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Entropy of black holes in the deformed Hořava–Lifshitz gravity

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ABSTRACT

We find the entropy of Kehagias–Sfetsos black hole in the deformed Hořava–Lifshitz gravity by using the first law of thermodynamics. When applying generalized uncertainty principle (GUP) to Schwarzschild black hole, the entropy $S = A/4 + (\pi/\omega)\ln(A/4)$ may be interpreted as the GUP-inspired black hole entropy. Hence, it implies that the duality in the entropy between the Kehagias–Sfetsos black hole and GUP-inspired Schwarzschild black hole is present.

1. Introduction

Recently Hořava has proposed a renormalizable theory of gravity at a Lifshitz point [1,2], which may be regarded as a UV complete candidate for general relativity. At short distances the theory of Hořava–Lifshitz (HL) gravity describes interacting non-relativistic gravitons and is supposed to be power counting renormalizable in $(1+3)$ dimensions. Recently, its black hole solutions has been intensively investigated [3–20].

Concerning the spherically symmetric solutions, Lü–Mei–Pope (LMP) have obtained the black hole solution with dynamical parameter λ in asymptotically Lifshitz spacetimes [3] and topological black holes were found in [4]. Its thermodynamics were studied in [7,8] but there remain unclear issues in defining the ADM mass and entropy. On the other hand, Kehagias and Sfetsos (KS) have found the "$\lambda = 1$" black hole solution in asymptotically flat spacetimes using the deformed HL gravity with parameter ω [10]. Its thermodynamics was defined in Ref. [11]. Also, Park has obtained the $\lambda = 1$ black hole solution with two parameters ω and Λ_W [15].

not confirm a solid connection between the GUP [21,22] and the black hole of deformed HL gravity, although partial connections were established between them. However, it was known that the GUP provides naturally a UV cutoff to the local quantum field theory as quantum gravity effects [23,24]. Also, the GUP density function may be replaced by a cutoff function for the renormalization group study of deformed HL gravity [25]. We have found GUP-inspired graviton propagators and compared these with UV-tensor propagators in the deformed HL gravity. Two were similar, but the p^5-term arisen from Cotton tensor was missed in the GUP-inspired graviton propagator. This shows that a power-counting renormalizable theory of the HL gravity is closely related to the GUP.

In this Letter, we will make a further progress on exploring the connection between the GUP and black hole of the deformed HL gravity. We obtain the entropy of KS black hole in the deformed HL gravity. This entropy may be interpreted as the GUP-inspired black hole entropy when applying the GUP to Schwarzschild black hole.

2. HL gravity

Introducing the ADM formalism where the metric is parameterized

$$ds^2_{ADM} = -N^2\,dt^2 + g_{ij}\big(dx^i - N^i\,dt\big)\big(dx^j - N^j\,dt\big), \tag{1}$$

the Einstein–Hilbert action can be expressed as

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\,\sqrt{g}N\big[K_{ij}K^{ij} - K^2 + R - 2\Lambda\big], \tag{2}$$

ature K_{ij} takes the

$$K_{ij} = \frac{1}{2N}\big(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\big). \tag{3}$$

Here, a dot denotes a derivative with respect to t. An action of the non-relativistic renormalizable gravitational theory is given by [1]

$$S_{HL} = \int dt\,d^3x\,[\mathcal{L}_K + \mathcal{L}_V], \tag{4}$$

where the kinetic terms are given by

$$\mathcal{L}_K = \frac{2}{\kappa^2}\sqrt{g}NK_{ij}\mathcal{G}^{ijkl}K_{kl} = \frac{2}{\kappa^2}\sqrt{g}N\big(K_{ij}K^{ij} - \lambda K^2\big), \tag{5}$$

with the DeWitt metric

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$$\mathcal{G}^{ijkl} = \frac{1}{2}\left(g^{ik}g^{jl} - g^{il}g^{jk}\right) - \lambda g^{ij}g^{kl} \tag{6}$$

and its inverse metric

$$\mathcal{G}_{ijkl} = \frac{1}{2}(g_{ik}g_{jl} - g_{il}g_{jk}) - \frac{\lambda}{3\lambda - 1}g_{ij}g_{kl}. \tag{7}$$

The potential terms is determined by the detailed balance condition as

$$\begin{aligned}\mathcal{L}_V &= -\frac{\kappa^2}{2}\sqrt{g}NE^{ij}\mathcal{G}_{ijkl}E^{kl} \\ &= \sqrt{g}N\left\{\frac{\kappa^2\mu^2}{8(1-3\lambda)}\left(\frac{1-4\lambda}{4}R^2 + \Lambda_W R - 3\Lambda_W^2\right)\right. \\ &\quad \left. - \frac{\kappa^2}{2w^4}\left(C_{ij} - \frac{\mu w^2}{2}R_{ij}\right)\left(C^{ij} - \frac{\mu w^2}{2}R^{ij}\right)\right\}.\end{aligned} \tag{8}$$

Here the E tensor is defined by

$$E^{ij} = \frac{1}{w^2}C^{ij} - \frac{\mu}{2}\left(R^{ij} - \frac{R}{2}g^{ij} + \Lambda_W g^{ij}\right) \tag{9}$$

with the Cotton tensor C_{ij}

$$C^{ij} = \frac{\epsilon^{ik\ell}}{\sqrt{g}}\nabla_k\left(R^j{}_\ell - \frac{1}{4}R\delta^j_\ell\right). \tag{10}$$

Explicitly, E_{ij} could be derived from the Euclidean topologically massive gravity

$$E^{ij} = \frac{1}{\sqrt{g}}\frac{\delta W_{TMG}}{\delta g_{ij}} \tag{11}$$

with

$$\begin{aligned}W_{TMG} &= \frac{1}{w^2}\int d^3x\,\epsilon^{ikl}\left(\Gamma^m_{il}\partial_j\Gamma^l_{km} + \frac{2}{3}\Gamma^n_{il}\Gamma^l_{jm}\Gamma^m_{kn}\right) \\ &\quad - \mu\int d^3x\sqrt{g}(R - 2\Lambda_W),\end{aligned} \tag{12}$$

where ϵ^{ikl} is a tensor density with $\epsilon^{123} = 1$.

In the IR limit, comparing $\mathcal{L}_0$ with Eq. (2) of general relativity, the speed of light, Newton's constant and the cosmological constant are given by

$$c = \frac{\kappa^2\mu}{4}\sqrt{\frac{\Lambda_W}{1-3\lambda}}, \qquad G = \frac{\kappa^2}{32\pi c}, \qquad \Lambda_{cc} = \frac{3}{2}\Lambda_W. \tag{13}$$

The equations of motion were derived in [26] and [3]. We would like to mention that the IR vacuum of this theory is Lifshitz spacetimes [7]. Hence, it is interesting to take a limit of the theory, which may lead to a Minkowski vacuum in the IR sector. To this end, one may deform the theory by introducing a soft violation term of "μ^4R" ($\tilde{\mathcal{L}}_V = \mathcal{L}_V + \sqrt{g}N\mu^4R$) and then, take the $\Lambda_W \to 0$ limit [10]. We call this as the "deformed HL gravity". This theory does not alter the UV properties of the HL gravity, while it changes the IR properties. That is, there exists a Minkowski vacuum, instead of Lifshitz vacuum. In the IR limit, the speed of light and Newton's constant are given by

$$c^2 = \frac{\kappa^2\mu^4}{2}, \qquad G = \frac{\kappa^2}{32\pi c}, \qquad \lambda = 1. \tag{14}$$

3. Entropy of KS black hole

A spherically symmetric solution to the deformed HL gravity was obtained by considering the line element

$$ds^2 = -N(r)^2\,dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right). \tag{15}$$

In this case, we have $K_{ij} = 0$ and $C_{ij} = 0$. Hence, it is emphasized that we have relaxed both the projectability restriction and detailed balance condition [1,27] since the lapse function N depends on the spatial coordinate r as well as a soft violation term of μ^4R is included. Substituting the metric ansatz (15) into $\tilde{\mathcal{L}}_V$ with $\mathcal{L}_K = 0$, one has the reduced Lagrangian

$$\begin{aligned}\tilde{\mathcal{L}}_V = \frac{\mu^4N}{\sqrt{f}}&\left[\frac{\lambda-1}{2\omega_\lambda}f'^2 - \frac{2\lambda(f-1)}{\omega_\lambda r}f'\right. \\ &\left. + \frac{(2\lambda-1)(f-1)^2}{\omega_\lambda r^2} - 2\left(1 - f - rf'\right)\right]\end{aligned} \tag{16}$$

where a parameter $\omega_\lambda = 8\mu^2(3\lambda-1)/\kappa^2$ specifies the deformed HL gravity.

For $\lambda = 1$, the KS solution is given by [10]

$$f_{KS} = N^2_{KS} = 1 + \omega r^2\left(1 - \sqrt{1 + \frac{4M}{\omega r^3}}\right) \tag{17}$$

with $\omega(=\omega_{\lambda=1}) = 16\mu^2/\kappa^2$. In the limit of $\omega \to \infty$ (equivalently, $\kappa^2 \to 0$), it reduces to the Schwarzschild metric function

$$f_{Sch}(r) = 1 - \frac{2M}{r}. \tag{18}$$

From the condition of $f_{KS}(r_\pm) = 0$, the outer (inner) horizons are given by

$$r_\pm = M\left[1 \pm \sqrt{1 - \frac{1}{2\omega M^2}}\right]. \tag{19}$$

In order to have a black hole solution, it requires that

$$M^2 \geqslant \frac{1}{2\omega}. \tag{20}$$

Furthermore, the extremal black hole is obtained from the condition of degenerate horizon ($f_{KS}(r_e) = 0$, $f'_{KS}(r_e) = 0$) as

$$r_e = M_e = \frac{1}{\sqrt{2\omega}} \tag{21}$$

with $f''_{KS}(r_e) = 4\omega/3$.

Thermodynamic quantities of mass, temperature, and heat capacity for the KS black hole are defined as [11]

$$\begin{aligned}&M(r_\pm) = \frac{1 + 2\omega r_\pm^2}{4\omega r_\pm}, \qquad T = \frac{2\omega r_+^2 - 1}{8\pi r_+(\omega r_+^2 + 1)}, \\ &C = -\frac{2\pi}{\omega}\frac{(\omega r_+^2 + 1)^2(2\omega r_+^2 - 1)}{2\omega^2 r_+^4 - 5\omega r_+^2 - 1}.\end{aligned} \tag{22}$$

In the limit of $\omega \to \infty$, these reduce to corresponding quantities of Schwarzschild black hole as

$$M \to \frac{r_+}{2}, \qquad T \to \frac{1}{4\pi r_+}, \qquad C \to -2\pi r_+^2. \tag{23}$$

Now we wish to derive the entropy by considering that the first law of thermodynamics holds for black hole in the deformed HL gravity:

$$dM = T\,dS. \tag{24}$$

Then, the entropy is calculated as

$$S = \int dr_+ \left[\frac{1}{T}\frac{\partial M}{\partial r_+}\right] + S_0, \tag{25}$$

which leads to [18]

$$S = \pi\left[r_+^2 + \frac{1}{\omega}\ln\left(r_+^2\right)\right] + S_0. \tag{26}$$

If one chooses

$$S_0 = \frac{\pi}{\omega}\ln\pi, \tag{27}$$

then we have a compact expression of the entropy

$$S = \frac{A}{4} + \frac{\pi}{\omega}\ln\left[\frac{A}{4}\right] \tag{28}$$

with $A/4 = \pi r_+^2$ and $G = 1$. We note that in the limit of $\omega \to \infty$, Eq. (28) reduces to the Bekenstein–Hawking entropy of Schwarzschild black hole as

$$S_{BH} = \frac{A}{4}. \tag{29}$$

It is clear that the logarithmic term represents the feature of KS black hole in the deformed HL gravity. Accordingly, we have to interpret this logarithmic term to understand why the entropy of KS black hole takes the form (28).

4. GUP-inspired Schwarzschild black hole

A meaningful prediction of various theories of quantum gravity (string theory and loop quantum gravity) and black holes is the presence of a minimum measurable length or a maximum observable momentum. This has provided the GUP which modifies commutation relations between position coordinates and momenta. Also the black hole solution of deformed HL gravity reminds us the Schwarzschild black hole modified with the GUP [11]. Hence, it is very interesting to develop a close connection between GUP and HL gravity. A generalized commutation relation[1] of

$$[x_i, p_j] = i\hbar\delta_{ij}\left(1 + \beta p^2\right) \tag{31}$$

leads to the generalized uncertainty relation

$$\Delta x\Delta p \geqslant \hbar\left[1 + \alpha^2 l_p^2\frac{(\Delta p)^2}{\hbar^2}\right] \tag{32}$$

with $l_p = \sqrt{G\hbar/c^3}$ the Planck length. Here a parameter $\alpha = \hbar\sqrt{\beta}/l_p$ is introduced to take into account the GUP effect. The Planck mass is given by $m_p = \sqrt{\hbar c/G}$. The above implies a lower bound on the length scale

$$\Delta x \geqslant (\Delta x)_{\min} \approx 2\hbar\sqrt{\beta} = 2\alpha l_p, \tag{33}$$

which means that the Planck length plays the role of a fundamental scale. On the other hand, Eq. (32) implies the upper bound on the momentum as

$$\Delta p \leqslant (\Delta p)_{\max} \approx \frac{2}{\sqrt{\beta}} = \frac{2m_p c}{\alpha}. \tag{34}$$

Furthermore, the GUP may be used to derive temperature for the modified Schwarzschild black hole by identifying Δp with the energy (temperature) of radiated photons [29]. The momentum uncertainty for radiated photons can be found to be

$$\frac{\Delta x}{2\alpha^2 l_p^2}\left[1 - \sqrt{1 - \frac{4\alpha^2 l_p^2}{(\Delta x)^2}}\right] \leqslant \frac{\Delta p}{\hbar} \leqslant \frac{\Delta x}{2\alpha^2 l_p^2}\left[1 + \sqrt{1 - \frac{4\alpha^2 l_p^2}{(\Delta x)^2}}\right]. \tag{35}$$

The left inequality implies small corrections to the Heisenberg's uncertainty principle for $\Delta x \gg \alpha l_p$ as $\Delta p \geqslant \hbar/\Delta x + \hbar\alpha^2 l_p^2/(\Delta x)^3 + \cdots$ [30]. On the other hand, the right inequality means that Δp cannot be arbitrarily large in order that the GUP in (32) makes sense. For simplicity, we use the Planck units of $c = \hbar = G = k_B = 1$ which imply that $l_p = m_p = 1$ and $\beta = \alpha^2$. Considering the GUP effect on the near-horizon and $\Delta x = 2r_+ = 4M$, the relation (35) reduces to

$$M\left[1 - \sqrt{1 - \frac{\beta}{4M^2}}\right] \leqslant \frac{\beta\Delta p}{2} \leqslant M\left[1 + \sqrt{1 - \frac{\beta}{4M^2}}\right]. \tag{36}$$

Replacing β with $2/\omega$, the above leads to a relation

$$r_- \leqslant \frac{\Delta p}{\omega} \leqslant r_+. \tag{37}$$

Here we wish to mention that a replacement of $\beta \to 2/\omega$ was performed because both sides of Eq. (36) have mathematically the same form as Eq. (19). It seems that Eq. (37) indicates a connection between quantum and classical properties of KS black holes in the deformed HL gravity.

Importantly, it was shown that based on the GUP [31], quantum correction to the Bekenstein–Hawking entropy S_{BH} of Schwarzschild black hole is given by [32,33]

$$S_{GUP} = S_{BH} + \pi\alpha^2\ln[S_{BH}] - \pi^2\alpha^4\frac{1}{2S_{BH}} + \cdots. \tag{38}$$

Considering the replacement of $\alpha^2 = \beta \to 2/\omega$, the above expression leads to

$$S_{GUP} = S_{BH} + \frac{2\pi}{\omega}\ln[S_{BH}] - \frac{4\pi^2}{\omega^2}\frac{1}{2S_{BH}} + \cdots \tag{39}$$

with $S_{BH} = \pi r_+^2 = A/4$.

Promisingly, for $\alpha^2 = \beta \to 1/\omega$, Eq. (38) recovers Eq. (28) up to the logarithmic term as

$$S_{GUP} \simeq \frac{A}{4} + \frac{\pi}{\omega}\ln\left[\frac{A}{4}\right]. \tag{40}$$

[1] We note that the GUP is in the heart of the quantum gravity phenomenology. Certain effects of quantum gravity are universal and thus, influence almost any system with a well-defined Hamiltonian [28]. In general, the GUP satisfies the modified Heisenberg algebra [23,24]

$$[x_i, p_j] = i\hbar\left(\delta_{ij} + \beta p^2\delta_{ij} + \beta' p_i p_j\right),$$
$$[x_i, x_j] = i\hbar\frac{(2\beta - \beta') + (2\beta + \beta')\beta p^2}{1 + \beta p^2}(p_i x_j - p_j x_i),$$
$$[p_i, p_j] = 0, \tag{30}$$

where p_i is considered as the momentum at high energies and thus, (30) can be interpreted to be the UV-commutation relations. In order to achieve the commutativity, we have to choose $\beta' = 2\beta$. In this case, the minimal length which follows from the modified Heisenberg algebra is given by $(\Delta x)_{\min} = \hbar\sqrt{5\beta}$. We emphasize that *the presence of the minimal length represents a feature of the GUP*. In order to see the GUP-inspired black holes, it is sufficient to consider βp^2 because this term determines the minimal size of the black hole.

5. Discussions

We have found the entropy of Kehagias–Sfetsos black hole in the deformed HL gravity by using the first law of thermodynamics. The presence of logarithmic term $\ln[A/4]$ seems to be universal for the HL gravity because it appeared in topological black hole solutions [4] and LMP solution [8].

We would like to mention that the GUP seems to be a powerful tool to study quantum gravity effects. The GUP with the relation $\Delta x = 2r_+ = 4M$ of the Schwarzschild radius r_+ and its ADM mass M make sense because quantum gravity effects of GUP is universal. Thus, the Schwarzschild black hole was modified if one assumes the GUP. It seems that the GUP explains a part of quantum gravity effects but not whole of these. A relevant relationship of the black hole entropy-area based on string theory and loop quantum gravity is given by [34]

$$S_{LQG} = \frac{A}{4} + \rho \ln\left[\frac{A}{4}\right] + \mathcal{O}\left(\frac{1}{A}\right), \tag{41}$$

where ρ is a model-dependent parameter. Therefore, the GUP was widely used to obtain quantum correction (41) to the Bekenstein–Hawking entropy of Schwarzschild black hole [31–33].

In this work, we have attempted to explain the logarithmic term $(\pi/\omega)\ln[A/4]$ for the entropy of black hole in the deformed HL gravity by considering the GUP-inspired entropy to Schwarzschild black hole. The corresponding quantities may be β in the generalized commutation relation (31) and $1/\omega$ of parameter in the deformed HL gravity (16). In the limit of $\beta \to 0$, we recover the Heisenberg uncertainty relation without quantum gravity effects, while in the limit of $\omega \to \infty$, the entropy of Schwarzschild black hole is recovered without the logarithmic term. This may imply a close connection between GUP and HL gravity.

However, we recognize that the entropy of KS black holes (standard black hole thermodynamics) was being compared with that of the GUP-inspired Schwarzschild black hole (non-standard black hole thermodynamics). This implies that there exists a sort of correspondence (duality) between two systems, but not that the GUP is a fundamental property of Hořava–Lifshitz gravity. Especially, we realize that logarithmic terms in black hole entropy appeared in many different models. At this stage, thus, the logarithmic term in the KS black hole entropy does not represent a definite signal that the GUP is an underlying principle of Hořava construction.

Consequently, we have shown that the duality in the entropy between the KS black hole from the HL gravity and GUP-inspired Schwarzschild black hole is present.

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