n f''(0))$. As we shall see in what follows, this case corresponds to $f''(0) < 0$ and, therefore, $\varepsilon = 1/(1 + 4\pi n f''(0)) > 1$.

Consider now the simplest example when all wormholes have equal values of $d = |\vec{R}_- - \vec{R}_+| = r_0$. In this case we can take $f(X) = \bar{a}/(4\pi r_0^2)\delta(X - r_0)$ and find $f(k) = \bar{a}(2\pi)^{-3/2} \sin(kr_0)/(kr_0)$ which defines the bias in the form

$$\bar{b}_1(k) = -4n\bar{a}(2\pi)^{-1/2} \frac{1}{k^2} \left( 1 - \frac{\sin(kr_0)}{kr_0} \right),$$

which for $kr_0 \ll 1$ gives $\bar{b}_1(k) \approx -\frac{4n\bar{a}}{(2\pi)^{1/2}} \frac{1}{6} r_0^2 (1 - \frac{1}{20}(kr_0)^2 + \cdots)$. Thus, we see that such bias produces a negative halo around a point source with the density

$$\bar{b}_1(r) = -\frac{n\bar{a}}{rr_0} \left( |r_0 - r| + r_0 - r \right) = -2n\bar{a} \left( \frac{1}{r} - \frac{1}{r_0} \right), \quad \text{as } r < r_0.$$

For $r < r_0$ it defines the scale-dependent renormalization of a source

$$\frac{\delta M(r)}{M} = 4\pi \int_0^r \bar{b}(r) r^2\, dr = -8\pi n\bar{a} \left( \frac{r^2}{2} - \frac{r^3}{3r_0} \right),$$

which for $r > r_0$ (where $b_1 = 0$) reduces to the constant negative shift $\delta M_{\text{tot}}/M = -\frac{4\pi}{3} n\bar{a} r_0^2$, i.e., we get $\varepsilon = 1/(1 - \frac{4\pi}{3} n\bar{a} r_0^2) > 1$. A more general case we obtain when considering an additional distribution $P(r_0)$ ($\int P(x)\, dx = 1$) over the parameter $r_0 = |\vec{R}_- - \vec{R}_+|$, which gives merely $\delta M_{\text{tot}}/M = -\frac{4\pi}{3} n \langle \bar{a} r_0^2 \rangle$ (where $\langle \bar{a} r_0^2 \rangle = \int \bar{a} x^2 P(x)\, dx$) and again we find that $\varepsilon > 1$.

We see that basic feature of wormholes is that the space possesses a specific polarizability of the topological origin. Moreover, such a polarizability exists in gravity as well. From electrodynamics we know that the polarizability of a medium leads to the screening (partial or total) of a source. We note that the screening can be effectively described by means of adding of the "mass-like" term to the Poisson equation $\Delta \to \Delta - m^2$ which transforms the Green function to the $G \sim -e^{-mr}/r$. By other words virtual photons or gravitons acquire in such a medium an effective mass. In particular, in Ref. [8] it was claimed that adding of the massive term allows to explain the rotation curve (i.e., the amount of dark matter) in any particular galaxy. However to explain the presence of DM in all galaxies (i.e., the variety of DM halos) the effective graviton mass should vary in space, ($m \to m(x)$) which is rather difficult to incorporate in the theory on the very fundamental level. In the presence of wormholes the bias (22) can also be interpreted as such an effective mass-like term which however turns out to be scale-dependent $m^2(k) = -k^2\bar{b}(k)/(1 + \bar{b}(k))$. Moreover, the sign of this term depends essentially on the interplay of the two parts $b = b_0 + b_1$, where $b_1 < 0$, while $b_0$ may in general have both signs. Thus, in a particular range of scales $\Delta k$ the effective mass-like term may have both signs. General consideration (12) shows that the sign of $b$ depends essentially on the behavior of the physical volume $V_{\text{phys}}(r)$ (i.e., of the physically admissible region of space). In the simplistic model considered $V_{\text{phys}}(r) < 4/3\pi r^3$ and therefore on sufficiently large distances $b$ is always positive. We also stress that in the case of a nontrivial topological structure on the very fundamental level gravitons remain massless (i.e., the theory does not change at all), while the effective graviton mass appears merely as the result of the topological polarization effects (i.e., due to the presence of a gas of wormholes).

## 4. Conclusions

In conclusion we briefly repeat basic results. First of all we have explicitly demonstrated that a static gas of wormholes leads indeed to the topological bias of point-like sources which can equally be interpreted as the presence of a "dark matter" halo around any point source. However in general, the halo density admits both (positive and negative) signs depending on scales and the specific features of the distribution of wormholes. By analogy with the magnetic media we can speak of dia- and para-susceptibilities of space.

The general geometric consideration has revealed that the sign of the bias (and that of the halo density) depends on the discrepancy between the behavior of the volume $V_{\mathrm{phys}}(r)$ of the physically admissible region of space and that of the coordinate space $V_{\mathrm{coor}}(r)$ (12) (which was confirmed by the subsequent rigorous calculations (19)). Moreover, a non-trivial halo (the dependence on the radius $r$) appears only due to the local inhomogeneity of the topological structure (e.g., see (19)). In particular, if we approximate $V_{\mathrm{phys}}(r) \sim r^D$, then (12) defines the behavior of the bias as $\bar{b}(r) \sim (3-D)1/r^D$, while at scales where the topological structure crosses over to homogeneity $V_{\mathrm{phys}}(r) \to r^3$ we get $\bar{b}(r) \to b\delta(r)$, i.e., the bias renormalizes merely the value of the source. We recall the observations evidence for the value $D \simeq 2$ starting from a few kpc up to at least 100 Mpc (e.g., see discussions in Refs. [10,14]).

We note that in the simplistic model considered $V_{\mathrm{phys}}(r) < 4/3\pi r^3$ and the total bias has always the positive sign. However, geometrically one can imagine a more complex topology (e.g., in multidimensional theories) for which we will get an excess of volume $V_{\mathrm{phys}}(r) > 4/3\pi r^3$ which will lead to a negative bias and a negative density of Dark halos. It is tempting to relate such a case to the Dark Energy phenomenon. However, it is clear that this cannot describe the total fraction of DE. An essential fraction should be also given by the vacuum zero-point fluctuations.[7] Indeed, let us prescribe some finite energy density to such fluctuations $\sigma_0$ (lambda term). In the flat space the vacuum density should disappear (the exact mechanism of the compensation or renormalization is not important here), while in the case of a non-trivial topology some portion of the volume cuts $V_{\mathrm{coor}} \to V_{\mathrm{phys}}$ and this gives an additional shift of the vacuum energy density $\sigma_0 \to \sigma = \sigma_0(V_{\mathrm{phys}} - V_{\mathrm{coor}})/V_{\mathrm{coor}}$, where the sign of $\sigma$ depends on the difference $(V_{\mathrm{phys}} - V_{\mathrm{coor}})$.

[7] We note that for macroscopic wormholes the Casimir energy density gives only a tiny contribution to DE.

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