The Detection and Prevention of Deadlock in Petri Nets

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Abstract

This article introduces the basic knowledge of Petri net at first, then analyzes two methods about the detection of deadlock in Petri nets. One is based upon the net structure; the other is based upon reachability tree and the article describes the steps of the arithmetic about it. Besides, the article summarizes the method based upon basic siphon theory for the prevention of deadlock in Petri nets and gives out examples for application.

Key words: Petri Net; Deadlock Detection; Deadlock Prevention; Deadlock

1 Introduction

Petri net is a graphical and formalized tool, which can describe and analyze systems with concurrency, asynchronous, indeterminacy abilities.

Petri net has many properties, like deadlock, activity, reachable and so on. These abilities are divided into two types. One type has nothing with the original status and it only reflects the properties of the net structure. While the other type is related to the original state, it reflects the dynamic properties of the Petri net.

Deadlock is an important property of Petri net. In the running process of concurrency system, we must solve the problems caused by deadlock. Only in this way can the system run well.

In this article, we talk about the detection and prevention of deadlock in Petri nets.

2 Basic Definition about Petri Net and Deadlock

2.1 Petri Net

A Petri net is a 3-tuple (N=(P,T;F)), where
(1) \( P = \{p_1, p_2, \ldots, p_m\} \) is a limited place set;
(2) \( T = \{t_1, t_2, \ldots, t_n\} \) is a limited transition set;
(3) \( P \cap T = \emptyset, P \cup T \neq \emptyset \);
(4) \( F \subseteq (P \times T) \cup (T \times P) \), F is a flow relation on the N, its elements were called arc;
(5) dom(F) \cup cod(F) = P \cup T

Besides,
\[ \text{dom}(F) = \{x \in P \cup T \mid \exists y \in P \cup T: (x,y) \in F\} \]
\[ \text{cod}(F) = \{x \in P \cup T \mid \exists y \in P \cup T: (y,x) \in F\} \]

In the graphic expressions of Petri net, places are presented by circles; transitions are presented by squares or short lines and directed arcs connect them.

2.2 Deadlock

If \( \Sigma = (P,T;F,M_0) \) is a Petri net and \( M_0 \) is a signal of it, we call \( (\Sigma, M_0) \) is a deadlock only on the condition that
Wensong Hu et al. / Physics Procedia 22 (2011) 656 – 659

2.3 Siphon

If a Petri net is $N=(P,T;F)$ and $(N,M_0)$ is a net system.

1. If and only if $S \subseteq P$ is founded, nonempty set $S \subseteq P$ is a siphon;
2. If and only if nonexistent other siphon is the proper subset of a siphon, we call this siphon is minimum.
3. A siphon who do not include any $P$-invariant is called strictly siphon; A siphon who is minimum and strictly is called strictly and minimum siphon (SMS).

3 The Detection of Deadlock in Petri Net

3.1 The Analysis of Deadlock Based on Petri Net Structure

Scientists like Barkau, Bermond have done many researches about the deadlock in the structure of Petri net, several important conclusions have put forward. Two of them are showed below.

Theorem 1: Suppose if $D$ is a deadlock of Petri net $\Sigma = (P,T;F,M_0)$, $G_D$ is a chart given by $D \cup \cdot D$ and $\cdot D$ is the input position of transition $t$ of $G_D$. If deadlock $D$ satisfies the condition that $|D| \geq 2$ is a minimum deadlock, if and only if $G_D$ contains an arc $\lambda$ which passes through all place of $G_D$ and $\lambda$ satisfies the condition that: all $t \in \lambda$, $|D| = 1$ or $|D| \geq 2$ and there is a commutative circle passes through all the place of $\cdot D$.

Theorem 2: Suppose if $D$ is a sub place set of Petri net $\Sigma = (P,T;F,M_0)$, if $D$ is a minimum deadlock, then if and only if the son charts derive from $(D,\cdot D)$ can compress into a single place and this place does not connect to any transitions.

The structure deadlock reflects some properties of Petri net and it contributes to the modeling and analysis for some systems. But this method has several disadvantages.

First, the structure deadlock is underlying, sometimes it does not influence the running of the Petri net.
Second, there is nothing between structure deadlock and the initial status. However the initial status influences the running status of Petri net much, the structure deadlock cannot reflect the dynamic properties of Petri net.
Last, the more complex the Petri net, the more deadlocks. So it is very difficult to modeling accord to the structure.

3.2 Using Reachability Tree to Analyze and Detect Deadlock

Reachability tree is an important tool to analyze the function of Petri net. It contains three properties showed below.

First, reachability tree covers the reachable set of the Petri net. If a Petri net contains deadlocks, its reachability tree must cover it.
Second, reachability tree classifies the reachable set. Different symbol classes was marked by different marks (which contain signal $\circ$). It is very effective for the locating of deadlock.
Last, according to the reachability tree, we can find the running process of Petri net.
But reachability tree is not perfect. Potential important information may loss for the input of signal $\circ$. In some degree, it is very difficult to judge whether a Petri net is dead. A combination between reachability tree and the net itself may be a good way to this problem.

4 The Prevention of Deadlock in Petri Nets

4.1 The Deadlock Prevention Strategy Based on Basic Siphon

Most control strategies for the prevention of deadlock are adding controllers to each SMS. However studies have shown that as the scale of the Petri net becomes bigger, the number of the SMS grows much faster. This situation makes the design of controller much hard.

Here we classify the SMS into basic siphons and additional siphons. Studies have shown that basic siphons are much less than the additional siphons in the SMS. So if we design controllers for the basic siphons and make the controllers control the additional siphons as well, the number of the controllers will be much small.

Scientists Li and Zhou had put forward the deadlock control arithmetic based on basic siphons and $S^3PR$ net. The arithmetic is showed below.

If Petri net $N(S^3PR)$ is existed:
Step 1: find all of the SMS in N;
Step 2: find the basic siphons from SMS and the left are additional siphons(SR);
Step 3: add control place (Vs) for each basic siphon(S).
(1) the output arcs of Vs connect to the source transitions of S;
(2) the input arcs of Vs link to the supplementary set of S;
(3) \( 1 \leq \xi_s \leq M_0(S) - 1, M_0(V_s) = M_0(S) - \xi; \)
(4) Repeat step 3, until all the basic siphons were added into the control place set.
(5) Adjust the value of \( \xi_1 \), until each additional siphon is under control.

4.2 Application Example of Arithmetic

Fig.1. initial Petri net

Step 1: find all the SMS from the picture above. We can find that there are three SMS in the picture. They are
\( S_1 = \{p_5, p_9, p_{12}, p_{13}\}, S_2 = \{p_4, p_{10}, p_{11}, p_{12}\}, S_3 = \{p_5, p_{10}, p_{11}, p_{12}, p_{13}\}. \)
Step 2: find the basic siphons from the SMS. From the definition of basic siphon, we can find two basic siphons.
They are \( S_1 = \{p_5, p_{10}, p_{12}, p_{13}\}, S_2 = \{p_4, p_{10}, p_{11}, p_{12}\}. S_3 = \{p_5, p_{10}, p_{11}, p_{12}, p_{13}\} \) is an additional siphon.
Step 3: now let us add the control place Vs, the method is showed below.
First we add control place Vs for basic siphon S1. At first, we must obtain the supplementary set of \( S_1 \), it is
\( C_{S1} = \{p_4, p_9\}. \) Then the output arcs of Vs connect to the source transition of the circle where \( C_{S1} \) is stayed, and its input arcs connect to the postset of \( C_{S1}. \) So we can get a conclusion that \( V_{S1} = \{t_1, t_8\}; V_{S2} = \{t_4, t_5, t_6\}. \) The same way we can get the control place Vs for S2, \( V_{S2} = \{t_1, t_8\}; V_{S3} = \{t_3, t_6, t_{10}\}. \) Adjusting value of \( \xi_1 \), we make it at \( M_0(V_{S1}) = 2, M_0(V_{S2}) = 2. \)
Last we get two control place showed as below.
5 Concludes

In this article, we introduce two methods for the detection of deadlock. One is based on the structure of Petri net and the other one is based on the reachability tree. The first method puts emphasis on the analysis of structure and it has nothing with the running process and initial status of Petri net. While the second method reflects the running process of the net, it can find the deadlock and its generating process. However some information is loss for the input of signal $\omega$. Each method has its advantages and disadvantages. Combining the two of them will be a good research direction.

Besides, the deadlock prevention arithmetic which base on basic siphons improves the traditional way and applies fewer controllers to prevent deadlock, reducing the complexity.

Reference