# Gamma-ray bursts and the relevance of rotation-induced neutrino sterilization 

D.V. Ahluwalia ${ }^{\mathrm{a}, \mathrm{b}, *}$, Cheng-Yang Lee ${ }^{\mathrm{a}}$<br>${ }^{\text {a }}$ Department of Physics and Astronomy, Rutherford Building, University of Canterbury, Private bag 4800, Christchurch 8140, New Zealand<br>${ }^{\mathrm{b}}$ Institute of Mathematics, Statistics and Scientific Computation, IMECC-UNICAMP CP 6065, 13083-859 Campinas, São Paulo, Brazil

## A R T I C L E I N F O

## Article history:

Received 11 October 2012
Accepted 23 January 2013
Available online 29 January 2013
Editor: S. Dodelson


#### Abstract

À la Pontecorvo when one defines electroweak flavour states of neutrinos as a linear superposition of mass eigenstates one ignores the associated spin. If, however, there is a significant rotation between the neutrino source, and the detector, a negative helicity state emitted by the former acquires a non-zero probability amplitude to be perceived as a positive helicity state by the latter. Both of these states are still in the left-Weyl sector of the Lorentz group. The electroweak interaction cross sections for such helicity-flipped states are suppressed by a factor of $\left(m_{v} / E_{v}\right)^{2}$, where $m_{v}$ is the expectation value of the neutrino mass, and $E_{v}$ is the associated energy. Thus, if the detecting process is based on electroweak interactions, and the neutrino source is a highly rotating object, the rotation-induced helicity flip becomes very significant in interpreting the data. The effect immediately generalizes to anti-neutrinos. Motivated by these observations we present a generalization of the Pontecorvo formalism and discuss its relevance in the context of recent data obtained by the IceCube neutrino telescope.


© 2013 Elsevier B.V. Open access under CCBY license.

In models of GRBs ( $\gamma$-ray bursts), ultrahigh energy neutrinos of several hundred TeV are expected to be emitted from accretion disk surrounding highly rotating black holes or neutron stars [1-3]. The emission in general is not isotropic. The IceCube neutrino detector has recently reported an absence of neutrinos associated with cosmic-ray acceleration in GRBs. The collaboration draws the conclusion that either GRBs are not the only source of cosmic rays with energies exceeding $10^{18} \mathrm{eV}$ or that efficiency of neutrino production is much lower, at least by a factor of 3.7 , than has been predicted [4]. Several objections have been raised to this interpretation of the data [5,6]. None of these works, however, incorporate the fundamental circumstance that the GRB neutrinos are produced in highly rotating frames while they are observed in a frame which may in comparison be considered as non-rotating. In conjunction with the observations contained in the Abstract above, this leads to a partial sterilization of the GRB neutrinos.

Motivated by these observation, we recall that in the standard neutrino-oscillation formalism à la Pontecorvo a flavour-eigenstate is a linear superposition of three mass eigenstates
$\left|\nu_{\ell}, \sigma\right\rangle=\sum_{j=1,2,3} U_{\ell j}^{*}\left|m_{j}, \sigma\right\rangle, \quad \ell=e, \mu, \tau, \quad \sigma=-\frac{1}{2}$.

[^0]Each of the underlying mass eigenstates corresponds to the same helicity, $\sigma$ (at this stage $\sigma=+1 / 2$ is suppressed by $m_{j} / E$ ). The $3 \times 3$ mixing matrix $U$ is determined from experiments as are the mass-squared differences $\Delta m_{j j^{\prime}}^{2}:=m_{j}^{2}-m_{j^{\prime}}^{2}$. For our purposes it suffices to assume that each of the mass eigenstates has fourmomentum $p_{\mu}$ with $\mathbf{p}_{j}=\mathbf{p}_{j^{\prime}}$. Thus flavour oscillations, in this working framework, reside in different $p_{0}$ associated with each of the mass eigenstates.

With the recent IceCube null result in mind, we now consider a set up in which the source of neutrinos resides in a highly rotating astrophysical object, say a GRB. To calculate flavour-oscillation probabilities for a neutrino detector on Earth we recall that under a space-time translation $a^{\mu}=(t, \mathbf{L})$, where $\mathbf{L}$ represents the source-detector separation,
$\left|m_{j}, \sigma\right\rangle \rightarrow e^{i p_{\mu} a^{\mu}}\left|m_{j}, \sigma\right\rangle$,
and each of the mass eigenstate picks up a $j$-dependent phase factor. It is this $j$ dependence that results in neutrino-flavour oscillations à la Pontecorvo [7,8]. If in the frame of the observer, the source rotates at an angular frequency, $\omega:=\omega \hat{\mathbf{n}}, \hat{\mathbf{n}}=$ $(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, then each of the mass eigenstate undergoes a $j$-independent transformation

$$
\begin{equation*}
\left|m_{j}, \sigma\right\rangle \rightarrow \sum_{\sigma^{\prime}}\left[\exp \left(i \frac{\sigma}{2} \cdot \hat{\mathbf{n}} \omega t\right)\right]_{\sigma^{\prime} \sigma}\left|m_{j}, \sigma^{\prime}\right\rangle, \quad \sigma^{\prime}= \pm \frac{1}{2} \tag{3}
\end{equation*}
$$

(where the change in momentum associated with the mass eigenstates is notationally suppressed). Because of the $j$-independence of this effect, the modified flavour-oscillation probability factorizes
$P\left(\ell, \sigma \rightarrow \ell^{\prime}, \sigma^{\prime}\right)=P\left(\sigma \rightarrow \sigma^{\prime}\right) P\left(\ell \rightarrow \ell^{\prime}\right)$.
In the above expression, $P\left(\ell \rightarrow \ell^{\prime}\right)$ is the usual flavour-oscillation probability of the standard formalism [9], while

$$
\begin{align*}
P\left(\sigma \rightarrow \sigma^{\prime}\right)= & {\left[\exp \left(i \frac{\sigma}{2} \cdot \hat{\mathbf{n}} \omega t\right)\right]_{\sigma^{\prime} \sigma}^{*} } \\
& \times\left[\exp \left(i \frac{\sigma}{2} \cdot \hat{\mathbf{n}} \omega t\right)\right]_{\sigma^{\prime} \sigma} \text { (no sum). } \tag{5}
\end{align*}
$$

Since $(\boldsymbol{\sigma} \cdot \hat{\mathbf{n}})^{2}$ is a $2 \times 2$ identity matrix, $\mathbf{I}$, the exponential enclosed in the square brackets reduces to ${ }^{1}$
$\cos \left(\frac{w L}{2}\right) \mathbf{I}+i \boldsymbol{\sigma} \cdot \hat{\mathbf{n}} \sin \left(\frac{w L}{2}\right)$.
A straightforward calculation then yields the modified expressions for flavour oscillations
$P\left(\ell,-\frac{1}{2} \rightarrow \ell^{\prime},+\frac{1}{2}\right)=\sin ^{2} \theta \sin ^{2}\left(\frac{\omega L}{2}\right) P\left(\ell \rightarrow \ell^{\prime}\right)$,
$P\left(\ell,-\frac{1}{2} \rightarrow \ell^{\prime},-\frac{1}{2}\right)=\left[1-\sin ^{2} \theta \sin ^{2}\left(\frac{\omega L}{2}\right)\right] P\left(\ell \rightarrow \ell^{\prime}\right)$.
These results are consistent with those found in the literature on magnetic resonance [10]. For the isotropically emitted neutrinos the standard averaging process over a sufficiently large patch of the sky gives
$\left\langle P\left(\ell,-\frac{1}{2} \rightarrow \ell^{\prime},+\frac{1}{2}\right)\right\rangle=\frac{1}{4} P\left(\ell \rightarrow \ell^{\prime}\right)$,
$\left\langle P\left(\ell,-\frac{1}{2} \rightarrow \ell^{\prime},-\frac{1}{2}\right)\right\rangle=\frac{3}{4} P\left(\ell \rightarrow \ell^{\prime}\right)$.

For models in which neutrinos are dominantly emitted perpendicular to the rotation axis one obtains ${ }^{2}$
$\left\langle P\left(\ell,-\frac{1}{2} \rightarrow \ell^{\prime},+\frac{1}{2}\right)\right\rangle=\frac{1}{2} P\left(\ell \rightarrow \ell^{\prime}\right)$,
$\left\langle P\left(\ell,-\frac{1}{2} \rightarrow \ell^{\prime},-\frac{1}{2}\right)\right\rangle=\frac{1}{2} P\left(\ell \rightarrow \ell^{\prime}\right)$.
Since the electroweak interaction cross section for the helicityflipped states are suppressed by a factor of $\left(m_{\nu} / E_{\nu}\right)^{2}$, rotation acts to partially sterilize neutrinos. In consequence the interpretation of the data reported by IceCube suffers a modification and the expected neutrino events are reduced by the above-indicated factors (modulo the remark made in footnote 2). As a final remark we note that similar effects also arise via gravitationally induced helicity transitions and these, together with the purely kinematical effect discussed here, show that the Pontecorvo formalism must be taken only as a first approximation in the neutrino-oscillation phenomenology. Failure to do so can result in significant misinterpretation of the data.

## Acknowledgements

We thank Sebastian Horvath and Dimitri Schritt for discussions.

## References

[1] E. Waxman, Phys. Rev. Lett. 75 (1995) 386, astro-ph/9505082.
[2] M. Vietri, Astrophys. J. 453 (1995) 883, astro-ph/9506081.
[3] M. Milgrom, V. Usov, Astrophys. J. 449 (1995) L37, astro-ph/9505009.
[4] IceCube Collaboration, R. Abbasi, et al., Nature 484 (2012) 351, arXiv:1204. 4219.
[5] A. Dar, Neutrinos and cosmic rays from gamma ray bursts, arXiv:1205.3479.
[6] P. Meszaros, N. Gehrels, Res. Astron. Astrophys. 12 (2012) 1139, arXiv:1209. 1132.
[7] B. Pontecorvo, Sov. Phys. JETP 26 (1968) 984.
[8] S.M. Bilenky, B. Pontecorvo, Phys. Rept. 41 (1978) 225.
[9] Particle Data Group Collaboration, J. Beringer, et al., in: Review of Particle Physics (RPP), Phys. Rev. D 86 (2012) 010001.
[10] I.I. Rabi, N.F. Ramsey, J. Schwinger, Rev. Mod. Phys. 26 (1954) 167.

[^1][^2]
[^0]:    * Corresponding author at: Institute of Mathematics, Statistics and Scientific Computation, IMECC-UNICAMP CP 6065, 13083-859 Campinas, São Paulo, Brazil.

    E-mail address: dvahluwalia@ime.unicamp.br (D.V. Ahluwalia).

[^1]:    ${ }^{1}$ Where we have set $t=L$ for ultrarelativistic neutrinos.

[^2]:    ${ }^{2}$ If the dominant neutrino emission is along the axis of rotation, the resulting flavour-oscillation probability is roughly the same as in the Pontecorvo formalism.

