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## A fuzzy approach to R&D project portfolio selection <sup>☆</sup>

Christer Carlsson <sup>a</sup>, Robert Fullér <sup>b</sup>,  
Markku Heikkilä <sup>c,a</sup>, Péter Majlender <sup>a,\*</sup>

<sup>a</sup> *Institute for Advanced Management Systems Research, Åbo Akademi University, Lemminkäinengatan 14B, FIN-20520 Åbo, Finland*

<sup>b</sup> *Department of Operations Research, Eötvös Loránd University, Pázmány Péter sétány 1C, H-1117 Budapest, Hungary*

<sup>c</sup> *Turku Centre for Computer Science, Åbo Akademi University, Lemminkäinengatan 14A, FIN-20520 Åbo, Finland*

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### Abstract

A major advance in the development of project selection tools came with the application of options reasoning in the field of *Research and Development* (R&D). The options approach to project evaluation seeks to correct the deficiencies of traditional methods of valuation through the recognition that managerial flexibility can bring significant value to projects. Our main concern is how to deal with non-statistical imprecision we encounter when judging or estimating future cash flows. In this paper, we develop a methodology for valuing options on R&D projects, when future cash flows are estimated by trapezoidal fuzzy numbers. In particular, we present a fuzzy mixed integer programming model for the R&D optimal portfolio selection problem, and discuss how our methodology can be used to build decision support tools for optimal R&D project selection in a corporate environment.

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*Keywords:* Fuzzy number; Possibilistic mean; Possibilistic variance; Real option valuation

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\* Corresponding author.

*E-mail addresses:* [christer.carlsson@abo.fi](mailto:christer.carlsson@abo.fi) (C. Carlsson), [rfuller@cs.elte.hu](mailto:rfuller@cs.elte.hu) (R. Fullér), [markku.heikkila@abo.fi](mailto:markku.heikkila@abo.fi) (M. Heikkilä), [peter.majlender@abo.fi](mailto:peter.majlender@abo.fi) (P. Majlender).

## 1. Introduction to R&D management

Usually, new production technologies are developed infrequently, and they often evolve in uneven pace. Innovations are unpredictable, and thus involve large uncertainties with respect to both the development of opportunities in existing product markets and those in production processes. Corporate R&D management, supporting the maximal use of new innovations and technologies, always tries to keep the company up with the pace of technological development. R&D projects are tools for the company's management to outpace competitors and obtain new information about promising technologies and methods. With such new information, companies aim to defend and build sustainable competitive advantages [25].

Due to their proactive nature, R&D projects are sometimes hard to evaluate. It is often the case that information required for the valuation is actually revealed gradually during the project, and at the beginning of the development opportunity there are no cash flow estimates available that would either justify or invalidate the evaluation of the project. In a sense of information quality, knowledge about the project's profitability is seldom precise, let alone that sometimes it is not even measurable. However, even in such situations the R&D management has to commit itself to either a positive decision to launch the project, a negative decision to abandon the project, or, which seems most plausible, a decision to wait and see if the information quality improves as time passes. This management position can be described as if the management had some information *hidden* or in *shadow*, and it had to make a decision about consuming some resources in order to uncover the information. The decision to use resources for information retrieval leads to the launch of the investment, when the *option to start* the project is used. On the other hand, the decision to deny resources leads to the abandonment of the underlying investment, when the *option to abandon* the project is used. Finally, the decision to stand by and wait for new information leads to *waiting* and *deferring* the investment opportunity, where both the option to start and the option to abandon are kept alive. In the absence of *quantitative* value-based statements represented by the cash flows, the R&D management often relies on *qualitative* statements made by the technological experts.

In the framework of R&D portfolio selection with real options, this kind of a management approach has a natural appeal. In 1993, Bowman and Hurry described how strategic management can be represented in the light of option theory [4]. Following their approach, we can view a strategic management process as a chain of options, where options have not been identified and are not known initially; this type of options are called *shadow options*. They become *real options* when real assets and the possible future use of real assets get connected to the options of starting R&D projects. With real options, the management has tangible strategic alternatives, such as real investment possibilities or joint ventures that can be exercised or put aside until a better time for entering into. In the case the strategic real option is expected to supply no further options, or the options are extinguished by the changes in the markets and technologies, the strategy is annulled. However, if the launch of the project supplies further options or managerial flexibility, the management can commit itself to either a strategy of *incremental continuation* or a strategy of *radical change*.

R&D management has several common features with strategic management. It actively aims at utilizing possibilities supplied by new technologies and innovations in business operations. Similarly to strategic management, R&D management also has to define objectives for the R&D operations. Following the basic R&D management approach,

we have created a support tool for evaluating R&D opportunities – the *Extended Project Portfolio Tool* (XPT) – for the following purposes:

- (1) to detect shadow options that are not yet measurable in terms of cash flows, and
- (2) to include such options into the decision making with R&D portfolios.

This kind of decision support approach is natural from the point of view of how various types of uncertainties are realized and resolved during the R&D projects. In several cases, uncertainties related to R&D are not only *systematic*, like the financial portfolio risk, or *dynamic*, like the volatility of the return on a stock option. Instead, for the management of R&D options, it is essential to recognize that uncertainty can also originate in discontinuities and discrepancies of the market and technological dynamics. Even though these factors do not directly affect the financial or operational variables of the portfolio, it is clear that they can make significant impact on the design of optimal strategy. In such a situation, the R&D management has to optimize the expected operational and financial benefits of the R&D portfolio by incorporating market and technology-based uncertain strategic effects into the portfolio selection process.

The approach we applied in XPT balances the portfolio selection process with respect to the various modes of strategic change, including

- the *incremental continuous strategy*,
- the *radical changing strategy* and
- the *explorative pilot strategy*.

Obviously, the optimality of the portfolio is implied by the decision. The R&D management has to choose a portfolio, where the aggregate contribution of the individual R&D projects (that define and correspond with certain R&D strategies) present a reasonable outcome with respect to the overall strategic control mechanisms. Such mechanisms can be set up by for instance presenting budgets that are allocated to specific strategies. There can be various kinds of budgets, such as

- financial,
- technological,
- market-based, or
- budgets based on the availability of some critical resources.

Furthermore, instead of the *mean-variance-based risk-return* criteria, the management has to deal with the following three criteria:

- (1) *return*,
- (2) *uncertainty* and
- (3) *strategic fit*

(see Figs. 1 and 2). The choice is subject to the particular mechanism applied by the management in practice when determining the budgets that allocate capital and other resources. It is also important to know how exogenous strategic knowledge and endogenous knowledge about expected project contribution can change the budgetary limits.

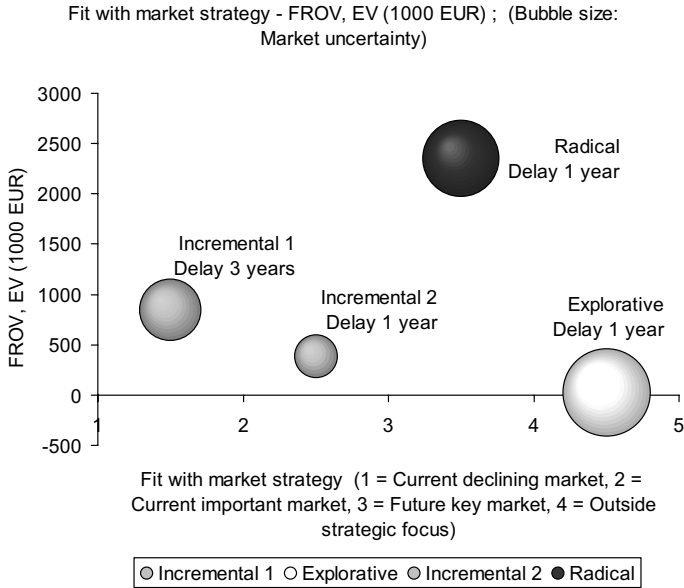


Fig. 1. Bubble diagram of four strategic R&D options in terms of profitability by *real option valuation*, market strategy and market uncertainty.

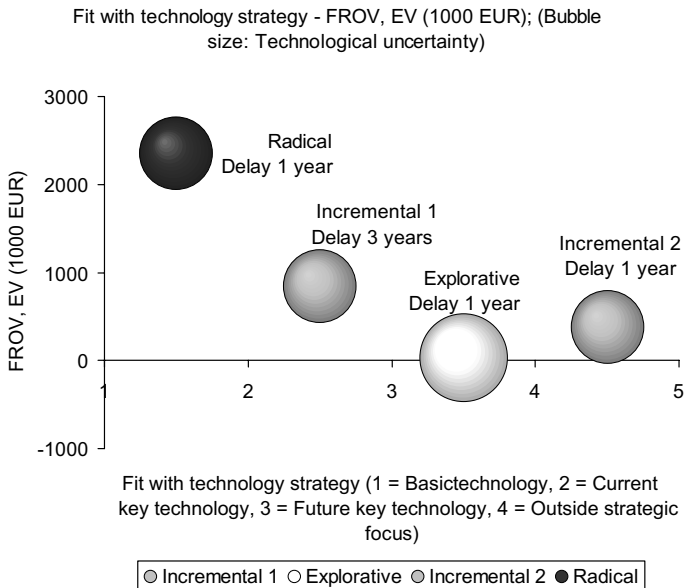


Fig. 2. Bubble diagram of four strategic R&D options in terms of profitability by *real option valuation*, technology strategy and technology uncertainty.

The use of *uncertainty* instead of *risk* as a criterion may require some more explanation. This choice addresses the situation, where the decision maker has to evaluate economic

benefits in lack of some well-functioning efficient market mechanism which could integrate the expected financial benefits and perceived financial risks into a single *risk-return* framework.

For example, a technological expert may state that if the project were to start now, then it would be possible to apply for a patent after a few years to protect a new production method. However, the expert cannot give any detail (e.g. probability distribution) about the likelihood that the patent will be granted or that the technological foresight will eventually come true. In such a case the patent is a “pre R&D option” that becomes a “real R&D option” when the uncertainty about the underlying technological foresight is determined. It would not make much sense if the technological expert presented some subjective probabilities to characterize outcomes that are unknown (not only for him or her but also for the whole community of technological experts). Thus, an expectation involving this kind of uncertainty cannot be represented as a *probability* of some known future state. However, it can be considered as a *possibility*, that is, a foresight characterizing the currently unknown state. In this case, the R&D management can

- (1) start the investment in order to resolve the uncertainty of the underlying foresight,
- (2) abandon the project as a sign of disbelief in its future success or profitability or
- (3) wait to stand by and analyze further the current situation by collecting more information.

In practice, R&D project portfolio selection problems are complicated due to the fact that the *quality* as well as the estimated *numerical data* of cash flows vary at every stage of the R&D development. When setting up an R&D project by defining its goals and deliverables, the project team essentially offers prospective new innovations. At this stage there may be (and quite often *is*) no knowledge about the revenue that would be generated when the project finishes. As time passes, and relevant information is retrieved and processed, the quality of cash flow estimates gradually improves. At that stage the project can be evaluated and analyzed by using its representative cash flow data based on net present valuation (NPV) or real option valuation (ROV) methods. However, to include the notion of *shadow options* into the decision making process, we have to define some characteristic criteria to recognize them in their embedded form. These criteria are based on the dynamics and uncertainties of both market and technology. In the process of characterizing R&D projects, the management translates the performance attributes of each project into portfolio criteria, such as budget limits. This stage can involve the representation of conflicting goals and interests, and it is carried out as an interactive negotiation process that aims at finding an imprecise but operational consensus.

Formulating from this point of view, we seek to correct the deficiencies of traditional investment valuation methods by incorporating the managerial flexibility that can (and usually does) bring significant value to projects. From our experience, we found that the main issue in the options approach to strategic project valuation is the correct characterization of the non-statistical imprecision that we encounter when judging or estimating future cash flows. Working out schemes for phasing and scheduling systems of interrelated projects, we will develop a basic model for valuing options on R&D investment opportunities, when future revenues are estimated by trapezoidal possibility distributions. Furthermore, drawing on our results, we shall present a fuzzy mixed integer programming model for the R&D optimal project portfolio selection problem.

## 2. Properties and valuation of R&D projects

Investment opportunities of the R&D type compete for major portions of the risk-taking capital, and as their outcome is particularly uncertain, compromises have to be made on their productivity. The short-term productivity may not be high, although the overall return of the investment program can be forecasted as very good. Another way of motivating an R&D investment is to point to strategic advantages, which would not be possible without the knowledge that the investment yields. Thus, R&D projects do offer some indirect (intangible) returns as well.

However, there are other issues. Global financial markets make sure that capital cannot be used non-productively, as its owners are offered other opportunities, and the capital will move (often quite fast) to capture these opportunities. The capital market has learned “the American way”, i.e. there is a shareholder dominance among the actors, which has often brought short-term shareholder return to the forefront as a key indicator of success, profitability and productivity. There are also lessons learned from the Japanese industry, which point to the importance of *immaterial investments*. They show that investments in buildings, production and supporting technologies become enhanced with immaterial investments, and that these are even more important for further investments and gradually growing maintenance investments.

With the core products and services created by R&D investments, markets are enhanced with lifetime services and gradually more advanced maintenance and financial add-in services. These features make it difficult to actually assess the productivity and profitability of the original R&D project, especially if the products and services are repositioned to serve new (e.g. emerging) markets. New technology and rapid technological innovations can change the life cycle of R&D investments, even as they are planned and evaluated. The challenge is to find the right time and the right innovation to modify the life cycle in an optimal way. Technology providers are actively involved throughout the life cycle of R&D projects, which actually changes the way we assess the profitability and the productivity of such investments.

R&D projects, and in particular, portfolios of R&D projects generate commitments, which possess the following properties:

- (i) long life cycles (taking into account their possible impacts on other investments),
- (ii) uncertain (i.e. *vague*), sometimes overly optimistic or pessimistic future cash flow estimates,
- (iii) uncertain (i.e. *biased*), sometimes questionable profitability estimates,
- (iv) imprecise assessments of future effects on productivity, market positions, competitive advantages and shareholder value, and
- (v) the ability to generate series of further investments.

Jensen and Warren [15] propose to use options theory to value R&D in the telecom service sector. The reasons are rather similar to those we identified above: research managers are under pressure to explain the value of R&D programs to senior management, and at the same time they need to evaluate individual projects to make management decisions on their own R&D portfolio. The research in real options theory has evolved from general presentations of flexibility of investments in industrial cases to more theoretical contributions, which resulted in the application of real option valuation methods to industrial R&D pro-

jects. The term real option was introduced by Kester [16] and Myers [21] in 1984. The option to postpone an investment opportunity was discussed by McDonald and Siegel [23]. Pakes [24] considered patents as options. Siegel et al. [26] discussed the option valuation of offshore oil properties. Majd and Pindyck [22] analyzed the optimal time and computed the option value of building operations in investment decisions. A fundamental book on managerial flexibility and strategy in resource allocation, written by Trigeorgis [27], presented a theory of real options. Abel et al. [1] discussed a theory of option valuation of real capital investments. Faulkner [14] discussed the application of real options to the valuation of R&D projects at Kodak. Kulatilaka et al. [17] discussed a capability-based real options approach to managing information technology investments.

The use of fuzzy sets to work with real options is a novel approach, which has not been considered and analyzed widely so far. One of the first results to apply fuzzy mathematics in finance was presented by Buckley [5], where he worked out how to use fuzzy sets to formulate the concepts of future value, present value and internal rate of return. Carlsson and Fullér [6] also dealt with fuzzy internal rate of return in the context of investment decisions for paper mills in the forest industry. Later, Carlsson and Fullér [7] developed a method for managing capital budgeting problems with fuzzy cash flows. However, there are a growing number of papers in the intersection of the disciplines of *real options* and *fuzzy sets*. In one of the first papers on developing the fuzzy Black-Scholes model, Carlsson and Fullér [8] presented a fuzzy real option valuation method. Muzzioli and Torricelli [20] used fuzzy sets to frame the binomial option pricing model. Carlsson and Fullér [9] analyzed the optimal timing of investment opportunities with fuzzy real options. Carlsson et al. [11,13] developed and tested a method for project selection with optimal timing and scheduling by using the methodology of fuzzy real options. Majlender [19] presented a comprehensive framework of the development of investment valuation methods in a possibilistic environment.

### 3. Real options for R&D portfolios

The options approach to R&D project valuation seeks to correct the deficiencies of traditional methods of valuation that are based on the methodologies of Net Present Valuation (NPV) and Discounted Cash Flow (DCF) analysis, through the recognition of managerial flexibility and its interaction with the underlying investment opportunities. This uncertainty can bring significant value to projects.

Real options in option thinking are based on the same principles as financial options. In real options, the options involve “real” (i.e. productive) assets as opposed to financial ones, where the options relate to some financial instruments [2]. To have a “real option” means to have the possibility for a certain period of time to either choose for or against something, without binding ourselves up front. The value of a real option is computed by [18]

$$\text{ROV} = S_0 e^{-\delta T} N(d_1) - X e^{-r_f T} N(d_2),$$

where

$$d_1 = \frac{\ln(S_0/X) + (r_f - \delta + \sigma^2/2)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln(S_0/X) + (r_f - \delta - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T},$$

and where  $S_0$  is the present value of expected cash flows,  $X$  is the nominal value of fixed costs,  $\delta$  is the value lost over the duration of the option,  $r_f$  is the annualized continuously compounded rate on a safe asset,  $T$  is the time to maturity of the option in years, and  $\sigma$  stands for the uncertainty of the expected cash flows potentially involved in  $S_0$ ; finally,  $N(d)$  denotes the probability that a random draw from a standard normal distribution will be less than  $d$ .

The main question that a firm must answer for a deferrable investment opportunity is the following:

*How long should we postpone the investment up to  $T$  time periods?*

To answer this question, Benaroch and Kauffman [3] suggested the following decision rule for an optimal investment strategy:

Where the maximum deferral time is  $T$ , make the investment (i.e. exercise the real option) at time  $t^*$ ,  $0 \leq t^* \leq T$ , for which the value of the option  $C_{t^*}$  is positive and attains its maximum value. That is,

$$C_{t^*} = \max_{t=0,1,\dots,T} \{V_t e^{-\delta t} N(d_1) - X e^{-r_f t} N(d_2)\} > 0, \tag{1}$$

where

$$\begin{aligned} V_t &= \text{PV}(\text{cf}_0, \text{cf}_1, \dots, \text{cf}_T; r) - \text{PV}(\text{cf}_0, \text{cf}_1, \dots, \text{cf}_{t-1}; r) = \text{PV}(\text{cf}_t, \dots, \text{cf}_T; r) \\ &= \sum_{j=0}^T \frac{\text{cf}_j}{(1+r)^j} - \sum_{j=0}^{t-1} \frac{\text{cf}_j}{(1+r)^j} = \sum_{j=t}^T \frac{\text{cf}_j}{(1+r)^j}, \end{aligned}$$

and where  $\text{cf}_t$  denotes the expected cash flows at time  $t$ ,  $t = 0, 1, \dots, T$ , and  $r$  is the project-specific risk-adjusted discount rate.

Of course, this decision rule has to be reapplied every time when new information arrives during the deferral period to see how the optimal investment strategy changes in the light of the new information. From a real option perspective, it can be worthwhile to undertake R&D investments with a negative Net Present Value (NPV), when early investments can provide information about future benefits or losses of the whole investment program.

#### 4. A hybrid approach to real option valuation

A fuzzy set  $\tilde{A}$  on the real line  $\mathbb{R}$  is called a trapezoidal fuzzy number with core  $[a, b]$ , left width  $\alpha \geq 0$  and right width  $\beta \geq 0$ , if its membership function is of the following form:

$$\tilde{A}(t) = \begin{cases} 1 - \frac{a-t}{\alpha} & \text{if } a - \alpha < t < a, \\ 1 & \text{if } a \leq t \leq b, \\ 1 - \frac{t-b}{\beta} & \text{if } b < t < b + \beta, \\ 0 & \text{otherwise} \end{cases}$$

and we use the notation  $\tilde{A} = (a, b, \alpha, \beta)$ .

Usually, the present value of the expected cash flows cannot be characterized by a single number. However, they can be estimated by a trapezoidal possibility distribution of the form

$$\tilde{S}_0 = (a, b, \alpha, \beta).$$



That is, the most possible values of the present value of the expected cash flows lie in the interval  $[a, b]$  (which is the core of the trapezoidal fuzzy number  $\tilde{S}_0$ ), and  $(b + \beta)$  is the upward potential and  $(a - \alpha)$  is the downward potential for the present value of the expected cash flows. In a similar manner, we can estimate the nominal value of the expected costs by using a trapezoidal possibility distribution of the form

$$\tilde{X} = (a', b', \alpha', \beta').$$

That is, the most possible values of the expected costs lie in the interval  $[a', b']$  (which is the core of the trapezoidal fuzzy number  $\tilde{X}$ ), and  $(b' + \beta')$  is the upward potential and  $(a' - \alpha')$  is the downward potential for the expected costs.

In 2003, Carlsson and Fullér [12] suggested the use of the following hybrid (fuzzy-probabilistic) formula for computing fuzzy real option values:

$$\tilde{C}_0 = \tilde{S}_0 e^{-\delta T} N(d_1) - \tilde{X} e^{-r_f T} N(d_2), \tag{2}$$

where

$$d_1 = \frac{\ln(E(\tilde{S}_0)/E(\tilde{X})) + (r_f - \delta + \sigma^2/2)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln(E(\tilde{S}_0)/E(\tilde{X})) + (r_f - \delta - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T},$$

and where  $E(\tilde{S}_0)$  denotes the possibilistic mean value of the present value of the expected cash flows,  $E(\tilde{X})$  stands for the possibilistic mean value of the expected costs and  $\sigma = \sigma(\tilde{S}_0)$  is the possibilistic variance of the present value of the expected cash flows [10]. Based on (2), Carlsson and Fullér derived a similar formula to (1) for the optimal investment strategy in a possibilistic setting [12].

### 5. A possibilistic approach to R&D portfolio selection

Facing a set of project opportunities of R&D type, the company is usually able to estimate the expected investment costs of the projects with a high degree of certainty. Thus, in the following we will assume that  $\tilde{X} = X \in \mathbb{R}$  is a crisp number. However, the cash flows received from the projects do involve uncertainty, and they are modelled by trapezoidal possibility distributions. Let us fix a particular project of length  $L$  and maximum deferral time  $T$  with cash flows

$$\tilde{cf}_i = (A_i, B_i, \Phi_i, \Psi_i), \quad i = 0, 1, \dots, L.$$

Now, instead of the *absolute* values of the cash flows, we shall consider their *fuzzy returns on investment* (FROI) by computing the return that we receive on investment  $X$  at year  $i$  of the project as

$$FROI_i = \tilde{R}_i = \frac{\tilde{cf}_i}{X} = \left( \frac{A_i}{X}, \frac{B_i}{X}, \frac{\Phi_i}{X}, \frac{\Psi_i}{X} \right) = (a_i, b_i, \alpha_i, \beta_i).$$

**Example 1.** Let  $\tilde{cf}_i = (0.9, 8.4, 3.9, 5.6)$  and  $X = 6$ . Then

$$\tilde{R}_i = (15\%, 140\%, 65\%, 93\%)$$

with possibilistic mean value

$$E(\tilde{R}_i) = \frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{6} = \frac{15 + 140}{2} + \frac{93 - 65}{6} = 82.17\%,$$

and (possibilistic) standard deviation

$$\begin{aligned} \sigma(\tilde{R}_i) &= \sqrt{\left(\frac{b_i - a_i}{2} + \frac{\alpha_i + \beta_i}{6}\right)^2 + \frac{(\alpha_i + \beta_i)^2}{72}} = \sqrt{\left(\frac{140 - 15}{2} + \frac{65 + 93}{6}\right)^2 + \frac{(65 + 93)^2}{72}} \\ &= 90.76\%. \end{aligned}$$

We compute the fuzzy net present value of the project by

$$\text{FNPV} = \left[ \sum_{i=0}^L \frac{\tilde{R}_i}{(1+r)^i} - 1 \right] \times X.$$

If a project with fuzzy returns on investment

$$\{\tilde{R}_0, \tilde{R}_1, \dots, \tilde{R}_L\}$$

can be postponed by a *maximum* of  $T$  years, then we will define the value of its *possibilistic deferral flexibility* by

$$\mathcal{F}_T = (1 + \sigma(\tilde{R}_0)) \times (1 + \sigma(\tilde{R}_1)) \times \dots \times (1 + \sigma(\tilde{R}_{T-1})) \times \text{FNPV},$$

where  $1 \leq T \leq L$ . If a project cannot be postponed then its possibilistic flexibility equals to its fuzzy net present value. That is, if  $T=0$  then  $\mathcal{F}_T = \text{FNPV}$ .

Considering a single R&D project with a maximum deferral flexibility of  $T$  time periods, the ultimate questions that a company has to address are the following:

*Should we undertake the R&D project? If we should, then how long do we need to postpone it before entering into it to utilize its potential and generate maximum profit?*

Applying the notion of possibilistic deferral flexibility, and adopting the methodology of [3], we can use the following decision rule for an optimal investment strategy:

If the maximum deferral time is  $T$ , launch the R&D investment (i.e. exercise the real option) at time  $t^*$ ,  $t^* \in \{0, 1, \dots, T\}$ , for which the value of the possibilistic deferral flexibility  $\mathcal{F}_{t^*}$  is positive and reaches its maximum. Namely,

$$\mathcal{F}_{t^*} = \max_{t=0,1,\dots,T} \{(1 + \sigma(\tilde{R}_0)) \times \dots \times (1 + \sigma(\tilde{R}_{t-1})) \times \text{FNPV}\} > 0,$$

where FNPV stands for the fuzzy net present value of the project, and  $\tilde{R}_i$  denotes the rate of return on investment at year  $i$ ,  $i = 0, 1, \dots, L$ .

Gradually, this decision rule has to be reapplied every time when new information arrives during the deferral period to see how the optimal investment strategy changes in the light of the new information. Taking into consideration that a large value of possibilistic deferral flexibility implies a large potential that the R&D project becomes profitable in the future, the R&D management can justify the support of R&D investments with small net present values and big deferral flexibilities. Keeping those opportunities alive, the management can digest information about future benefits or losses associated with the whole investment program.

The basic optimal R&D project portfolio selection problem can be formulated as the following fuzzy mixed integer programming problem:

$$\begin{aligned} & \text{maximize} && \mathcal{F} = \sum_{i=1}^N u_i \mathcal{F}_i \\ & \text{subject to} && \sum_{i=1}^N u_i X_i + \sum_{i=1}^N (1 - u_i) c_i \leq B, \\ & && u_i \in \{0, 1\}, \quad i = 1, \dots, N, \end{aligned} \quad (3)$$

where  $N$  is the number of R&D projects;  $B$  is the whole investment budget;  $u_i$  is the decision variable associated with project  $i$ , which takes value one if project  $i$  starts now (i.e. at time zero) and takes value zero if it is postponed and is going to start at a later time;  $c_i$  denotes the cost of postponing project  $i$  (i.e. the capital expenditure required to keep the associated real option alive); finally,  $X_i$  and  $\mathcal{F}_i$  stand for the investment cost and the possibilistic deferral flexibility of project  $i$ , respectively,  $i = 1, \dots, N$ .

In our approach to fuzzy mathematical programming problem (3), we have used the following defuzzifier operator for  $\mathcal{F}$ :

$$v(\mathcal{F}) = (E(\mathcal{F}) - \tau \times \sigma(\mathcal{F})) \times X,$$

where  $0 \leq \tau \leq 1$  denotes the decision maker's risk aversion parameter.

Since R&D projects are characterized by long planning horizons and very high levels of uncertainty, the value of managerial flexibility can be substantial. Therefore, the fuzzy real options model is quite practical and useful. The standard work in the field uses probability theory to account for the uncertainties involved in future cash flow estimates. This may be defended for financial options, for which we can assume the existence of an efficient market with numerous players and numerous stocks for trading, which in turn justifies the assumption of the validity of the laws of large numbers and thus the use of statistical methods. The situation for real options associated with an investment opportunity of R&D type is quite different. The option to postpone an R&D project does have consequences, which differs from efficient markets, as the number of players as well as the number of consequences produced are quite small. The imprecision we encounter when judging or estimating future cash flows is non-stochastic by nature, and the use of probability theory can give us a misleading level of precision and a notion that the consequences are somehow repetitive. This is not the case, since in our case the uncertainty is genuine, i.e. we simply do not know the exact level of future cash flows. Without introducing fuzzy real option models, it would not be possible to formulate this genuine uncertainty.

The proposed model that incorporates subjective judgments as well as statistical uncertainties can give investors a better understanding of the problem when making R&D investment decisions.

## 6. Summary

Multinational enterprises with large R&D departments often face the difficulty of selecting an appropriate portfolio of research projects. The cost of developing a new product or technology is low as compared to the cost of its introduction to the global market. The NPV rule and other discounted cash flow techniques for making R&D investment

decisions seem to be inappropriate for selecting a portfolio of R&D projects, as they favor short term projects in relatively certain markets over long term projects in relatively uncertain markets. Since many new products are identified as failures during the R&D stages, the possibility of refraining from market introduction can add a significant value to the NPV of R&D projects. Therefore, R&D investments can be interpreted as the price of an option on major follow-on investments.

In our *OptionsPort* project,<sup>1</sup> we represented the optimal R&D portfolio selection problem as a fuzzy mathematical programming problem, where the optimal solutions defined the optimal portfolios of R&D projects with the largest (aggregate) possibilistic deferral flexibilities.

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