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Procedia Materials Science 3 (2014) 1566 – 1572
Procedia Materials Science
www.elsevier.com/locate/procedia

20th European Conference on Fracture (ECF20)

Crack growth rate model under constant cyclic loading and effect of different singularity fields

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Abstract

The effect of two singularity fields: the Hutchinson-Rice-Rosengren (HRR) fields and the Rice-Kujawski-Ellyin (RKE) fields, is investigated in the present study to discuss a former fatigue crack growth (FCG) model based on energy balance during growth of the crack. A parameter which can demonstrates the effect of different types of singularity fields is included in the crack propagation model which can predict stage-II FCG behavior independently from the basic low cycle fatigue properties. Good agreement between experimental and theoretical results is obtained.

Selection and peer-review under responsibility of the Norwegian University of Science and Technology (NTNU), Department of Structural Engineering

Keywords: Fatigue crack growth, Low cycle fatigue, Process zone, Cyclic energy density;

1. Introduction

Fatigue crack growth has been studied in various types of structural materials in recent decades. It is generally accepted that the local nature of the phenomenon of fatigue crack propagation rate, da/dN, may be described by a sigmoidal curve in log(da/dN) vs log($\triangle K$) coordinate scale. In the intermediate $\triangle K$ range, log(da/dN) is almost linearly related to log($\triangle K$), hence the semi-empirical relation proposed by Paris and Erdogan, da/dN=C($\triangle K$)^m, where C and m are termed material constants. It is well-known that fatigue crack growth occurs because of local reversed plastic yielding of a material near the crack tip [1-3]. Based on nonlinear fracture mechanics and low cycle

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Selection and peer-review under responsibility of the Norwegian University of Science and Technology (NTNU), Department of Structural Engineering
doi:10.1016/j.mspro.2014.06.253

fatigue concepts, the crack growth law is desirable to be formulated and some attempts were made scholars. It would be an advantage to predict the FCG behavior chiefly from the low cycle fatigue (LCF) properties since the LCF test is easier to conduct and the LCF properties, can also be estimated from the monotonic tensile data [4, 5] and the hardness data [6]. A series of fatigue crack growth model [7-16] are generally formulated with the help of the stress and strain range ahead of the crack tip and using a suitable failure criterion to predict fatigue crack growth behavior. All failure criteria are found that energy-based criteria are more suitable than others [17-20].

Based on the cyclic HRR fields near the crack tip, a model developed by Pandey [14] predicts the fatigue crack growth rate using the equation

$$\frac{da}{dN}=\frac{\phi_p}{W_c}=\frac{\left(1-n^{'}\right)\cdot\left[\tilde{\sigma}_\theta\big|_{\theta=0}(\tilde{\sigma}_\theta\big|_{\theta=0}-\frac{1}{2}\tilde{\sigma}_r\big|_{\theta=0})\right]}{4EI_{n^{'}}\sigma_f^{'}\varepsilon_f^{'}}\cdot\left(\Delta K-\Delta K_{th}\right)^2 \tag{1}$$

In Eq. (1), $\sigma_f^{'}$ is the fatigue strength coefficient, $\varepsilon_f^{'}$ is the fatigue ductility coefficient, E is the elastic modulus, $\triangle K$ is the stress intensity range, $\triangle K_{\mathrm{th}}$ is the threshold stress intensity range, W_c is the area below the cyclic stress strain curve, Φ_p is the plastic dissipated energy per cycle unit growth, $I_{n^{'}}$ is a non-dimensional parameter of exponent $n^{'}$, $\tilde{\sigma}_\theta$ and $\tilde{\sigma}_r$ are non-dimensional functions of cyclic strain hardening exponent $n^{'}$. Eq. (1) prediction results are, however, dependent upon the material constitutive relation and the crack-tip fields.

The present study is aimed to examine these effects of different singularity fields ahead of the crack tip for the model proposed by Pandey [14] and propose a fatigue crack growth model. The resulting prediction is then examined by the experimental FCG data which can be easily found in the literature. Cyclic Stress-strain distribution near the crack tip

2. Cyclic Stress-strain distribution near the crack tip

The monotonic solution for the elastic-plastic field near a crack tip were given by Hutchinson [21] and Rice and Rosengren [22], which are now commonly referred to as the HRR singularity fields. The HRR fields are for the monotonically increasing load. To extend the response to unloading, reloading and cyclic loading, we may use Rice's plastic superposition method [23]. The fundamental assumption is that the components of the plastic strain tensor remain in constant proportion to on another at each point in the plastic region. The method may be taken as a first approximation in the absence of a more accurate theory. The cyclic stress-strain range along crack line (θ=0) is given by Eq. (2).

$$\begin{aligned}
&\Delta\sigma\left(\theta=0,N^{'}\right)=\sigma_y^{'}\left[\tilde{\sigma}_\theta\left(\theta=0,N^{'}\right)\cdot(\frac{1}{\alpha^{'}})^{1/\left(N^{'}+1\right)}\cdot\left(\frac{\pi}{I_{N^{'}}}\right)^{1/\left(N^{'}+1\right)}\cdot\left(\frac{\Delta K^2}{\sigma_y^2\pi x}\right)^{1/\left(N^{'}+1\right)}\right]\\
&\Delta\varepsilon\left(\theta=0,N^{'}\right)=\Delta\varepsilon_e\left(\theta=0,N^{'}\right)+\Delta\varepsilon_p\left(\theta=0,N^{'}\right)\\
&\Delta\varepsilon_e\left(\theta=0,N^{'}\right)=\frac{\sigma_y^{'}}{E}\left[\left(\tilde{\sigma}_\theta\left(\theta=0,N^{'}\right)-\nu\tilde{\sigma}_r\left(\theta=0,N^{'}\right)\right)\cdot(\frac{1}{\alpha^{'}})^{1/\left(N^{'}+1\right)}\cdot\left(\frac{\pi}{I_{N^{'}}}\right)^{1/\left(N^{'}+1\right)}\cdot\left(\frac{\Delta K^2}{\sigma_y^2\pi x}\right)^{1/\left(N^{'}+1\right)}\right]\\
&\Delta\varepsilon_p\left(\theta=0,N^{'}\right)=\frac{\sigma_y^{'}}{E}\left[\left(\tilde{\sigma}_\theta\left(\theta=0,N^{'}\right)-\frac{1}{2}\tilde{\sigma}_r\left(\theta=0,N^{'}\right)\right)\cdot\left(\alpha^{'}\right)^{1/\left(N^{'}+1\right)}\cdot\left(\frac{\pi}{I_{N^{'}}}\right)^{N^{'}/\left(N^{'}+1\right)}\cdot\left(\frac{\Delta K^2}{\sigma_y^2\pi x}\right)^{N^{'}/\left(N^{'}+1\right)}\right]
\end{aligned} \tag{2.a}$$

where

$$N^{'} = 1/n^{'} \tag{2.b}$$

In Eq. (2), r and θ are polar coordinates, x is the distance from the ahead of the crack tip, $N^{'}(N^{'}=1/n^{'})$ is the cyclic strain hardening exponent, $\sigma^{'}_{y}$ is a reference stress. Hutchinson has provided numerical values for the dimensionless parameters mentioned above. For convenience, the parameters required in applying the plane stress HRR theory can be obtain by curve fitting the tables provided by Shih [24].

$$\tilde{\sigma}_{\theta}\left(\theta=0,N^{'}\right) = -0.56168\exp(-0.99531N^{'}) - 0.03151\exp(-0.16508N^{'}) + 1.15310 \quad 2 \le N^{'} \le 100 \tag{3.a}$$

$$\tilde{\sigma}_{r}\left(\theta=0,N^{'}\right) = 0.44386\exp(-0.45292N^{'}) + 0.09435\exp(-0.08423N^{'}) + 0.58278 \quad 2 \le N^{'} \le 100 \tag{3.b}$$

$$I_{N^{'}} = 0.69424\exp(-0.06251N^{'}) + 2.18980\exp(-0.36520N^{'}) + 2.54772 \quad 2 \le N^{'} \le 100 \tag{3.c}$$

The cyclic stress-stain curve can be described as

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K^{'}}\right)^{1/n^{'}} \tag{4.a}$$

So, the parameter $\alpha^{'}$ in Eq. (2) is given in the form

$$\alpha^{'} = \frac{2E}{\left(2K^{'}\right)^{N^{'}}\sigma_{y}^{1-N^{'}}} \tag{4.b}$$

Closed form solutions for the stresses and strains in the plastic zone have not yet been obtained for strain hardening materials in the Mode I and Mode II. However, Rice's solution for Mode III (anti-plane shear) [25] can be obtain the stresses and strains within the plastic zone for the tensile loaded crack (Mode I). Such a method was developed by Kujawski and Ellyin [26] for a Ramberg-Osgood material, based on an energy interpretation of the strain hardening exponent. The stress-strain fields are termed RKE fields. Based on Rice's plastic superposition method [23], the cyclic stress-strain components normal to the crack plane, in the plane stress condition, can be derived from RKE fields. So, the cyclic fields can be described as

$$\begin{aligned} \Delta\sigma &= \frac{\sigma^{'}_{y}}{E}\left[\frac{\Delta K^{2}}{(1+n^{'})\pi\left(\sigma^{'}_{y}\right)^{2}x}\right]^{n^{'}/(1+n^{'})} \\ \Delta\varepsilon &= \frac{\sigma^{'}_{y}}{E}\left[\frac{\Delta K^{2}}{(1+n^{'})\pi\left(\sigma^{'}_{y}\right)^{2}x}\right]^{n^{'}/(1+n^{'})} + \frac{\sigma^{'}_{y}}{E}\left[\frac{\Delta K^{2}}{(1+n^{'})\pi\left(\sigma^{'}_{y}\right)^{2}x}\right]^{1/(1+n^{'})} \end{aligned} \tag{5}$$

The product of stress range and plastic range is then given by

$$\Delta\sigma\Delta\varepsilon_{p} = \frac{\Delta K^{2}}{\psi Ex} \tag{6}$$

where

$$\psi = \left(1 + n^{'}\right) \cdot \pi \tag{6.a}$$

The cyclic HRR fields are transformed according to the product of stress range and plastic range, the constant Ψ in Eq. (6.a) is given by

$$\psi = I_{n^{'}} / \left[\tilde{\sigma}_{\theta}\big|_{\theta=0} (\tilde{\sigma}_{\theta}\big|_{\theta=0} - \frac{1}{2}\tilde{\sigma}_{r}\big|_{\theta=0}) \right] \tag{6.b}$$

Substituting Eq. (6.b) into Eq. (1), Eq. (1) can be rewritten as

$$\frac{da}{dN} = \frac{\phi_p}{W_c} = \frac{\left(1-n^{'}\right)}{4E\psi\sigma^{'}_{f}\varepsilon^{'}_{f}} \cdot \left(\Delta K - \Delta K_{th}\right)^2 \tag{7}$$

where Ψ is a function of cyclic exponent $n^{'}$. Based on the cyclic RKE fields, a new model as similar with Eq. (1) can also be described Eq. (7) where the Ψ is Eq. (6.a).

3 Comparison with experimental data for difference singularity fields

The crack growth model developed is compared with the experimental results available in the literature for the materials given in Table 1 in which the mechanical and fatigue properties of the above materials are also listed.

Figure 1 shown that the Ψ parameter in Eq. (7) for two types singularity fields near the crack tip. Ψ value is calculated form Eq. (6.a) and Eq. (6.b). The Ψ value estimated from HRR Fields is higher than the Ψ value estimated from RKE Fields.

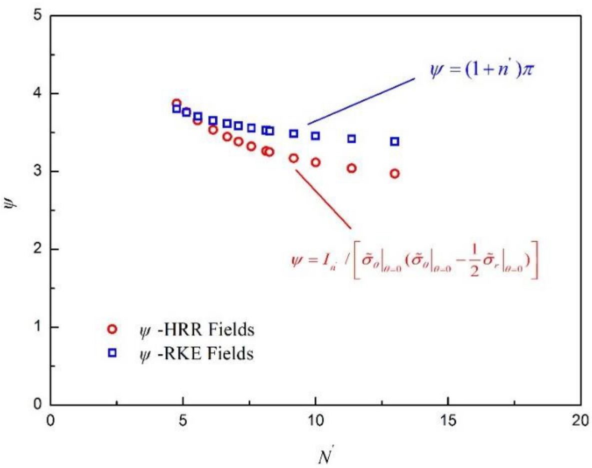

Figure 1. The Ψ parameter in Eq. (7) for two types singularity fields.

Table 1.Mechanical and fatigue properties

Material	E /GPa	$\sigma^{'}$ /MPa	$K^{'}$ /MPa	$n^{'}$	$\sigma_f^{'}$ /MPa	b	$\varepsilon_f^{'}$	c	R	$\triangle K_{th}$ /MPa·m[1] /2
7075 T6 alloy [27]	71	469	781	0.088	781	-0.045	0.19	-0.52	0.5	1.98
4340 steel [27]	200	889	1910	0.123	1879	-0.0895	0.64	-0.636	0.7	4.56

A5333-B1 steel [13]	200	345	1047	0.163	869	-0.085	0.32	-0.52	0.1	7.7
SAE 1020 steel [28]	205	270	941	0.18	815	-0.114	0.25	-0.54	0.1	11.6
API5L X60 steel [28]	200	370	840	0.132	720	-0.076	0.31	-0.53	0.1	8.0
E36 steel [10, 14, 29, 30]	206	350	1255	0.21	1194	-0.124	0.60	-0.57	0.0	5.0
10Ni steel [14, 31]	207	1106	2177	0.109	2019	-0.08	0.54	-0.645	0.1	5.0
2219 T 851 Al [9, 10, 14, 17, 29]	71	334	710	0.121	613	-0.0756	0.35	-0.55	0.1	2.7
8630 steel [14, 29, 32]	207	661	2267	0.195	1936	-0.121	0.42	-0.693	0.5	10
C-Mn steel [14, 29, 32]	208	372	896	0.141	868	-0.101	0.15	-0.514	0.0	13
35CD4 steel [29, 30]	209	800	1180	0.15	1818	-0.1	1.15	-0.71	0.0	3.0
TA12 steel [15]	113	903	1494	0.077	1609	-0.081	0.29	-0.662	0.1	8.0
35 NDC 16 steel [10]	190.6	1405	3580	0.15	3050	-0.11	0.58	-0.76	0.1	5.0
2024 T351 Al [27]	70	404	751	0.1	738	-0.081	0.3	-0.6	0.0	2.68
Spring steel (500℃) [13]	210	1490	3190	0.15	2970	-0.106	0.62	-0.709	0.1	2.05

To study detailed the prediction model Eq. (7), Figure 2 gives the impact factor in Eq. (7) based on fifteen metal fatigue properties. The maximum difference in impact facto (within the circle), which is affected different singularity fields ahead of crack tip, is found for 7075-T6 AL alloy in Figure 2.

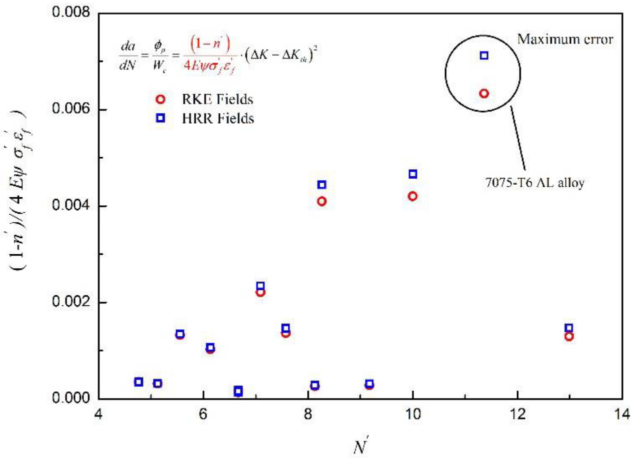

Figure 2. Impact factor in Eq. (7) with different fatigue properties of materials.

Figure 3 shows Eq. (7) prediction result for 7075-T6 AL alloy with two types singularity fields. Good agreement between experimental and Eq. (7) result is obtained.

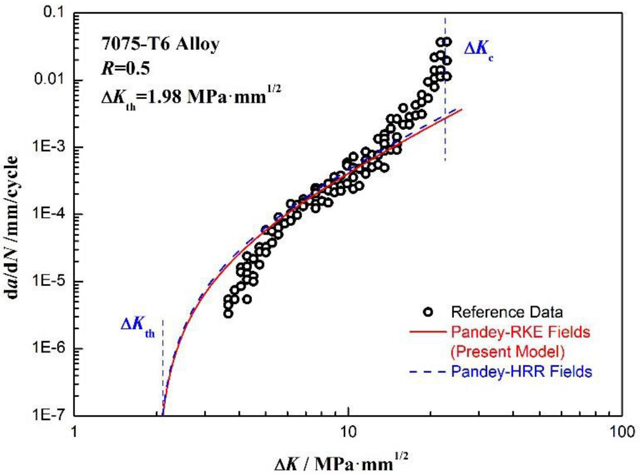

Figure 3. Comparison between Eq. (7) predictions and experimental data of 7050-T6 AL alloy

4 Conclusions

This work is a further develop about Pandey's previous work [14] where an expression for the fatigue crack growth rate is derived. Two types of the crack-tip, HRR Fields and RKE Fields, are employed in the present study. Hutchinson gives some numerical values for the dimensionless parameters mentioned Eq. (2). For convenience, the dimensionless parameters are estimated by curve fitting the numerical values at θ=0. The RKE Fields are considered ahead of the crack tip. The choice of the singularity fields (HRR or RKE) is contained in the express for the parameter for Eq. (7).

The parameter Ψ in Eq. (7) is studied based on the low cycle fatigue properties of fifteen metal materials, as shown in Figure 1. Figure 1 shown that Ψ value from Eq. (6.a) is lower than Ψ value from Eq. (6.b). Therefore, Eq. (7) prediction results may be effects based on different singularity fields. Figure 2 shown the impact factor of two kinds of singularity fields.

The prediction of the rate of crack propagation model is compared with the 7075-T6 alloy, and the agreement is found to be fairly good for two types of the crack tip, as shown in Figure 3. Figure 3 shown that Eq. (7) prediction result has a little drop, but the effect can be negligible.

Acknowledgements

The authors gratefully acknowledge financial support for this work form National Nature Science Foundation of China (No. 11072205).

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