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## Parameter identification of nonlinear viscoelastic model with impact area parameter for sport surface by using multi-intensity multi-area impact test

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### Abstract

In previous studies, we proposed various models of material and methods for identifying the parameters of these models by multi-intensity impact test. Especially, the exponential function type nonlinear Voigt model shows an excellent accuracy for identifying the property of material behavior. Although other papers show that this model is applicable to the case of varying impact duration and various temperatures, we have not yet discussed how the properties would change as the impact area changed. The purpose of this study is to propose an identification technique of nonlinear Voigt model with hysteresis and dependence on impact area by multi-intensity multi-area impact test. The nonlinear elastic element of this model is expressed by the exponential function based on displacement and impact area. And the nonlinear viscous element of the model is expressed by the product of the exponential function based on displacement and the exponential function based on velocity.

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### 1. Introduction

Some sports governing bodies have their own standards for shock attenuation. In these standards, the test has a unique dropping mass-spring model and an impact velocity which regulates the impact force applied to the surface. The dropping mass-spring and impact velocity determines the impact force, but there exists infinite combinations which produce the same force. Additionally, as shown in Figure 1, there are differences in shock absorption between the two sets of dropping mass-spring onto the same surface even if these two sets create exactly the same force [1]. It means that a dropping mass-spring test indicates the shock absorption characteristics only when a specific combination of mass-spring and velocity are used. And it is clear that human running and other activity have various types of impact forces. This issue will be solved by incorporating computer simulation method for evaluating shock attenuation instead of mechanical tests. In previous studies, we proposed various models of material and methods for identifying the parameters of these models by multi-intensity multi-impact test [2] [3]. Especially, the exponential function type nonlinear Voigt model shows an excellent accuracy for identifying the property of material behavior [4]. Other papers show that this model is applicable to the case of varying impact duration, various temperatures and varying impact areas [5]. For impact area, parameter identification was performed on each impact area data to determine a unique parameter set for each test foot [6]. In other words, eight parameter sets were obtained for eight impact areas. In this paper, we propose a new rubber viscoelastic model incorporating an impact area parameter. With this model, only a set of parameters is required for representing the behaviour of a material with various impact areas. The purpose of this study is to propose a nonlinear Voigt model applicable to the cases of varying impact areas and identification method of this model by multi-intensity multi-area impact test.

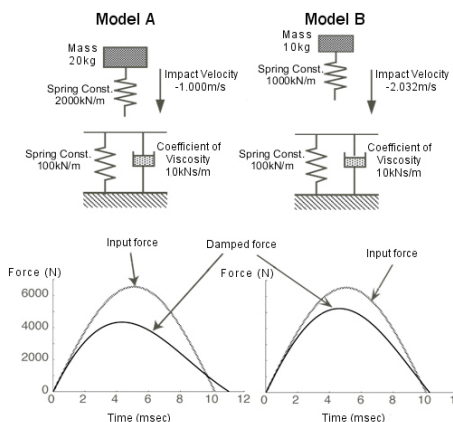


Fig.1. Simulated damped force produced by different set of mass-spring models on a same viscoelastic models

### 2. Experiment

#### 2.1 Shock tester

As shown in Figure 2, the shock tester is constructed of three parts, test foot, measurement instrument and dropping weight. A small force transducer and accelerometer are mounted in the impact sensor and non-contact displacement sensor is mounted for confirming the displacement data from the acceleration sensor. With this impact sensor, the impact force, the acceleration and the displacement of the test foot

are measured when the impact weight is dropped onto the impact sensor. Impact velocity and impact displacement are calculated by numerical integration of the acceleration. The displacement data acquired from the displacement sensor is compared with the results of the numerical integration.

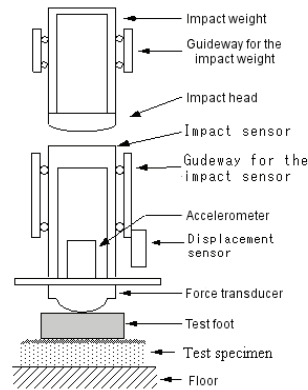


Fig. 2. Shock tester

## 2.2 Test foot

The test foot is a cylindrical object made of polyacetal resin, which has sufficient stiffness compared to the test material. There were eight different test feet whose impact areas varied from 706 mm<sup>2</sup> to 2780 mm<sup>2</sup>. Table 1 shows the specifications of each test foot.

Table 1. Impact area and mass of the test foot

Test foot	No.1	No.2	No.3	No.4	No.5	No.6	No.7	No.8
Impact area [mm <sup>2</sup> ]	707	990	1288	1590	1886	2206	2507	2780
Diameter [mm]	30	35.5	40.5	45	49	53	56.5	59.5
Mass [g]	19.7	27.6	36.1	44.6	52.9	61.8	70.3	77.9
Thickness [mm]	20							

## 2.3 Test specimen

The test specimen is made of 14 mm thick urethane and has an embossed surface. This specimen is one of the track surfacing products certified by I.A.A.F.

### 3. Nonlinear Voigt Model for Material

Figure 3 and the following equations describe the nonlinear exponential function type Voigt model proposed in this paper:

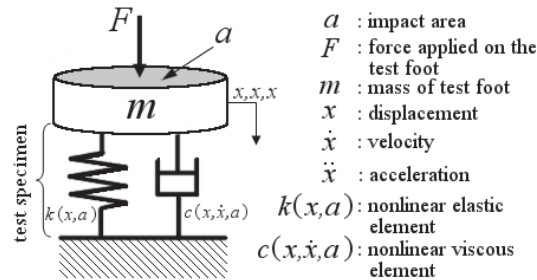


Fig.3. Nonlinear Voigt model

$$F - m\ddot{x} = f = f_k + f_c \quad (1)$$

$$f_k = \text{sign}(x)e^A|x|^B$$

$$A = k_0 + k_1x + k_2a^\alpha$$

$$B = p_0 + p_1x + p_2a^\beta \quad (2)$$

$$f_c = \text{sign}(\dot{x})e^D|x|^E|\dot{x}|^G|a|^H$$

$$D = c_0 + c_1x + c_2\dot{x}$$

$$E = q_0 + q_1x + q_2\dot{x}$$

$$G = r_0 + r_1x + r_2\dot{x}$$

$$H = s_0 + s_1x + s_2\dot{x} \quad (3)$$

where  $F$  is the impact force applied on the test foot,  $f$  is the force applied to the specimen surface,  $m$  is the test foot mass,  $x$  is the displacement of test foot,  $\dot{x}$  is the velocity of test foot,  $\ddot{x}$  is the acceleration of test foot,  $f_k$  is the reaction force by nonlinear elastic element, and  $f_c$  is the reaction force by nonlinear viscous element.

## 4. Method

### 4.1 Experimental method

Twenty impact tests were performed by dropping an impact weight (5kg) from a height of from 5 mm to 100 mm in 5 mm steps. This set of impact tests was performed for each test foot. In all the impact tests, impact forces, the impact accelerations and impact displacement were measured.

#### 4.2 Identification method by multi-intensity impact test

Impact force, impact acceleration, impact velocity and impact deformation are obtained by the experimentation of eight impact areas. Longitudinal data of each is combined as unified data. The impact force and deformation are extracted where the velocity is 0 m/s in each trial in order to decide the elastic coefficient prior to the viscous coefficients (Equation set 2). To calculate the elastic coefficients, a least-square method is used for the twenty pairs of the  $f_k$  data of eight sets different test feet. After calculating the elastic coefficients,  $f_c$  can be calculated by subtracting  $f_k$  from  $f$  in each of the sampled points. Then the coefficient of the viscous element is calculated by searching for optimum value satisfying Equation set 3.

#### 4.3 Evaluation of identification accuracy

Relative standard error (RSE) between the experimental force and the estimated force of each impact test is used for the evaluation of identification accuracy.

$$RSE = \frac{1}{\bar{f}} \left\{ \frac{\sum_{i=1}^n (\hat{f}_i - f_i)^2}{n-1} \right\}^{\frac{1}{2}} \quad (4)$$

Where,  $f$  is the experimental force and the  $\hat{f}$  is the estimated force calculated from the model.  $\bar{f}$  is the mean value of experimental force. The identification accuracy of the model is evaluated against each trial by Equation 4. And the overall identification accuracy of all impact area is evaluated by MRSE. MRSE is defined as an average of RSEs.

### 5. Results and Discussion

#### 5.1 Identification accuracy

Figure 4 shows the RSE against the mean impact force and impact area. Table 2 shows the MRSE of 8 different impact areas. As shown in Figure 4 and Table 2, high identification accuracy was obtained in the multi-intensity multi-area impact test.

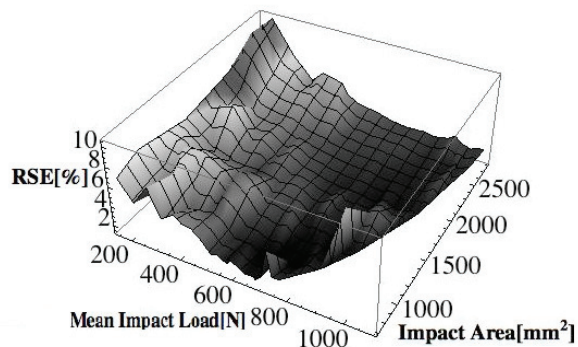


Fig.4 RSE vs. Mean impact load vs. Impact area

Table 2. MRSE for each test foot

Test foot	Mean relative standard error [%]
No.1	3.50
No.2	2.75
No.3	3.50
No.4	3.12
No.5	2.90
No.6	3.01
No.7	3.61
No.8	4.01

5.2 Elastic element and viscous element

Figure 5 shows the extracted experimental force and deformation when the deformation velocity is 0 in each trial and the estimated curve of elastic element in each impact area. Fig. 6 shows the estimated force produced by the nonlinear viscous element of the highest impact force in test foot No.1, 4 and 8. As shown in Fig. 5 and Fig. 6, not only the elastic element but also the viscous element has high accuracy of identification. Especially in the viscous element in Fig.6, this model can represent the hysteresis curve characterized as the typical rubber behaviour even in multi-intensity multi-area impact test.

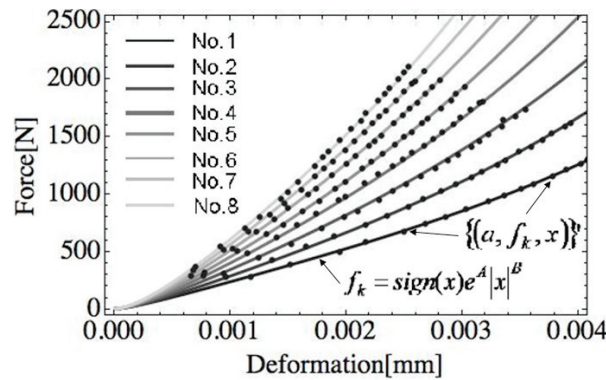


Fig.5. Reaction force of nonlinear elastic element

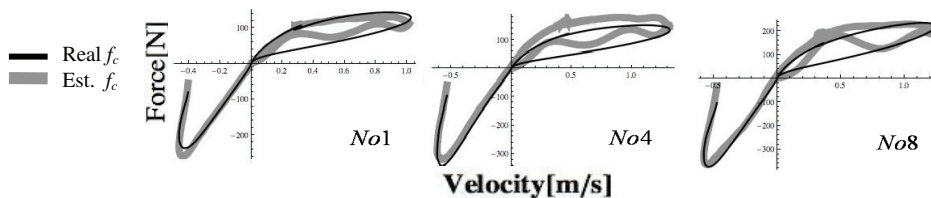


Fig. 6. Reaction force of nonlinear viscous element

## 6. Conclusion

Previous research has identified and measured the parameters corresponding to impact areas with high accuracy. In this paper, a nonlinear viscoelastic model corresponding to the change in the impact area and identification method of the previous model by multi-intensity multi-area impact test is proposed. It has been shown that this model is able to obtain high identification accuracy and is applicable to a wide impact range.

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