Wave interaction with porous vertical breakwater in two-layer fluid

Menghua Zhu, Hua Huang, Jiemin Zhan, Lin Guo, Ruizhi Chen

Department of Applied Mechanics and Engineering, College of Engineering, Sun Yat-sen University, Guangzhou, 510275, China

Abstract

In this paper, the two-layer fluid model is used to investigate the wave interaction with porous vertical breakwater resting on permeable seabed in two-layer fluid. Based on the Biot seepage consolidation theory and linear wave theory, the analytical solutions to the wave potentials and the seepage pressures inside permeable seabed induced by surface wave and inner wave are derived by applying eigenfunction expansion approach. And accordingly, the analytical expressions of wave-induced uplift force and overturning moment acting on the bottom of porous breakwater are obtained. Numerical results are presented to illustrate the seepage loads caused by inner wave may have same order of magnitude as that induced by surface wave, and the wave-induced seepage loads may have the same order of magnitude as direct wave loads. It is found that the porosity of the breakwater surface may lead an obvious reduction in direct wave loads on breakwater and a certain reduction in wave-induced seepage loads on the bottom of breakwater, especially, lead to a significant reduction in the inner wave-induced overturn moment.

Keywords: Two-layer Fluid; Porous Vertical Breakwater; Permeable seabed; Wave-induced seepage loads

1. Introduction

Vertical breakwaters are widely used to provide protection from waves in harbor and marinas. Considerable works have been done on wave field with breakwaters in uniform fluid. Such as Morgan and Firat [1] studied wave reflection by submerged vertical and semicircular breakwater. However, the ocean is not uniform in density actually, which is usually formulated with a two-layer fluid model. There are inner waves on the interface between the two fluid layers because of density stratification. Many studies reveal that the effects of inner waves on structures may have the same magnitude as the effects of the surface wave [2], thus they can’t be neglected. However, for breakwater resting on permeable elastic seabed in two-layer fluid, there may be several different seepage loads

* Corresponding author. Tel.: +86-1353-527-9557
E-mail address: tsyhh1982@163.com
acting on the bottom of breakwater induced by surface wave and inner wave. The wave-induced seepage pressure forces can not only cause erosion around the foundations of the breakwater but may also produce periodic uplift force and overturn moment on the underside of the breakwater [3]. Furthermore, to obtain better design, many investigations have been done in the interaction with porous structures. It reveals that the structures with porosity will significantly reduce the exerted loads by waves [4].

In this paper, the attention is mainly focused on the surface and inner wave-induced seepage effects on porous vertical breakwater resting on permeable seabed in two-layer fluid. Since the seepage field within the seabed is induced by wave pressure on the bottom of the sea, the wave field problems will also be mentioned, but briefly.

2. Theoretical analysis

As shown in Fig.1, the porous vertical breakwater with width $b$ resting on permeable elastic seabed with depth $h$ in two-layer fluid with depth $d$. And the density of the upper layer is $\rho_1$, the density of the lower layer is $\rho_2$, the depth of the upper layer is $h_1$, the depth of the lower layer is $h_2$. A rectangular coordinate system $(x, z)$ is defined with the origin location and the positive direction shown in Fig.1. There are two kinds of water wave modes in two layer fluid including surface wave with wave frequency $\omega^+$ and inner wave with wave frequency $\omega^-$. 

![Fig.1. Porous vertical breakwater resting on permeable seabed in two-layer fluid](image)

Considering the boundary layer of the breakwater is thin, so the analysis is based on these assumptions, the fluid in the system is incompressible, inviscid and the flow field is irrotational. Based on Biot seepage consolidation theory, it is assumed that there is no relative sliding between the breakwater and the seabed. The seabed is isotropic, permeable, linear elastic and the bottom of seabed is impermeable, without deformation.

Let $\Phi^{(j)}$ denotes total wave velocity potentials in region $\Omega_j$ ($j=1,2,3$). Where $\Phi^{(1)} = \text{Re}(\Phi^{(1)}), \Phi^{(j)} = \Phi^{(j)} + \Phi^{(j)}$, and in which $\Phi^{(j)}$ and $\Phi^{(j)}$ are the corresponding spatial incident velocity potential and spatial reflected velocity potential. The velocity potentials in domain must satisfy the boundary conditions,

$$
\lim_{x \to \pm \infty} \frac{\partial \Phi^{(j)}}{\partial x} + ik \Phi^{(j)} = 0, \quad \lim_{y \to \pm \infty} \frac{\partial \Phi^{(j)}}{\partial y} - ik \Phi^{(j)} = 0, \quad \lim_{y \to \pm \infty} \frac{\partial \Phi^{(j)}}{\partial y} - ik \Phi^{(j)} = 0, \quad \lim_{x \to \pm \infty} \frac{\partial \Phi^{(3)}}{\partial x} - ik \Phi^{(3)} = 0
$$

(1)

When the flow passing through the porous breakwater, it can be assumed to obey Darcy’s law. Hence,
\[
\begin{align*}
\frac{\partial \phi^{(1)}}{\partial x} &= \frac{\partial \phi^{(2)}}{\partial x} = iGk \frac{\omega^2}{\omega} (\phi^{(1)} - \phi^{(2)}) \quad (x = 0, 0 \leq z \leq d) \\
\frac{\partial \phi^{(2)}}{\partial x} &= \frac{\partial \phi^{(3)}}{\partial x} = iGk \frac{\omega^2}{\omega} (\phi^{(2)} - \phi^{(3)}) \quad (x = b, 0 \leq z \leq d)
\end{align*}
\]

where \( k \) denotes wave number, \( \frac{\omega}{\omega} = \sqrt{\frac{T_1(k)}{\tanh kd}} \), \( G = \frac{\gamma \rho_0}{\mu k} \) is porous-effect parameter, in which \( \gamma \) is a material constant having length dimension, \( \rho \) is the density of sea water and \( \mu \) is the coefficient of dynamic viscosity.

Applying the eigenfunction expansion approach, the solutions for \( \phi^{(j)} \) are given as,

\[
\begin{align*}
\phi^{(1)} &= \frac{2\rho A^+}{\omega^2} Z^+(z) \cos k_z x e^{ik_y y} e^{-i\omega t} \\
\phi^{(2)} &= \left( \frac{-i \rho A^+}{\omega^2} \right) Z^+(z) \left( C_2^+ e^{-ik_z z} + D_2^+ e^{ik_z z} \right) e^{ik_y y} e^{-i\omega t} \\
\phi^{(3)} &= \left( \frac{-i \rho A^+}{\omega^2} \right) Z^+(z) E_3^+ e^{ik_z z} e^{ik_y y} e^{-i\omega t}
\end{align*}
\]

\[
Z^+(z) = \begin{cases} \cosh k(z-d) + T^+(k) \sinh k(z-d) & (h_2 \leq z \leq d) \\
E^+(k) \cos k z & (0 \leq z < h_2) \end{cases}
\]

where \( A^+ \) and \( A^- \) denote the amplitude of surface wave and inner wave, respectively. \( C_2, D_2 \) and \( E_3 \) are unknown coefficients. \( E^+(k) = \frac{T^+(k) \cosh kh - \sinh kh}{\sinh kh} \), \( T^+(k) = \frac{T_1(k) + T_2(k) + \sqrt{(T_1(k) + T_2(k))^2 - 4(1-\lambda)T_1(k)T_2(k)}}{2(1+\lambda T_2)} \), in which \( \lambda = \rho_1/\rho_2 \), \( T_1 = \tanh kh_1 \), \( T_2 = \tanh kh_2 \).

With assumptions mentioned above, let \( p^{(j)} \) denotes the seepage pressures induced by surface wave and inner wave respectively in region \( \nu_j \quad (j=1,2,3) \). Where \( \Re(p^{(j)}) = \Re(p^{(j)}) \), then based on Biot seepage consolidation theory, the seepage pressures are governed by,

\[
\nabla^2 p^{(j)} = C_s \frac{\partial p^{(j)}}{\partial t} - C_s = \frac{\rho g}{k_s} (n_0 \beta_j + 1 - \frac{2\nu}{G_0} ) \frac{1}{2-2\nu} \frac{\partial p^{(j)}}{\partial z} = 0 \quad (z = -h), \frac{\partial p^{(j)}}{\partial z} = 0 \quad (z = 0, 0 \leq x \leq b)
\]

\[
p^{(1)} = p^{(2)}, \frac{\partial p^{(1)}}{\partial x} = \frac{\partial p^{(2)}}{\partial x} \quad (x=0, -h \leq z \leq 0), \quad p^{(2)} = p^{(3)} \quad (x=b, -h \leq z \leq 0)
\]

where \( C_s \) is the comprehensive effect coefficient of porous media and sea water, \( k_s \) is the permeability coefficient of soil, \( \beta_j \) is the compressibility coefficient of pore fluid, \( n_0 \) is the porosity of sea bed, \( G_0 \) is the shear modulus, \( \nu \) is the Possion ratio. Applying the eigenfunction expansion approach, the solutions are given as,

\[
\begin{align*}
p^{(2)} &= 2A^+ \rho_2 g e^{-i\omega t} e^{ik_y y} E^+(k) \sum_{m=0}^{\infty} \left( B_m e^{km_x} + C_m e^{-km_x} \right) \cos \frac{m\pi z}{h} \quad (C_s \neq 0) \\
p^{(3)} &= 2A^+ \rho_2 g e^{-i\omega t} E^+(k) \left[ B_0^c + C_0^c x + \sum_{m=1}^{\infty} \left( B_m^c e^{km_x} + C_m^c e^{-km_x} \right) \cos \frac{m\pi z}{h} \right] \quad (C_s = 0)
\end{align*}
\]
Where \( B_0 \), \( C_0 \) and \( B_m \), \( C_m \) are unknown coefficients, \( k_{3m}^2 = \sqrt{(m \pi / h)^2 + k_{c}^2 - i \omega_L C_s} \). By applying interface conditions equation (6) and Fourier series expansion, it is easy to obtain the full algebraic equations for coefficients \( B_0 \), \( C_0 \) and \( B_m \), \( C_m \).

Formulas mentioned above are the mathematical expressions for pressure under the base of the breakwater. And the mathematical expressions for wave-induced seepage uplift forces \( F_v^\pm \) and overturn moments \( M_v^\pm \) exerting on the bottom of breakwater can be obtained by,

\[
F_v^\pm = \text{Re}(f_v^\pm), f_v^\pm = \int_{0}^{b} p^{\pm(z)} \big|_{z=0} dx \quad M_v^\pm = \text{Re}(m_v^\pm), m_v^\pm = \int_{0}^{b} p^{\pm(z)} \big|_{z=0} (x-\frac{b}{2}) dx
\]

3. Results and conclusions

Computer programs have been written to facilitate the analysis of the above mathematical expressions, and the seepage effects for several configurations have been investigated. Assuming the seabed is not deformable, then \( C_v = 0 \). Let the amplitude of wave \( A' = A' = A \). Figure 2 shows the influence of porous-effect parameter on surface wave loads and inner wave loads, the porosity of the breakwater leads to a slight reduction in the seepage forces, but the seepage overturn moment has a great decrease. Figure 3 shows the comparison of seepage loads induced by surface wave and inner wave. It demonstrates that the seepage loads induced by inner wave or surface wave may have same order. Figure 4 shows the comparison of wave-induced seepage overturn moment and direct horizontal wave moment. It indicates that seepage loads and direct horizontal wave loads may have the same order.

Fig.2. Uplift force and overturn moment for \( \lambda = 0.9, h / d = 0.4, b / d = 3, b / h = 0.4 \)
Fig. 3. Comparison of seepage loads induced by surface wave and inner wave for \( G=1, \lambda = 0.9, \frac{h}{d} = 0.4, b \rightarrow d = 3, b / h = 0.4 \)

Fig. 4. Comparison of overturn moment and wave moment for \( G=1, \lambda = 0.9, \frac{h}{d} = 0.4, b \rightarrow d = 3, b / h = 0.4 \)

**Acknowledgements**

This work was financially supported by the National Marine Public Welfare Research Projects of China. (Project No. 201005002)

**References**


