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Reconstruction of inhomogeneous properties of orthotropic viscoelastic layer

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ABSTRACT

The consecutive scheme of reconstructing the functions characterizing the instantaneous and long-termed modules, for inhomogeneous orthotropic viscoelastic layer whose properties continuously vary through a thickness is proposed. The identification problem is solved on the basis of the additional information on the integral characteristics of displacement fields measured on top border of a layer.

Iterative process of reconstruction of six unknown functions is formulated. To reconstruct the rest six functions, systems of Fredholm's integral equations of the first kind with smooth kernels are received.

Computational experiment on a reconstruction of various laws of inhomogeneity is conducted; a comparative analysis of the results obtained is carried out.

1. Introduction

The present century is marked by the prosperity of new production technologies of functionally graded materials (FGM) (Hirai, 1996; Aboudia et al., 1999) which properties continuously vary depending on one or several coordinates and that successfully ousts layered composites. FGM are manufactured by means of mechanical bonding of materials with differing values of physical and chemical parameters. As a rule, change of FGM properties is associated with the corresponding variation of the chemical composition or physical structure of the material. To obtain such material they use layer-by-layer coating with changing composition or sintering of several plates or tablets of various compositions. Properties gradient through volume can be implemented by change of degree of linking (for polymers) and filling; and gradient through volume surface – by varying the degree of modification, for instance. Regularity of such characteristics leads to the uniformity of properties variation, and the irregularity leads to uneven form of dependence of properties on composition or structure. The basic operating characteristic of FGM is the possibility of using such materials in high-duty devices being exploited in extreme conditions (high gradients of mechanical loads and temperatures).

One of the most widespread ways to describe FGM properties is to use the theory of mixtures and introduction of effective modules which are obtained by using homogenization of heterogeneous mechanical properties of a sample. Usually, as functions describing heterogeneous mechanical properties of an object of research, exponential and power laws are used.

In this way, in the paper (Lu et al., 2008) the procedure of calculation of natural frequencies of a three-dimensional functionally graded cantilever beam is presented with consideration of various laws describing the properties of FGM: linear, exponential and homogeneous laws.

Exponential and power laws of effective properties for FGM beams are also used in the papers (Ray and Sachade, 2006; Simsek et al., 2012). In Guo and Noda (2008) a finite element model for static analysis of functionally graded plates integrated with a layer of piezoelectric fiber-reinforced composite (PFRC) material was developed. The layer of the PFRC material acted as distributed actuator of plates. The PFRC material considered there was a new smart material with enhanced effective piezoelectric coefficient as compared to its constituent monolithic counterpart, and piezoelectric fibers were oriented longitudinally in a plane of FG plates.

The above researches are devoted to direct problems study for bodies and structures of FGM, in which laws of variation of materials properties are assigned a priori. It should be noted that in production of functionally graded composites for the realization of quality control, the reliable techniques of identification of physical

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properties are required, which would allow to confirm a coincidence of real properties obtained with those that were designed (Yu and Wu, 2009). It is important for such technologies to be fast, simple and inexpensive. The acoustic method of sounding satisfies such requirements.

Models of inhomogeneous bodies, except the application described above, are used in many other fields of mechanics: biomechanics, mechanics of composite materials, and mechanical engineering.

Therefore, a problem of identification of inhomogeneous properties is one of the most important problems allowing carrying out correct calculations of durability, stability and oscillations of constructions of such materials.

In case, when laws of the theory of mixtures are used to describe properties of FGM, the problem of identification is reduced to the determination of characteristic parameters (which are constants) of distribution laws that are known a priori. As a rule, such inverse coefficient problems lead to the problem of residual functional minimization in a finite dimensional space. The implementation of minimization methods has its own specificity and can be carried out on the basis of gradient methods or on the basis of evolutionary algorithms. The problem of the viscoelastic properties identification of traditional multilayer composite structures is also reduced to the same problem of residual functional minimization, in which homogeneous elastic or viscoelastic layers are connected to each other, forming a structure with improved mechanical and thermal properties (Dietrich et al., 1998; Elkhaldi et al., 2012). In the study (Al-Khoury et al., 2002), the inverse problem of identification of 4 viscoelastic parameters (constants) for isotropic viscoelastic layered medium was considered. The reconstruction was realized by the minimum condition of the residual functional on the basis of displacement fields measured under a sensor. In the paper (Lecompte et al., 2007) the inverse coefficient problem of identification of homogeneous elastic properties of cross-shaped sample within the framework of an orthotropic material models is studied.

In case of inhomogeneous materials, inverse coefficient problem is much more complicated. The difficulties are caused both by solving direct problem that can be solved only numerically with using of finite difference or finite element schemes, and complexity of the inverse problems solving, in particular, by formulation of operator relations between functions to reconstruct and traces of solutions at the boundary. In the paper (Chen et al., 2013) an inverse algorithm based on the conjugate gradient method and the discrepancy principle was employed to estimate the unknown space-dependent thermal conductivity of functionally graded hollow cylinder.

A distinctive feature of the identification scheme proposed is the possibility to reconstruct laws unknown a priori that describe inhomogeneous properties of materials. The proposed method was successfully applied for a reconstruction of inhomogeneous properties of elastic (Vatulyan et al., 2011) and electroelastic (Vatulyan et al., 2012) layered media, and of inhomogeneous residual stress state in beams and plates (Nedin and Vatulyan, 2011, 2013a,b). In the present research, the viscosity which is typical for real bodies' and structures' behavior is considered. The problem on reconstruction of inhomogeneous viscoelastic properties of orthotropic layer is considered on the basis of the data on integral characteristics of displacements fields measured on the top border of layer under frequency sounding mode.

The proposed problem is urgent because of the need to create efficient methods of reconstruction of inhomogeneous mechanical properties of medium with layered structure which are extensively used during the modeling of different objects, such as biological tissues, layered composites, and new functionally gradient materials.

2. Statement of the inverse coefficient problem

Let us consider the steady-state oscillations of inhomogeneously thick viscoelastic orthotropic layer $S = \{x_1, x_2 \in (-\infty, \infty), x_3 \in [0, h]\}$ under in-plane and out-of-plane deformations. The bottom side of a layer is clamped; a part of upper border is subjected to a load applied with the frequency ω and determined by vector $P(x_1, t) = p(x_1)e^{i\omega t},\ x_1 \in [a, b]$.

Within the context of this formulation, after elimination of the time factor, the boundary value problem takes the form

$$\sigma_{ij,j} + \rho\omega^2 u_i = 0, \tag{1}$$

$$\begin{aligned} &x_3 = 0:\ \ u_i = 0 \\ &x_3 = h:\ \ \sigma_{i3} = \begin{cases} p_i(x_1),\ \ x_1 \in [a, b] \\ 0,\ \ x_1 \notin [a, b] \end{cases} \end{aligned} \tag{2}$$

In the case of in-plane deformation $(i, j = 1, 3)$ the nonzero components of the displacement vector are $u_1(x_1, x_3),\ u_3(x_1, x_3)$, the constitutive relations take form

$$\begin{aligned} \sigma_{11} &= C^*_{11}u_{1,1} + C^*_{13}u_{3,3} \\ \sigma_{33} &= C^*_{13}u_{1,1} + C^*_{33}u_{3,3} \\ \sigma_{13} &= C^*_{55}(u_{1,3} + u_{3,1}) \end{aligned} \tag{3}$$

In out-of-plane case $(i = 2, j = 1, 3)$ the displacement vector has the only nonzero component $u_2(x_1, x_3)$ and the constitutive relations are

$$\sigma_{21} = \sigma_{21} = C^*_{66}u_{2,1}, \quad \sigma_{23} = C^*_{44}u_{2,3} \tag{4}$$

The relations represent a transfer of the model of standard viscoelastic body to the three-dimensional case; and besides, the body is inhomogeneous, i.e. both instant and long modules are the functions of transverse coordinate. Accounting of a viscosity is realized in the framework of the model of standard viscoelastic body based on the correspondence principle (Christensen, 1971); in accordance with this principle, the elastic modules in constitutive relations of elasticity theory are replaced by complex modules, i.e. now, in spite of the elastic function case, the characterizing physico-mechanical properties of the viscoelastic material depend not only on the coordinate x_3, but also on the vibration frequency.

$$C^*_{lm}(x_3, i\omega) = \frac{i\omega n C'_{lm}(x_3) + C''_{lm}(x_3)}{1 + i\omega n} \tag{5}$$

were $C'_{lm}(x_3)$ and $C''_{lm}(x_3)$ are the functions characterising dependencies of the instantaneous and long-termed modules on cross-section coordinate x_3, n is the relaxation time. While carrying out computing experiments it is important to consider a model condition $C'_{lm}(x_3) > C''_{lm}(x_3)$.

The inverse coefficient problem is reduced to determination of inhomogeneous characteristics $C'_{lm}(x_3)$ and $C''_{lm}(x_3)$ from additional information on the displacement fields measured at the upper boundary of the layer under the frequency probing mode in case of in-plane (6) and out-of-plane (7) oscillations:

$$u_1(x_1, \omega) = f_1(\omega),\ \ u_3(x_1, \omega) = f_3(\omega), \tag{6}$$

$$u_2(x_1, \omega) = f_2(\omega) \tag{7}$$

3. Solution of the inverse coefficient problem. Reduction to a system of Fredholm's integral equations of the first kind

The scheme of inhomogeneous properties reconstruction for orthotropic layer that presented in Vatulyan et al. (2013) is further developed for the case of viscoelastic heterogeneous layered medium.

Let us apply the integral Fourier transform on spatial coordinate x_1 and denote the corresponding Fourier transforms as

$$\tilde{u}_j(x_3,\alpha) = \int_{-\infty}^{+\infty} u_j(x_1,x_3,\alpha)e^{i\alpha x_1}dx_1.$$

Let us introduce the dimensionless variables

$$x_3 = hx, \quad \tilde{u}_s = hU_s, \quad k^2 = \rho\omega^2 h^2/C'_{55}(0),$$

$$C^*_{lm} = C'_{55}(0)c^*_{lm}, \quad \tilde{p}_s = C'_{55}(0)P_s, \quad s = 1,2,3, \quad l,m = 1,3,4,5,6$$

$$v = n\sqrt{\frac{C'_{55}(0)}{\rho h^2}}, \quad c^*_{lm}(x,ik) = \frac{ikvc'_{lm}(x) + c''_{lm}(x)}{1+ikv},$$

$$c''_{lm}(x) = \frac{C''_{lm}(x_3)}{C'_{55}(0)}, \quad c'_{lm}(x) = \frac{C'_{lm}(x_3)}{C'_{55}(0)}$$

By applying the known analytical properties of the Fourier transform of bounded functions, let us represent them as decompositions by powers of α degree, where the coefficients of decompositions are the moments of different orders of the displacement vector components.

$$\begin{aligned}
&U_j(x,\alpha) = U_j^{(0)}(x) + i\alpha U_j^{(1)}(x) + \alpha^2 U_j^{(2)}(x) + \cdots,\\
&P_j(\alpha) = P_j^{(0)} + i\alpha P_j^{(1)} + \alpha^2 P_j^{(2)} + \cdots, \ j = 1,2,3\\
&P_j^{(0)} = \int_{-\infty}^{+\infty} P_j(x)dx, \ P_j^{(1)} = \int_{-\infty}^{+\infty} P_j(x)xdx, \ P_j^{(2)} = -\frac{1}{2}\int_{-\infty}^{+\infty} P_j(x)x^2dx\\
&U_j^{(0)} = \int_{-\infty}^{+\infty} U_j(x)dx, \ U_j^{(1)} = \int_{-\infty}^{+\infty} U_j(x)xdx, \ U_j^{(2)} = -\frac{1}{2}\int_{-\infty}^{+\infty} U_j(x)x^2dx
\end{aligned} \tag{8}$$

After representing the problems (1)–(6) in dimensionless form and equating operator coefficients by the same degrees α to each other, the decomposition (8) allows to obtain the chain of boundary value problems separated with respect to unknown functions $U_j^{(m)}(x)$ and $c^*_{ij}(x,ik)$, which are solved sequentially (Vatulyan et al., 2013).

As a result, when recovering the complex modules $c^*_{55}(x,ik)$, $c^*_{44}(x,ik)$, we have the same boundary value problems as in the boundary problem on longitudinal vibration of an inhomogeneous viscoelastic rod, that was studied earlier in Anikina et al. (2011):

$$\begin{cases}
(g(x,ik)y')' + k^2y = 0\\
y(0) = 0\\
g(1,ik)y'(1,k) = p\\
y(1,k) = f(k)
\end{cases} \tag{9}$$

It is worth noting that in boundary value problem (9) the function $y(x,k)$ in general is a complex function of two arguments. To get back to the original problems, we should put

$$g(x,ik) = c^*_{jj}(x,ik), \quad y(x,k) = U_s^{(0)}(x,k),$$

$$p = P_s^{(0)}, \quad f(k) = f_s(k), j = 3,4,5, \quad s = 1,2,3$$

Solving of inverse coefficient problem of functions reconstruction

$$g(x,ik) = \frac{ikvg'_{ij}(x) + g''_{ij}(x)}{1+ikv}$$

can be reduced to the regularization iterative process (Vatulyan et al., 2011, 2013), on each step of which the corrections $\delta g(x,ik)$ in a neighborhood of some initial approximation $g^{(0)}(x,ik)$ are determined.

Thus, at the first step it is possible to reconstruct the instantaneous and long-termed modules $c'_{33}(x)$, $c''_{33}(x)$, $c'_{55}(x)$, $c''_{55}(x)$, $c'_{44}(x)$, $c''_{44}(x)$.

Further reconstruction of viscoelastic properties is performed by taking into account the functions which were restored at the first step.

Hence, with regard to the complex modules $c^*_{13}(x,ik)$, $c^*_{11}(x,ik)$, $c^*_{66}(x,ik)$ we have the following system of boundary value problems:

$$\begin{cases}
\left(c^*_{55}U_1^{(1)'}\right)' + k^2U_1^{(1)} - c^*_{13}U_3^{(0)'} - \left(c^*_{55}U_3^{(0)}\right)' = 0\\
\left(c^*_{33}U_3^{(1)'}\right)' + k^2U_3^{(1)} - c^*_{55}U_1^{(0)'} - \left(c^*_{13}U_1^{(0)}\right)' = 0\\
U_1^{(1)}(0) = U_3^{(1)}(0) = 0\\
c^*_{55}(1)U_1^{(1)'}(1) - c^*_{55}(1)U_3^{(0)}(1) = P_1^{(1)}\\
-c^*_{13}(1)U_1^{(0)}(1) + c^*_{33}(1)U_3^{(1)'}(1) = P_3^{(1)}\\
U_1^{(1)}(1,k) = f_1^{(1)}(k), \ U_3^{(1)}(1,k) = f_3^{(1)}(k)
\end{cases} \tag{10}$$

$$\begin{cases}
\left(c^*_{55}U_1^{(2)'}\right)' + k^2U_1^{(2)} - c^*_{11}U_1^{(0)} + c^*_{13}U_3^{(1)'} + \left(c^*_{55}U_3^{(1)}\right)' = 0\\
\left(c^*_{33}U_3^{(2)'}\right)' + k^2U_3^{(2)} + c^*_{55}U_1^{(1)'} - c^*_{55}U_3^{(0)} + \left(c^*_{13}U_1^{(1)}\right)' = 0\\
U_1^{(2)}(0) = U_3^{(2)}(0) = 0\\
c^*_{55}(1)U_1^{(2)'}(1) + c^*_{55}(1)U_3^{(1)}(1) = P_1^{(2)}\\
c^*_{33}(1)U_3^{(2)'}(1) + c^*_{13}(1)U_1^{(1)}(1) = P_3^{(2)}\\
U_1^{(2)}(1,k) = f_1^{(2)}(k), \ U_3^{(2)}(1,k) = f_3^{(2)}(k)
\end{cases} \tag{11}$$

$$\begin{cases}
\left(c^*_{44}U_2^{(2)'}\right)' - c^*_{66}U_2^{(0)} + k^2U_2^{(0)} = 0\\
U_2^{(2)}(0) = 0\\
c^*_{44}(1)U_2^{(2)'}(1) = P_2^{(2)}\\
U_2^{(2)}(1,k) = f_2^{(2)}(k)
\end{cases} \tag{12}$$

To realize sequence reconstruction of inhomogeneous viscoelastic properties $c^*_{13}(x,ik)$, $c^*_{11}(x,ik)$, $c^*_{66}(x,ik)$ (in the specified order), let us consider the first equations of the systems (10)–(12) and two differential equations with respect to the functions $c^*_{55}(x,ik)$, $U_1^{(0)}(x)$ and $c^*_{44}(x,ik)$, $U_2^{(0)}(x)$ obtained from the system (9).

Let us multiply the differential equation (10) by $U_1^{(0)}(x)$, and multiply the differential equation for the functions $c^*_{55}(x,ik)$, $U_1^{(0)}(x)$ by $U_1^{(1)}(x)$, then integrate each equation on the segment $[0,1]$ and subtract the second equation from the first one. Taking into account the boundary and additional conditions, after simple transformations we obtain the integral Fredholm equations of the first kind with respect to the functions $c^*_{13}(x,ik)$.

Performing actions similar to those described above, it is possible to receive the following two integral Fredholm equations of the first kind with respect to the functions $c^*_{11}(x,ik)$ and $c^*_{13}(x,ik)$. Further, after separation of material and imaginary parts, we will have three systems of two integral Fredholm equations of the first kind with smooth kernels for the functions characterizing the corresponding instant and long modules.

$$\int_0^1 \mathbf{c}_{13}(x)\mathbf{K}\left(U_3^{(0)'}(x,k), U_1^{(0)}(x,k)\right)dx = \mathbf{R}_3(k), \quad k \in [k_1,k_2], \tag{13}$$

$$\int_0^1 \mathbf{c}_{11}(x)\mathbf{K}\left(U_1^{(0)}(x,k), U_1^{(0)}(x,k)\right)dx = \mathbf{R}_1(k), \quad k \in [k_3,k_4], \tag{14}$$

$$\int_0^1 \mathbf{c}_{66}(x)\mathbf{K}\left(U_2^{(0)}(x,k), U_2^{(0)}(x,k)\right)dx = \mathbf{R}_2(k), \quad k \in [k_5,k_6], \tag{15}$$

$$\mathbf{c}_{ij}(x) = \left(c'_{ij}(x)c''_{ij}(x)\right), \quad \mathbf{R}_i(k) = (R_{1i}(k)R_{2i}(k))$$

$$\mathbf{K}(U(x,k),V(x,k)) = K_1(U(x,k),V(x,k))K_2(U(x,k),V(x,k))$$

$$K_3(U(x,k),V(x,k))K_4(U(x,k),V(x,k))$$

$$K_1(U(x,k),V(x,k)) = z_1(k)U^{(R)}(x,k) - z_2(k)U^{(I)}(x,k),$$

$$K_2(U(x,k),V(x,k)) = z_3(k)U^{(R)}(x,k) + z_2(k)U^{(I)}(x,k)$$

$$K_3(U(x,k),V(x,k)) = z_2(k)U^{(R)}(x,k) + z_1(k)U^{(I)}(x,k),$$

$$K_4(U(x,k),V(x,k)) = -z_2(k)U^{(R)}(x,k) + z_3(k)U^{(I)}(x,k)$$

$$U^{(R)}(x,k) = Re(U(x,k)V(x,k)),\ U^{(I)}(x,k) = Im(U(x,k)V(x,k))$$

$$z_1(k) = \frac{k^2v^2}{1-k^2v^2},\ z_2(k) = \frac{kv}{1-k^2v^2},\ z_3(k) = \frac{1}{1-k^2v^2}$$

$$R_{13}(k) = P_1^{(0)}Re(f_3^{(1)}(k)) - P_1^{(1)}Re(f_3^{(0)}(k)) + \int_0^1 \left(c'_{55}K_1\left(U_3^{(0)},U_1^{(0)\prime}\right) + c''_{55}K_2\left(U_3^{(0)},U_1^{(0)\prime}\right)\right)dx$$

$$R_{23}(k) = P_1^{(0)}Im(f_3^{(1)}(k)) - P_1^{(1)}Im(f_3^{(0)}(k)) + \int_0^1 \left(c'_{55}K_3\left(U_3^{(0)},U_1^{(0)\prime}\right) + c''_{55}K_4\left(U_3^{(0)},U_1^{(0)\prime}\right)\right)dx$$

$$R_{11}(k) = P_1^{(2)}Re(f_1{}^{(0)}(k)) - P_1^{(0)}Re(f_1{}^{(2)}(k)) - \int_0^1 [\left(c'_{13}K_1\left(U_3^{(1)\prime},U_1^{(0)}\right) + c''_{13}K_2\left(U_3^{(1)\prime},U_1^{(0)}\right)\right) - \left(c'_{55}K_1\left(U_3^{(1)},U_1^{(0)\prime}\right) + c''_{55}K_2\left(U_3^{(1)},U_1^{(0)\prime}\right)\right)]dx$$

$$R_{21}(k) = P_1^{(2)}Im(f_1{}^{(0)}(k)) - P_1^{(0)}Im(f_1{}^{(2)}(k)) - \int_0^1 \left(c'_{13}K_3\left(U_3^{(1)\prime},U_1^{(0)}\right) + c''_{13}K_4\left(U_3^{(1)\prime},U_1^{(0)}\right)\right) - \left(c'_{55}K_3\left(U_3^{(1)},U_1^{(0)\prime}\right) + c''_{55}K_4(U_3^{(1)},U_1^{(0)\prime})\right)dx$$

$$R_{12}(k) = -P_2^{(0)}Re(f_2^{(2)}(k)) + P_2^{(2)}Re(f_2^{(0)}(k)),$$

$$R_{22}(k) = -P_2^{(0)}Im(f_2^{(2)}(k)) + P_2^{(2)}Im(f_2^{(0)}(k)).$$

After we reconstruct $c^*_{13}(x,ik)$ from the systems of integral equations (13) it is possible then to determine the function $c^*_{11}(x,ik)$ from the system (14) solution.

From the out-of-plane problem analysis, the reconstruction of the complex module $c^*_{66}(x,ik)$ may be carried out from the system (14) solution.

Therefore, from the systems of integral equations (13) and (14) it is possible to consistently restore functions characterizing the instantaneous and long-termed modules: $c'_{13}(x),\ c''_{13}(x),\ c'_{11}(x),\ c''_{11}(x),\ c'_{66}(x),\ c''_{66}(x)$.

The integral equations kernel analysis at the point of $x = 0$ revealed that according to the boundary conditions the kernels vanish. That means that the reconstruction at this point of the instantaneous and long-termed modules will be carried out with higher error due to the "loss of information". We can also predict the error accumulation in the final steps of a reconstruction, and mostly it will be observed while recovering the modules $c'_{11}(x),\ c''_{11}(x)$.

4. Computational experiments on solving the inverse coefficient problem: model examples

Following the suggested scheme, we conducted a computational experiment on a reconstruction of unknown functions characterizing an instantaneous and long-termed modules for a layer of thickness $h = 1$. In a series of calculations, the density was taken as constant: $\rho = 1$. Relaxation time was considered known: $\tau = 0.1$.

The frequency range was chosen during the analysis of the modulus and the argument of the frequency–response function of the displacement transforms for $x_3 = h$, that allows to reveal the most efficient frequency ranges for the point of view of the identification accuracy. In the computational experiments, it was found that the most effective identification is when you select the frequency range between the extremums of the frequency response function in the left neighborhood of the second or the third extremum.Thereby, a frequency range was chosen in the experiments in accordance with this approach.

Figures show graphs of the exact solution (solid line), initial approximation (dash-dotted line), and reconstructed function (squares).

In the first two experiments, to reconstruct $c'_{55}(x),\ c''_{55}(x),\ c'_{33}(x),\ c''_{33}(x)$ by an iterative processes, the initial approximations was chosen from the conditions of minimum of the discrepancy functional in the class of linear functions. From an analysis of the frequency response function, the frequency range was chosen between the first and second resonance frequencies, which turned out to be the most informative from the point of view of the reconstruction accuracy.

At the third and the fourth steps the proposed computing scheme does not demand a choice of the initial approximation.

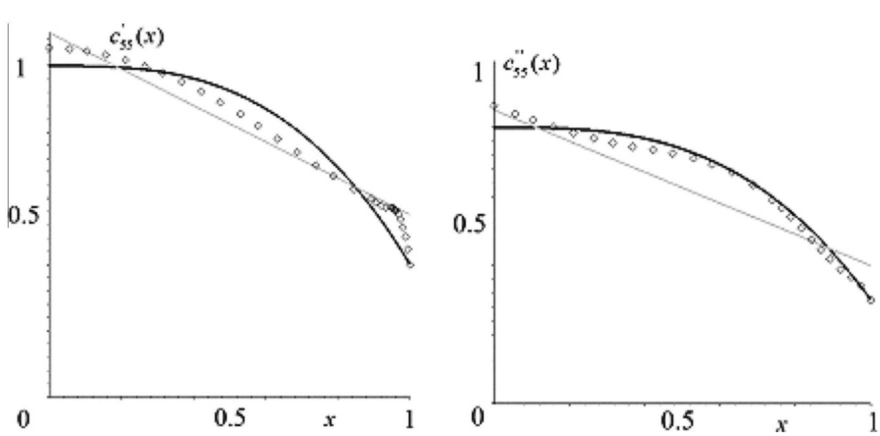

Fig. 1. Reconstruction of monotonic functions $c'_{55}(x)$ and $c''_{55}(x)$.

In view of restrictions on the amount of represented results, the reconstruction results of the functions $c'_{44}(x),\ c''_{44}(x),\ c'_{66}(x),\ c''_{66}(x)$ are not presented; note, that its reconstruction procedure is identical to the reconstruction procedure of the other functions.

First step. Reconstruction of the functions $c'_{55}(x),\ c''_{55}(x)$.

For the functions $c'_{55}(x) = 1 - 0.6x^3,\ c''_{55}(x) = 0.8 - 0.5x^3$, the initial approximations are found in the form

$$\delta c'_{55}(x) = 1.1 - 0.55x, \quad \delta c''_{55}(x) = 0.85 - 0.45x.$$

The frequency range was set as follows: $k \in [2, 3]$.

To reconstruct the functions, 5 iterations were required. The reconstruction error did not exceed 7% in extreme points (see Fig. 1).

Second step. Reconstruction of the functions $c'_{33}(x),\ c''_{33}(x)$.

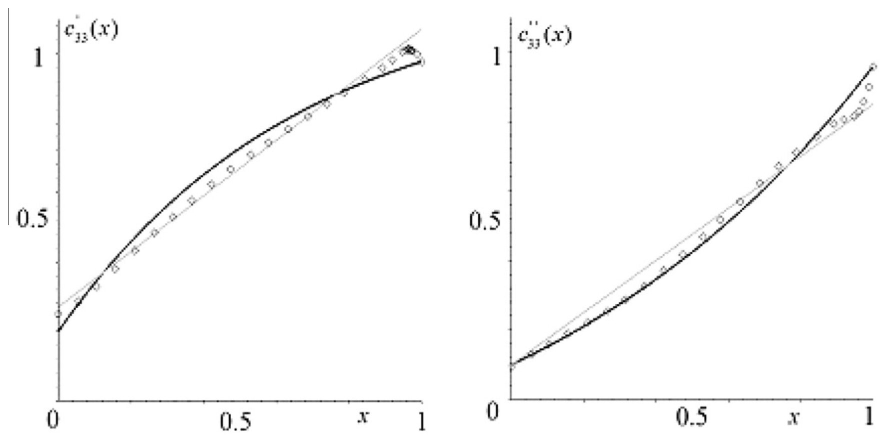

Fig. 2. Reconstruction of monotonic functions $c'_{33}(x)$ and $c''_{33}(x)$.

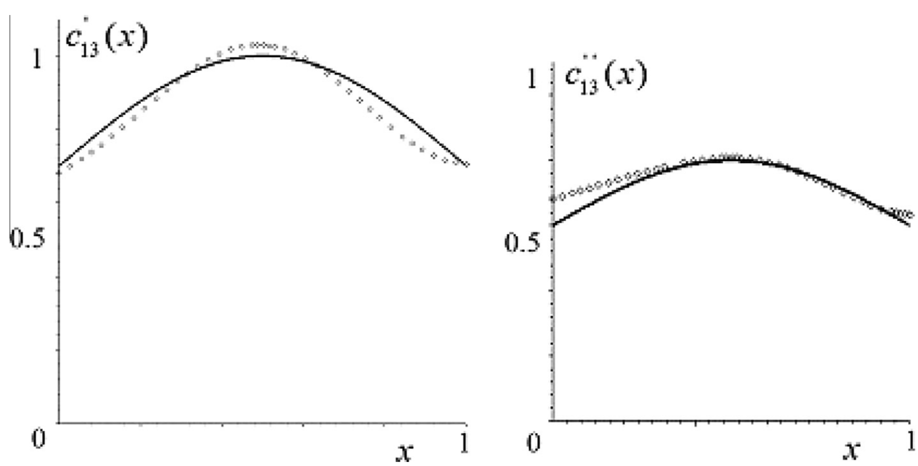

Fig. 3. Reconstruction of nonmonotonic functions $c'_{13}(x)$ and $c''_{13}(x)$.

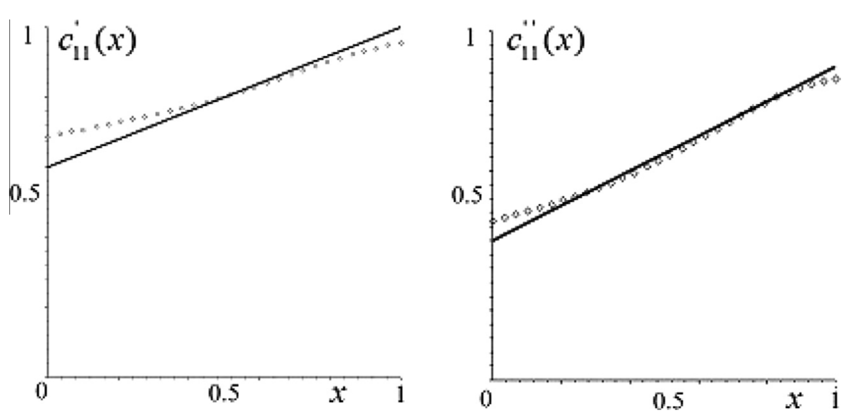

Fig. 4. Reconstruction of monotonic functions $c'_{11}(x)$ and $c''_{11}(x)$.

For the functions $c'_{33}(x) = 1.2 - e^{-1.5x}$, $c''_{33}(x) = 0.5e^x - 0.4$, the initial approximations are found in the form:

$$\delta c'_{33}(x) = 0.8x + 0.27, \quad \delta c''_{33}(x) = 0.75x + 0.1.$$

The frequency range was set as follows: $k \in [1.6, 2.2]$.

To reconstruct the functions, 8 iterations were required. The reconstruction error did not exceed 6% in extreme points (See Fig. 2).

Third step. Reconstruction of the functions $c'_{13}(x)$ *and* $c''_{13}(x)$.

For the functions $c'_{13}(x) = 0.3\sin(\pi x) + 0.7$, $c''_{13}(x) = 0.2\sin(\pi x) + 0.6$ the frequency range was set as follows: $k \in [1.49, 2.6]$. The reconstruction error did not exceed 10% in extreme points.

Fourth step. Reconstruction of the functions $c'_{11}(x)$, $c''_{11}(x)$. For the functions $c'_{11}(x) = 0.6 + 0.4x$, $c''_{11}(x) = 0.4 + 0.5x$ the frequency range was chosen as follows: $k \in [1.7, 2.6]$. The reconstruction error did not exceed 11% in extreme points. In that way, the computing experiment results testify an efficiency of the algorithm proposed and confirm its operability for various laws of variation of the viscoelastic characteristics of a medium (see Figs. 3 and 4).

5. Conclusions

The effective scheme of numerical research of inverse coefficient problem for inhomogeneously thick viscoelastic orthotropic layer on the basis of acoustic sounding data is offered. The consecutive reconstruction algorithm for the functions characterizing the instantaneous and long-termed modules of the layered viscoelastic medium from the solution of systems of Fredholm's integral equations of the first kind with smooth kernels and iterative process is formulated. Computing experiments for modelling examples are made.

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