Optimization method for last train coordination plan of urban rail transit based on network operation

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Abstract

With the generation of urban rail transit network operation mode, the coordination of network operation plan becomes a key factor for improving the overall level of service, in which the last train coordination plan is the most important. There is some research on the coordination program of network operation at home and abroad, but mainly focused on the coordination of a single point (transfer station). In fact, there are usually no less than two transfer stations between one line and other lines. Therefore, the coordination of a single point cannot meet the needs of making the network operation plan. In this paper, a last train coordination model considered multi-point transfer for rail line is established. Because of the model’s characteristics of global nonlinear but local linear and monotonous, an algorithm based on section is set up. The result of case calculation shows that the proposed model and algorithm are simple and practical, which can provide an optimal coordination plan of last train for single line.

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Key Words: Network Operation, Coordination Plan, Last Train, Multi-point Transfer

1. Introduction

1.1. Background

With the continuous expansion of urban rail transit network, network operation has become the main mode of operation & management, and “one ticket transfer” brings great convenience for passengers. In order to reduce the transfer waiting time of passengers, urban rail transit operators pay attention to the network train diagram, which requires the operation planners to develop optimum network coordination plan. The purpose of network coordination plan is to make trains between different lines coordinate at the transfer stations, and minimize the average waiting time of all passengers on the network.

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An important task in the program of network coordination is the last train coordination plan. The coordinate state of last train between different lines determines whether the passengers can transfer to the last train of the destination line, which directly influences the service level of urban rail transit.

1.2. Literature Review

The study on network coordination at home and abroad mainly focuses on two aspects: operation coordination plan and real-time control. The operation coordination plan is to limit and adjust the network diagram in programmatic terms, and reduce the waiting time of the passengers in the transfer process. For example, Bassel J. Younan\(^1\) introduces the coordination plan between urban rail transit and bus, which can reduce the waiting time of passengers by adjusting the departure time, stop time; Drs. Rob MP Goverde\(^2\) does analysis on waiting time of passengers on the network with low departure frequency or train delays, and proposes the coordination optimization method; Lee, KKT\(^4-5\) also studies the coordination between urban rail transit and bus at the terminal station, and proposes the corresponding model and algorithm; Zhang\(^6\) proposes a calculation method of coordination plan from the network level, which aims to reduce the entire waiting time of passengers in the transfer stations, and achieves certain results. Real-time control refers to using control measures, such as adjustment of departure time, stop time and speed, to ensure the effective coordination between different lines or different modes of transport. For example, Eui-Hwan Chung\(^3\) does research on the coordination between the intermodal transport and make a real-time CP (connection protection) control mode to ensure a reliable time different lines or different modes of transport.

By the overview of the research above, domestic and foreign scholars mainly focuses on the interface between the various modes of transport in only one single convergence point. However, for the urban rail transit network, there are usually some transfer stations between one line and other lines instead of one. Therefore, multiple points should be considered in coordination plan. As a result of the urban rail transit departure interval is short, except the first and last train, the waiting time for passengers in transfer station is short. So the network coordination should focus on the coordination plan of the first and last train, which unfortunately few research scholars at home and abroad pays attention to.

This paper focuses on the last train coordination plan for single line considering multi-point transfer, that is to develop the optimal last train timetable for single line and make the waiting time of all passengers on the line shortest under the constraint of multiple convergence points.

2. Related Concepts

(1) Coordination relationship

Coordination relationship refers to the coordination of trains of different lines and directions at the transfer station. As shown in Fig.1, station A is the transfer station between Line 1 and Line 2, the black arrow dashed line means: trains of Line 1 from down direction coordinate trains of Line 2 from up direction at the transfer station A.
(2) Active and passive coordination relationship
In the coordination relationship, the former line is active line and the latter is passive line. So do the trains. In one adjustment, if the train time of active/passive line should be modified, the coordination relationship is active/passive one.

(3) Graphic description of coordination relationship
The graphic description of coordination relationship is shown as Fig.2. Train 1 is the last train of the Line 1 from down direction; Train 2 is the last train of the Line 2 from up direction; the horizontal axis is the time axis; \( t_1, t_2 \) are the arrival and departure time of Train 1; \( t_3, t_4 \) are the arrival and departure time of Train 2; the arrow dotted line means the coordination relationship: Train 1 of Line 1 from down direction coordinate Train 2 of Line 2 from up direction at the transfer station A.

(4) Coordination cost
Coordination cost is the comprehensive cost caused to passengers under certain determined coordination relationship. If the coordination relationship is satisfied, coordination cost is the waiting time at the transfer station; otherwise, coordination cost can be the maximum waiting time tolerated by passengers.
3. Model Construction

While determining the last train time of a line from specific direction, the last train coordination relationship between the line and other lines needs to be considered. In this section, the calculation of first and last train time based on network operation for single line is set up, which takes the minimum cost of all the coordination relationship as objective. The specific details are as follows:

Objective function:

$$\min \sum_i k_i T_i + \delta W_i \quad W_i \in W$$

Constraints:

$$T_i = \begin{cases} 
    t^D_{i} - t^A_{r_i} - t^r_{r_i}, & r_i \text{ is satisfied, } r_i \in r \\
    0, & r_i \text{ is not satisfied, } W_i \in W 
\end{cases}$$

$$\delta_i = \begin{cases} 
    0, & r_i \text{ is satisfied} \\
    1, & r_i \text{ is not satisfied} 
\end{cases}$$

If \( t^D_{\text{nast}_{r_i}} \geq t^A_{r_{r_i}} + t^r_{r_i} \), then \( r_i \) is satisfied; Otherwise \( r_i \) is not satisfied

\[
\begin{align*}
    & r = \{r_1, r_2, r_3, \ldots, r_m\} \\
    & W = \{W_1, W_2, W_3, \ldots, W_m\} \\
    & t^D_r = t^D_{l_{r} p_{r} S_{r}}, \quad t^D_{p_{r} S_{r}} \in T_{l_{r} p_{r}} \\
    & t^A_r = t^A_{l_{r} p_{r} S_{r}}, \quad t^A_{l_{r} p_{r} S_{r}} \in T_{l_{r} p_{r}} \\
    & T_i = \{T^A_{l_{r} S_{r}}, T^D_{l_{r} S_{r}}, T^A_{l_{r} S_{r}}, T^D_{l_{r} S_{r}}, \ldots, T^A_{l_{r} S_{r}}, T^D_{l_{r} S_{r}}\} \\
    & T^r_{l_{r} S_{r}} = \{T^r_{l_{r} S_{r}}, T^r_{l_{r} S_{r} S_{r}}, T^r_{l_{r} S_{r} S_{r}}, T^r_{l_{r} S_{r} S_{r} S_{r}}, \ldots, T^r_{l_{r} S_{r} S_{r}}, T^r_{l_{r} S_{r}}\} \\
    & T^{r S}_{l_{r} S_{r}} = T^{D}_{l_{r} S_{r}} - T^{A}_{l_{r} S_{r}}, \quad \forall i \in (1, 2, 3, \ldots n) \\
    & T^{r S}_{l_{r} S_{r}} = T^{A}_{l_{r} S_{r}} - T^{D}_{l_{r} S_{r}}, \quad \forall i \in (1, 2, 3, \ldots n) 
\end{align*}
\]

Where,

\( r_i \) — NO. \( i^{th} \) coordination relationship;  \\
\( r \) — the collection of coordination relationships selected;  \\
k_i — important coefficient of \( r_i \);  \\
\( T_i \) — comprehensive cost of \( r_i \);  \\
\( W \) — the collection of penalty values if the coordination relationships is not satisfied;  \\
t^r_{r_i} — the transfer time of \( r_i \);  \\
l^p_{r_i} — the arrival time of the active train in \( r_i \) at the transfer station;
The departure time of the passive train in \( R_i \) at the transfer station;  
\( T_i \) —timetable of Train \( I \);  
\( l^* \) —the train whose timetable needs to be calculated in the coordination relationships;  
\( T_{i,R}^s \) —the runtime collection of Train \( l^* \);  
\( T_{i,S_n}^s \) —the stop time of Train \( l^* \) at Station \( S_n \);  
\( T_{i,S_{n-1},S_n}^r \) —the runtime of Train \( l^* \) in Section \( S_{n-1}, S_n \).

4. Algorithm Design

According to the model’s characteristics of non-continuity and local optimum, this paper attempts to change the feasible solution domain into a one-dimensional interval, and calculate the global optimal solution by sections.

The main calculation steps for single line are as follows:

Step1: According to the coordination relationships, calculate the arrival and departure time \( (T_{i,S_n}^A, T_{i,S_n}^D) \) of Train \( l^* \) at the transfer station \( S_i \) when the coordination relationships are just to meet. The formulas are as following:

Active coordination relationship:
\[
T_{i,S_n}^A = i_{i,S_n}^D - t_i
\]

Passive coordination relationship:
\[
T_{i,S_n}^D = i_{i,S_n}^A + t_i
\]

Step2: According to the arrival and departure time \( (T_{i,S_n}^A, T_{i,S_n}^D) \) of Train \( l^* \) at the transfer station \( S_n \) and the runtime collection \( T_{i,R}^s \), calculate the set of departure time of Train \( l^* \) at the starting station \( T_{i,S_n}^D = \{T_{i,S_{n_1}}^D, T_{i,S_{n_2}}^D, \cdots, T_{i,S_{n_r}}^D\} \); \( T_{i,S_n}^D \) is the departure time of Train \( l^* \) when coordination relationships \( \mathcal{R}_i \) is just to meet; The collection \( T_{i,S_n}^D \) is a set of non-continuous points. Then, the feasible interval domain of the departure time of Train \( l^* \) can be obtained as \( [\min, \max, T_{i,S_n}^{D_{-}}, T_{i,S_n}^{D_{+}}] \). Thus, the feasible solution domain of the model has been changed into a one-dimensional interval. The feasible solution domain is showed in Fig.3:
Step 3: Divide the feasible solution domain into several sections,

\[ \text{Sec} = \left( T^{D+}_{\ell_S,1}, T^{D+}_{\ell_S,2}, \ldots, T^{D+}_{\ell_S,n} \right) \]

Obviously, the sections are continuously monotonic.

Step 4: Calculate the optimal time of each section. According to the characteristics of continuously monotonic, the optimal target value of the section is calculated as follows:

\[ \text{Opt}_{f_{\text{Seci}}} = \min \left( f \left( T^{D+}_{\ell_S,(i-1)} \right), f \left( T^{D+}_{\ell_S,i} \right) \right), \text{Seci} \in \text{Sec} = \left( 1, 2, 3 \ldots n \right) \]

Then, the optimal target value of the optimal target value can be obtained by the following formula.

\[ \text{Opt} f = \min_{\text{Seci} \in \text{Sec}} \text{Opt}_{f_{\text{Seci}}} \]

5. Numerical Example

Take Beijing Metro Line 1 for example, and the coordination objective is that the last train from down direction can coordinate the last trains of other lines at all transfer stations of Line 1. According to the proposed model and calculation process, part of the optimal last train timetable of Line 1 can be obtained as Tab.1. The results are simple and practical.
Tab.1 Part of the last train timetable of Line 1

<table>
<thead>
<tr>
<th>Station</th>
<th>Sihui East</th>
<th>Guomao</th>
<th>Jianguomen</th>
<th>Dongdan</th>
<th>Xidan</th>
<th>Pingguoyuan</th>
</tr>
</thead>
</table>

6. Conclusion

In this paper, firstly, a model for last train coordination plan for single line based on network operation is set up, which has the characteristics of global nonlinear but local linear and monotonous. Then, an algorithm based on section is proposed to solve the model. Also, at the end of the paper, a numerical example is given. The results show that the proposed model and algorithm can provide an optimal coordination plan of last train for single line, which is helpful to improve the service level of urban rail transit.

References