An accurate 3D inspection system using heterodyne multiple frequency phase-shifting algorithm

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Abstract

This paper presents an accurate 3D inspection system for industrial applications, which uses digital fringe projection technology. The system consists of two CCD cameras and a DLP projector. The mathematical model of the 3D inspection system with 10 distortion parameters for each camera is proposed. A heterodyne multiple frequency phase-shifting algorithm is employed for overcoming the unwrapping problem of phase functions and for a reliable unwrapping procedure. The redundant phase information is used to increase the accuracy of the 3D reconstruction. To demonstrate the effectiveness of our system, a standard sphere was used for testing. The verification test for the 3D inspection systems are based on the VDI standard 2634. The result shows the proposed system can be used for industrial quality inspection with high measurement precision.

Keywords: 3D inspection System; Digital Fringe Projection; Phase-Shifting; Camera Calibration

1. Introduction

The optical techniques for three-dimensional (3D) shape measurement have been extensively studied for various applications, including quality control, archeology, industrial inspection, reverse engineering, etc. [1-3]. Among all the techniques, the fringe projection methods, such as phase-shift, are promising since they have the advantages of high precision, high resolution, full field and easy implementation.

In the measuring process, the resolution of the phase map is one of the key problems. The main phase solving methods include the Fourier transform method and phase-shifting method. The Fourier transform method applies Fourier transform on the fringe image to obtain the phase map of the image [4], and it assumes that the phase varies much slower than the carrier frequency of the fringe pattern, that means the surface of the tested object should be gently and not various acutely. The phase-shifting method is widely used for its high precision and robustness [5] and [6]. It projects several sinusoidal phase-shifting fringes, and from the fringe images we can easily get the
primary phase, which means the fractional phase value that ranges in \((0, 2\pi]\). In phase measuring interferometry, the primary phase value can be calculated by either phase-shifting or Fourier transform technique, but the intrinsic periodic nature of the fringe function means that the strip order information is lost. This fundamental limitation should be resolved in order to obtain absolute phase measurement. A common method to solve this problem consists in applying additional fringe patterns, such as the Gray-code method [7], time dimension method and Chinese remainder theorem algorithm, which provide the wanted strip orders. At the same time, the projection of additional fringe patterns increase the time cost and it becomes an obstacle to rapid phase measurement. On the other hand, some algorithms without additional fringe patterns are developed, this method usually get the final phase map by the property of the periodic fringe, such as \(2\pi\) discontinuity, reliability, and so on. These methods use only one primary phase map, and sometime lead to mistakes since the lack of additional information.

In this paper, an inspection system consists of two digital cameras and a DLP projector is developed. The mathematical model of the 3D inspection system with 10 distortion parameters for each camera is proposed. A heterodyne multiple frequency phase-shifting algorithm is employed for overcoming the unwrapping problem of phase functions and for a reliable unwrapping procedure. The redundant phase information is used to increase the accuracy of the 3D reconstruction.

2. System Methodology

2.1 Mathematical model of the inspection system

The inspection vision system includes two cameras and a Digital Light Procession (DLP) projector. Fig. 1 shows the 3D mathematical model of the stereo system. In the field of computer vision, the camera model is used to solve the correspondence problem between the 3D object points and 2D image points, for which the perspective projection model is widely used [2].

In this paper, we also consider the tangential distortion and thin prism distortion of camera lens. The new distortion model used in our method is
\[ dx = A_1 x(r^2 - r_0^2) + A_2 x(r^4 - r_0^4) + A_3 x(r^6 - r_0^6) + B_1 (r^2 + 2x^2) + 2xyB_2 + C_1 x + C_2 y \]
\[ dy = A_1 y(r^2 - r_0^2) + A_2 y(r^4 - r_0^4) + A_3 y(r^6 - r_0^6) + B_2 (r^2 + 2y^2) + 2xyB_1 \]

(1)

Where \( A_1, A_2, A_3 \) are radial distortion parameters, \( B_1 \) and \( B_2 \) are tangential distortion parameters, and \( C_1 \) and \( C_2 \) are thin prism distortion parameters, \( r \) is the image radius and \( r_0 \) is the second zero crossing of the distortion curve.

System calibration is a key issue that affects the accuracy of measurement system. The stereo vision system should be calibrated by calculating two cameras’ internal and external parameters, and the relative position of the two cameras is obtained. The rotation matrix \( R \) and translation matrix \( T \) are:

\[
R = \begin{pmatrix}
  r_1 & r_2 & r_3 \\
  r_4 & r_5 & r_6 \\
  r_7 & r_8 & r_9 \\
\end{pmatrix}
\]
\[
T = \begin{pmatrix}
  t_x \\
  t_y \\
  t_z \\
\end{pmatrix}
\]  

(2)

The point coordinates of the object is then calculated as

\[
\begin{pmatrix}
  X \\
  Y \\
  Z \\
\end{pmatrix} = (A^T A)^{-1} A^T B
\]

(3)

Where

\[
A = \begin{bmatrix}
  1 & 0 & -x_1 \\
  0 & 1 & -y_1 \\
  r_1 - r_7 x_2 & r_2 - r_8 x_2 & r_3 - r_9 x_2 \\
  r_4 - r_7 y_2 & r_5 - r_8 y_2 & r_6 - r_9 y_2 \\
\end{bmatrix}
\]
\[
B = \begin{bmatrix}
  0 \\
  x_2 t_z - t_x \\
  y_2 t_z - t_y \\
\end{bmatrix}
\]

The phase-shifting and heterodyne principle

2.2 Phase-shifting technology is a general technique to deal with fringe image. It relies on consecutive illumination of the tested object with sinusoidal fringes and recording of the fringes by CCD camera at each illumination step. With a phase-shift of \( \pi / 2 \), the mathematical representation of the fringe image can be expressed in the general form

\[
I_i(x, y) = I'(x, y) + I''(x, y) \cos[\phi(x, y) + i \pi / 2], \quad i = 0, 1, 2, 3
\]

(4)

Where \( I_i(x, y) \) is the intensity of the \( i \) frame, \( I'(x, y) \) is the illumination intensity, \( I''(x, y) \) is the modulation coefficient, \( \phi(x, y) \) is the phase function, which can be used to get the shape information of the object. From Eq.(4) we can get

\[
\phi(x, y) = \tan^{-1}\left[\frac{I_1(x, y) - I_2(x, y)}{I_3(x, y) - I_4(x, y)}\right]
\]

(5)

\( \phi(x, y) \) is called the primary phase, which ranges in \([0, 2\pi]\). To get the absolute phase value \( \phi(x, y) \), we need the order of the fringes as
\[
\varphi(x, y) = \Phi(x, y) + O(x, y) \ast 2\pi
\]

where \(O(x, y)\) is an integer that denotes the fringe orders.

The multiple frequency heterodyne principle can be used to obtain the fringe orders, which comes from optical interference. Two different phase function \(\varphi_1(x)\) and \(\varphi_2(x)\) with different frequencies \(\lambda_1\) and \(\lambda_2\) are superposed, shown in Fig. 2. The superposition means the phase-correct subtraction of these two functions getting the desired phase \(\Phi(x)\) with the frequency \(\lambda_b\).

In this way, three phase functions \(\varphi_1(x), \varphi_2(x)\) and \(\varphi_3(x)\) are created with the following frequencies:

\(\lambda_1 = 1/81, \lambda_2 = 1/72, \lambda_3 = 1/64\). From the phase functions \(\varphi_1(x), \varphi_2(x)\) and \(\varphi_3(x), \varphi_3(x)\), new phase functions \(\varphi_{12}(x)\) and \(\varphi_{23}(x)\) are calculated with the resulting frequencies \(\lambda_{12} = 1/9, \lambda_{23} = 1/8\). Then, from the phase functions \(\varphi_{12}(x)\) and \(\varphi_{23}(x)\), the unambiguous phase \(\varphi_{123}(x)\) is calculated with the correct frequency \(\lambda_{123} = 1\). The whole process of unwrapping is shown in Fig. 3.
3. Experiments and Result Analysis

In this work, our stereo vision system software is developed using VC ++6.0 in the Windows XP environment. The hardware platform consists of two 1.3 million-pixel CCD cameras and a DLP projector. The CCD size is 6.6 mm × 5.3 mm, the pixel size is 5.2 μm × 5.2 μm, the resolution is 1280 × 1024, and the frame rate is 15 frames/s. The lens we used is Schneider Lens with focal length fixed at 16mm. As shown in Fig. 4, the measuring devices are installed on a frame, the two cameras are located on both ends of a beam and the projector is in the middle.

Fig. 4 The vision inspection system

3.1. System calibration

Parameters need to calibrate of the vision system illustrated in section 2.1. Fig. 5 shows a pair of images used in the calibration procedure. The calibrated interior orientation and lens distortion parameters of two cameras are listed in Table 1 respectively.

Fig. 5 Images used in calibration

Tab1. Interior orientation and lens distortion parameters of two cameras

<table>
<thead>
<tr>
<th></th>
<th>Left camera</th>
<th>Right camera</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>-2.088668e-010</td>
<td>-1.081924e-009</td>
</tr>
<tr>
<td>A2</td>
<td>6.218764e-015</td>
<td>1.668371e-014</td>
</tr>
<tr>
<td>A3</td>
<td>-6.682331e-021</td>
<td>-2.002049e-020</td>
</tr>
<tr>
<td>B1</td>
<td>2.135709e-007</td>
<td>3.216488e-007</td>
</tr>
<tr>
<td>B2</td>
<td>2.475766e-007</td>
<td>-2.423612e-007</td>
</tr>
<tr>
<td>C1</td>
<td>-1.850259e-004</td>
<td>-1.384058e-004</td>
</tr>
</tbody>
</table>
Besides, the Rotation matrix $R$ and Translation matrix $T$ between the two camera coordinate systems are also obtained as follows:

$$
R = \begin{bmatrix}
9.404e-001 & -1.100e-002 & -3.398e-001 \\
1.100e-002 & 9.999e-001 & -4.428e-003 \\
3.398e-001 & 7.732e-004 & 9.405e-001 
\end{bmatrix},
T = \begin{bmatrix}
-144.746 \\
-1.647 \\
-22.531
\end{bmatrix}^T
$$

Re-project the 3D points of calibration target using the calibrated parameters, and the average re-project residual is about 0.02pixel. The result indicates that the proposed system has considerable precision.

### 3.2. Measurement test

To demonstrate the accuracy of the inspection system, a calibrated sphere was used for testing in our system. As shown in Figure 6. The verification test is based on the VDI standard 2634. A high precision Coordinate Measuring Machine (CMM) was used to obtain the radius value of the standard sphere.

<table>
<thead>
<tr>
<th>Tab2. Measurement result of CMM and inspection system</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMM[mm]</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>Stddev</td>
</tr>
</tbody>
</table>

![Fig.6 Cloud points of the standard sphere](image)

The obtained data from the CMM and the inspection system using the proposed method are listed in Table 2. The standard deviation of the CMM result is 0.001, while the standard deviation of our system is approximately 0.008. The maximum absolute error between CMM and our system is about 0.011 mm. Although the standard deviation of our inspection system is larger than CMM’s, it is accurate enough for many industrial on-site inspection fields.

### 4. Conclusion

We have developed an accurate inspection system using multiple frequency heterodyne method. Major issues of the vision system have been discussed in detail. The mathematical model of the 3D inspection system with 10 distortion parameters for each camera is proposed. A heterodyne multiple frequency phase-shifting algorithm is
employed for overcoming the unwrapping problem of phase functions and for a reliable unwrapping procedure. The redundant phase information is used to increase the accuracy of the 3D reconstruction. The experiment result shows that the maximum absolute error between CMM and our system is about 0.011 mm and the standard deviation of our system is approximately 0.008.

5. REFERENCES