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An Improved Definition of Proper Efficiency for Vector Maximization with Respect to Cones

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Recently Borwein has proposed a definition for extending Geoffrion's concept of proper efficiency to the vector maximization problem in which the domination cone S is any nontrivial, closed convex cone. However, when S is the nonnegative orthant, solutions may exist which are proper according to Borwein's definition but improper by Geoffrion's definition. As a result, when S is the nonnegative orthant, certain properties of proper efficiency as defined by Geoffrion do not hold under Borwein's definition. To rectify this situation, we propose a definition of proper efficiency for the case when S is a nontrivial, closed convex cone which coincides with Geoffrion's definition when S is the nonnegative orthant. The proposed definition seems preferable to Borwein's for developing a theory of proper efficiency for the case when S is a nontrivial, closed convex cone.

1. INTRODUCTION

Let $S \subseteq \mathbf{R}^p$ be any nontrivial cone where $p \geqslant 2$. Consider a vector-valued criterion function

$$f(x) = [f_1(x), f_2(x), \ldots, f_p(x)]$$

defined over a nonempty set $X \subseteq \mathbf{R}^n$ where $f_j\colon X \to \mathbf{R}\ \forall_j \in J = \{1, 2, \ldots, p\}$. For any $t, u \in \mathbf{R}^p$, let $t \geqslant_S u$ signify that $t - u \in S$. Then the vector maximization problem

$$VMAX\colon f(x) \text{ subject to } x \in X \tag{P}$$

is the problem of finding all solutions that are efficient in the sense of the following definition.

DEFINITION 1.1. A point $\bar{x}$ is said to be an *efficient* solution of (P) if $\bar{x} \in X$ and $f(x) \geqslant_S f(\bar{x})$ for some $x \in X$ implies that $f(x) = f(\bar{x})$.

Adapting the terminology of Yu [11], S will be referred to as the *domination cone* for (P). For the case when $S = \mathbf{R}_+^p$, where $\mathbf{R}_+^p = \{t \in \mathbf{R}^p \mid t_j \geqslant 0\ \forall j \in J\}$, in order to eliminate anomalous solutions and to allow a more satisfactory

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characterization, Geoffrion [5] has suggested restricting attention to efficient solutions that are proper in the sense of the following definition. For any nonempty sets A and B, let A/B denote $\{x \mid x \in A, x \notin B\}$.

DEFINITION 1.2 (Geoffrion). When $S = \mathbf{R}_+^p$, a point $\bar{x}$ is a *properly efficient* solution of (P) when it is an efficient solution of (P) and there exists a scalar $M > 0$ such that for each $i \in J$ and each $x \in X$ satisfying $f_i(x) > f_i(\bar{x})$, there exists at least one $j \in J/\{i\}$ with $f_j(x) < f_j(\bar{x})$ and $[f_i(x) - f_i(\bar{x})]/[f_j(\bar{x}) - f_j(x)] \leqslant M$. Geoffrion referred to an efficient point that is not properly efficient as *improperly efficient*.

Recently Borwein [3] has proposed a definition for extending Definition 1.2 to the case when S is any nontrivial, closed convex cone. When $S = \mathbf{R}_+^p$, and for all $j \in J$, f_j is a concave function on the convex set X, Borwein's definition coincides with Geoffrion's [3, p. 61]. However, when $S = \mathbf{R}_+^p$ and at least one objective function is nonconcave on X, solutions may exist which are proper according to Borwein's definition but improper by Geoffrion's definition.

In this paper we propose a new definition for a properly efficient solution of (P) when S is a nontrivial, closed convex cone. By strengthening Borwein's requirement for properness, this definition coincides with Geoffrion's definition when S is the nonnegative orthant, even if some objective function is nonconcave. In Section 2 we present the new definition and explain how it strengthens Borwein's definition. In Section 3, we prove that when $S = \mathbf{R}_+^p$, our definition, unlike Borwein's, coincides with Geoffrion's definition. We also show the equivalence of our definition and Borwein's under an appropriate concavity assumption. In Section 4, properties of proper efficiency according to our proposed definition and according to Borwein's definition are compared. Although these definitions yield identical extensions of Geoffrion's fundamental results, when $S = \mathbf{R}_+^p$ all properties of proper efficiency as defined by Geoffrion hold under our proposed definition, but not under Borwein's. From these results we conclude, in Section 5, that our definition seems preferable to Borwein's for developing a theory of proper efficiency when S is a nontrivial, closed convex cone.

2. THE NEW DEFINITION OF PROPER EFFICIENCY

To extend Geoffrion's definition of proper efficiency to the case when S is a nontrivial, closed convex cone, Borwein used the tangent cone concept. Let $\mathbf{R}_+ = \{t \in \mathbf{R} \mid t \geqslant 0\}$.

DEFINITION 2.1. Let $C \subseteq \mathbf{R}^p$ and $w \in C$. The *tangent cone* to C at w, denoted $T(C, w)$, is the set of limits of the form $h = \lim \lambda_i(w^i - w)$, where $\{\lambda_i\}$ is a sequence in $\mathbf{R}_+$ and $\{w^i\}$ is a sequence in C with limit w.

Let $f(X) = \{f(x) \mid x \in X\}$ and, for any sets $A, B \subseteq \mathbf{R}^p$, let $A - B = \{a - b \mid a \in A \text{ and } b \in B\}$. Borwein's proposed definition is as follows.

DEFINITION 2.2 (Borwein). Let $S \subseteq \mathbf{R}^p$ be a nontrivial, closed convex cone. A point $\bar{x}$ is said to be a *properly efficient* solution of (P) when $\bar{x}$ is an efficient solution of (P) and $T[f(X) - S, f(\bar{x})] \cap S = \{0\}$.

Our proposed definition of proper efficiency strengthens Borwein's requirement for properness so that when $S = \mathbf{R}_+{}^p$, even if, for some $j \in J$, f_j is not concave on X, the new definition and Geoffrion's definition coincide. To accomplish this, the concept of a projecting cone will be used. For any set C, let cl C denote the closure of C.

DEFINITION 2.3. Let $C \subseteq \mathbf{R}^p$. The *projecting cone* of C, denoted $P(C)$, is the set of all points h of the form $h = \lambda \bar{w}$, where $\lambda \in \mathbf{R}_+$ and $\bar{w} \in C$.

The projecting cone of a set C is also known as the cone *generated* by C and the *conical extension* of C. Canon, Cullum, and Polak [4], Kelley [6], Wijsman [9, 10] and others have used the projecting cone concept.

Our proposed definition of a properly efficient solution is as follows.

DEFINITION 2.4. Let $S \subseteq \mathbf{R}^p$ be a nontrivial, closed convex cone. A point $\bar{x}$ is said to be a *properly efficient* solution of (P) when $\bar{x}$ is an efficient solution of (P) and $\operatorname{cl} P[f(X) - S - \{f(\bar{x})\}] \cap S = \{0\}$.

Following Geoffrion [5], an efficient solution which is not properly efficient will be referred to as *improperly efficient*.

Let $C \subseteq \mathbf{R}^p$ and $w \in C$. From Definitions 2.1 and 2.3, $T(C, w) \subseteq \operatorname{cl} P(C - \{w\})$, but the reverse containment may not hold. Therefore, Definition 2.4 strengthens Borwein's requirement for proper efficiency by using the concept of a projecting cone in place of the tangent cone concept. Geometrically, this stronger requirement can be interpreted as follows. By considering only properly efficient solutions as defined by Definition 2.4, any efficient solution $\bar{x}$ is excluded for which sequences $\{y^i\}$ in $f(X) - S$ and $\{\lambda_i\}$ in $\mathbf{R}_+$ exist such that the directions $h^i = \lambda_i[y^i - f(\bar{x})]$ have a nonzero limit which belongs to S. Under Borwein's requirement for proper efficiency, such a point $\bar{x}$ would not be excluded if the sequence $\{y^i\}$ could not be chosen so as to converge to $f(\bar{x})$.

3. EQUIVALENCE THEOREMS

Because of the strong requirement for properness imposed by Definition 2.4, this definition, unlike Borwein's, coincides with Geoffrion's definition of a properly efficient solution of (P) when $S = \mathbf{R}_+{}^p$, even if some objective function is nonconcave on X. Before proving this equivalence, consider the following result.

THEOREM 3.1. *Let $S \subseteq \mathbf{R}^p$ be a nontrivial, closed convex cone. Suppose that f is a concave function with respect to S on the convex set X.*[1] *Then Definitions 2.2 and 2.4 are equivalent.*

Proof. Let $\bar{x} \in X$. Since S is a nontrivial, closed convex cone and f is concave with respect to S on the convex set X, $V = f(X) - S$ is a convex set. From Varaiya [7, Remark 2.1], $\text{cl}\, P[V - \{f(\bar{x})\}] = T[V, f(\bar{x})]$ in this case.

From Theorem 3.1, the strengthening of Borwein's requirement for properness embodied in Definition 2.4 does not alter his definition when f is concave with respect to S on X. Therefore, in this case, all properties concerning proper and improper efficiency that have been developed using Borwein's definition [1, 3] also hold under Definition 2.4.

We now prove the equivalence of Definition 2.4 and Geoffrion's definition when $S = \mathbf{R}_+{}^p$.

THEOREM 3.2. *When $S = \mathbf{R}_+{}^p$, Definitions 1.2 and 2.4 are equivalent.*

Proof. To prove the theorem, we will show that (i) if $\bar{x}$ is properly efficient by Definition 1.2, then $\bar{x}$ is properly efficient by Definition 2.4 and that (ii) the converse to (i) holds.

(i) The proof of this proceeds, with minor modifications, as does the proof of a result shown earlier (see Borwein [3, Proposition 1]) and will therefore not be given.

(ii) Let $\bar{x}$ be an efficient solution of (P). Suppose that $\bar{x}$ is not properly efficient according to Definition 1.2. Let $\{M_i\}$ be an unbounded sequence of positive real numbers. Then, by reordering the objective functions, if necessary, one can assume that for each M_i there exists an $x^i \in X$ such that $f_1(x^i) > f_1(\bar{x})$ and

$$[f_1(x^i) - f_1(\bar{x})]/[f_j(\bar{x}) - f_j(x^i)] > M_i \tag{3.1}$$

for all $j \in J/\{1\}$ such that $f_j(x^i) < f_j(\bar{x})$. By choosing a subsequence of $\{M_i\}$, if necessary, one can assume that

$$\tilde{J} = \{j \in J/\{1\} \mid f_j(x^i) < f_j(\bar{x})\}$$

is constant for all i. ($\tilde{J} \neq \varnothing$ by Definition 1.1.) For each i, let

$$\lambda_i = [f_1(x^i) - f_1(\bar{x})]^{-1}. \tag{3.2}$$

Then λ_i is positive for all i. Either (a) $J/\tilde{J} = \{1\}$ or (b) $J/\tilde{J} \neq \{1\}$.

[1] A function $f: X \to \mathbf{R}^p$ is a *concave function with respect to a cone* $S \subseteq \mathbf{R}^p$ on X, where $X \subseteq \mathbf{R}^n$ is a nonempty convex set, when, for any α such that $0 \leqslant \alpha \leqslant 1$ and for any $x^1, x^2 \in X$, $f[\alpha x^1 + (1 - \alpha)x^2] - \alpha f(x^1) - (1 - \alpha) f(x^2) \in S$.

Case 1. $J/\tilde{J} = \{1\}$. For each $j \in J$, consider

$$h_j = \lim \lambda_i[f_j(x^i) - f_j(\bar{x})]. \tag{3.3}$$

From (3.2), $h_1 = 1$.
Suppose $j \in \tilde{J}$. From (3.1),

$$[f_j(x^i) - f_j(\bar{x})]/[f_1(x^i) - f_1(\bar{x})] > -(M_i)^{-1} \tag{3.4}$$

for all i. Since $j \in \tilde{J}$ and $f_1(x^i) > f_1(\bar{x})$ for all i, we have that

$$0 > [f_j(x^i) - f_j(\bar{x})]/[f_1(x^i) - f_1(\bar{x})] \tag{3.5}$$

for all i. Since $\{M_i\}$ is an unbounded sequence of positive real numbers, (3.4) and (3.5) together imply that

$$\lim[f_j(x^i) - f_j(\bar{x})]/[f_1(x^i) - f_1(\bar{x})] = 0.$$

From (3.2) and (3.3), this implies that $h_j = 0$. Therefore, $h_j = 0\ \forall j \in \tilde{J}$.
Since $J/\tilde{J} = \{1\}$, these results imply that

$$\lim \lambda_i[f(x^i) - f(\bar{x})] \in \mathbf{R}_+{}^p/\{0\}.$$

According to Definition 2.4, $\bar{x}$ is not a properly efficient solution of (P). By the contrapositive, the proof of (ii) for this case is complete.

Case 2. $J/\tilde{J} \neq \{1\}$. Suppose $j \in J/\tilde{J}$, $j \neq 1$. Since $j \notin \tilde{J}$ and $f_1(x^i) > f_1(\bar{x})$ for all i, we have that

$$[f_j(x^i) - f_j(\bar{x})]/[f_1(x^i) - f_1(\bar{x})] \geqslant 0$$

for all i. Therefore, by choosing a suitable subsequence I, if necessary, the limit given by

$$\lim_{i \in I}[f_j(x^i) - f_j(\bar{x})]/[f_1(x^i) - f_1(\bar{x})]$$

exists and is either a finite nonnegative number or $+\infty$. From (3.2), this implies that

$$\lim_{i \in I} \lambda_i[f_j(x^i) - f_j(\bar{x})] \in \mathbf{R}_+,$$

if one assumes that $+\infty$ is an element of $\mathbf{R}_+$. Since any subsequence of a convergent sequence converges to the same limit as the sequence, from the proof for Case 1 it follows that

$$\lim_{i \in I} \lambda_i[f_j(x^i) - f_j(\bar{x})] = 0$$

for all $j \in J$. From (3.2),

$$\lim_{i \in I} \lambda_i [f_1(x^i) - f_1(\bar{x})] = 1.$$

Therefore,

$$\lim_{i \in I} \lambda_i [f(x^i) - f(\bar{x})] \in \mathbf{R}_+^p / \{0\},$$

and $\bar{x}$ is not properly efficient for (P) according to Definition 2.4. By the contrapositive, the proof of (ii) for this case is complete.

From Theorem 3.2, when $S = \mathbf{R}_+^p$, and at least one objective function is nonconcave on X, no solution which is improper according to Geoffrion's definition can be proper by Definition 2.4. However, such a solution may be proper by Borwein's definition. For instance, consider the following example.

EXAMPLE. Let $S = \mathbf{R}_+^2$ and let $f_j(x_1, x_2) = x_j \; \forall j \in J = \{1, 2\}$ be defined on the nonconvex set X given by

$$X = [A_1 \cup A_2 \cup A_3] \cap \mathbf{R}_+^2,$$

where

$$A_1 = \{(x_1, x_2) \in \mathbf{R}^2 \mid x_1 + x_2 \leqslant 6\},$$

$$A_2 = \{(x_1, x_2) \in \mathbf{R}^2 \mid x_1 \leqslant 2\},$$

and

$$A_3 = \{(x_1, x_2) \in \mathbf{R}^2 \mid x_2 \leqslant 2\}.$$

By Definition 1.1, $\bar{x} = (3, 3)$ is an efficient solution of the corresponding vector maximization problem. For any $N > 0$, $x = (2, N) \in X$. Therefore, there is no $M > 0$ such that

$$[f_2(x) - f_2(\bar{x})]/[f_1(\bar{x}) - f_1(x)] \leqslant M$$

for each $x \in X$ satisfying $f_2(x) > f_2(\bar{x})$. By Geoffrion's definition, $\bar{x}$ is improperly efficient. By Definition 2.3,

$$\operatorname{cl} P[f(X) - S - \{f(\bar{x})\}] = B_1 \cup B_2,$$

where

$$B_j = \{(h_1, h_2) \in \mathbf{R}^2 \mid h_j \leqslant 0\}$$

$\forall j \in J$. Therefore, as required by Theorem 3.2, $\bar{x}$ is also improperly efficient according to Definition 2.4. However, from Definition 2.1,

$$T[f(X) - S, f(\bar{x})] = \{(h_1, h_2) \in \mathbf{R}^2 \mid h_1 + h_2 \leqslant 0\},$$

so that $\bar{x}$ is properly efficient according to Borwein's definition.

4. Comparison of Properties

Let us compare properties of proper efficiency according to Borwein's definition with properties according to our proposed definition.

Using Definition 2.2, Borwein [3, Theorems 1 and 2] has extended Geoffrion's fundamental characterizations of proper efficiency when $S = \mathbf{R}_+{}^p$ [5, Theorems 1 and 2] to the case when S is a nontrivial, closed convex cone. These extensions also hold under our proposed definition of proper efficiency. In particular, we have the following two results. Let (int S^*) denote the interior of the *dual cone* S^* for S, where

$$S^* = \{s^* \in \mathbf{R}^p \mid \langle s^*, s\rangle \geqslant 0 \ \forall s \in S\}.$$

THEOREM 4.1. *Assume S is a nontrivial, closed convex cone. Suppose that $\bar{x}$ is optimal for*

$$\max \langle s^*, f(x)\rangle \text{ subject to } x \in X \qquad (\mathrm{P}_{s^*})$$

for some $s^* \in (\text{int } S^*)$. Then $\bar{x}$ is a properly efficient solution for (P) *according to Definition* 2.4.

THEOREM 4.2. *Assume S is a nontrivial, closed convex cone. Suppose that* (int S^*) $\neq \varnothing$ *and that f is a concave function with respect to S on the convex set X. Then $\bar{x}$ is a properly efficient solution for* (P) *according to Definition* 2.4 *if and only if $\bar{x}$ is optimal for (P_{s^*}) for some $s^* \in (\text{int } S^*)$.*

The proof of Theorem 4.1 proceeds, with minor modifications, as does the proof of Theorem 1 of Borwein [3]. Theorem 4.2 is an immediate consequence of Borwein's Theorem 2 [3] and our Theorem 3.1.

Although both Definition 2.4 and Borwein's definition of proper efficiency yield identical extensions of Geoffrion's fundamental results, Definition 2.4 has a distinct advantage over Borwein's definition. From Theorem 3.2, when $S = \mathbf{R}_+{}^p$, all properties of proper efficiency as defined by Geoffrion also hold under Definition 2.4. However, certain properties of proper efficiency as defined by Geoffrion may not hold under Borwein's definition when $S = \mathbf{R}_+{}^p$ and at least one objective function is nonconcave on X. For example, consider the following theorem. (This theorem has been shown previously only for the case when, for all $j \in J$, f_j is a concave function on the convex set X [2, 8].)

THEOREM 4.3. *Let $S = \mathbf{R}_+{}^p$ and assume that the problem*

$$\sup \sum_{j \in J} f_j(x)$$

subject to

$$f_j(x) \geqslant f_j(\bar{x}) \ \forall j \in J, \qquad (\mathrm{P}_{\bar{x}})$$

and

$$x \in X,$$

is unbounded for some $\tilde{x} \in X$. *Then, according to Definition* 1.2, *no properly efficient solutions of* (P) *exist.*

Proof. Let $\bar{x}$ be an efficient solution of (P) and let $\{M_i\}$ be an unbounded sequence of positive real numbers. Since $(P_{\tilde{x}})$ is unbounded, for each $L > 0$, there exists an $x \in X$ such that

$$\sum_{j \in J} f_j(x) > L$$

and

$$f_j(x) \geqslant f_j(\tilde{x}) \; \forall j \in J.$$

Therefore, by reordering the objective functions, if necessary, for each M_i there exists an $x^i \in X$ such that

$$f_1(x^i) - f_1(\bar{x}) > M_i \tag{4.1}$$

and

$$f_j(x^i) \geqslant f_j(\tilde{x}) \; \forall j \in J. \tag{4.2}$$

By choosing a suitable subsequence of $\{M_i\}$, if necessary, one can assume that

$$\tilde{J} = \{ j \in J/\{1\} \mid f_j(x^i) < f_j(\bar{x}) \}$$

is constant for all i. Suppose $j \in \tilde{J}$. Then from (4.2) and the definition of $\tilde{J}$,

$$f_j(\tilde{x}) \leqslant f_j(x^i) < f_j(\bar{x})$$

for all i. Therefore, for each $j \in \tilde{J}$,

$$0 < f_j(\bar{x}) - f_j(x^i) \leqslant f_j(\bar{x}) - f_j(\tilde{x}) \tag{4.3}$$

for all i.

Let M be any positive scalar. Pick some $M_{i'} \in \{M_i\}$ such that $M_{i'} > M\theta$, where $\theta = \max_{j \in \tilde{J}}[f_j(\bar{x}) - f_j(\tilde{x})]$. Then, from (4.1), $f_1(x^{i'}) - f_1(\bar{x}) > M[f_j(\bar{x}) - f_j(\tilde{x})] \; \forall j \in \tilde{J}$. Using (4.3), this implies that

$$f_1(x^{i'}) - f_1(\bar{x}) > M[f_j(\bar{x}) - f_j(x^{i'})] \; \forall j \in \tilde{J}.$$

Rearranging, we have that

$$[f_1(x^{i'}) - f_1(\bar{x})]/[f_j(\bar{x}) - f_j(x^{i'})] > M \; \forall j \in \tilde{J}.$$

Since M is an arbitrary positive scalar, $\bar{x}$ is not properly efficient according to Definition 1.2.

Let $S = \mathbf{R}_+^p$. Theorem 4.3 gives a sufficient condition for concluding that no properly efficient solutions of (P) exist, according to Geoffrion's definition. However, if at least one objective function is nonconcave on X, this condition is *not* sufficient to conclude that no properly efficient solutions of (P) exist according to Borwein's definition of proper efficiency. For example, in the example in Section 3, the set of efficient points of the corresponding vector maximization problem is $\{(x_1, x_2) \in \mathbf{R}^2 \mid x_1 + x_2 = 6 \text{ and } 2 < x_1 < 4\}$. The problem

$$\sup x_1 + x_2$$

subject to

$$x_1 \geqslant 0,$$
$$x_2 \geqslant 0,$$

and

$$(x_1, x_2) \in X,$$

where X is as given in the example, is unbounded. As required by Theorem 4.3, each efficient point is improperly efficient according to Geoffrion's definition. However, each efficient point is properly efficient by Borwein's definition.

5. Conclusion

Using either Borwein's definition or our proposed definition, Geoffrion's fundamental results characterizing proper efficiency can be extended to the case when S is a nontrivial, closed convex cone. However, since our definition contains a stronger requirement for properness than Borwein's, our definition coincides with Geoffrion's definition when $S = \mathbf{R}_+^p$. Therefore, when $S = \mathbf{R}_+^p$, all properties of proper efficiency as defined by Geoffrion also hold under our proposed definition. Such properties may not hold under Borwein's definition when at least one objective function is nonconcave on X. Therefore, it seems preferable to use our proposed definition instead of Borwein's in developing a theory of proper efficiency for the case when S is a nontrivial, closed convex cone.

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