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Modified Stress Intensity Factor Equations for Semi-Elliptical Surface Cracks in Finite Thickness and width Plates

Yang PENG^{a†}, Lewei TONG^{a*}, Xiao-Ling ZHAO^b, Zhigang XIAO^c^a*Department of Building Engineering, Tongji University, China*^b*Department of Civil Engineering, Monash University, Australia;**Changjiang Professor, Tongji University, China*^c*School of Applied Sciences and Engineering, Monash University, Australia*

Abstract

Weld defects and severe variation of shape near the welds cause high stress concentration at weld toes or weld roots. This high stress concentration reduces fatigue lives of welded structures. A stress intensity factor (SIF) which includes this effect increases the accuracy of fatigue lives prediction. A magnification factor is commonly used to multiply the SIF of semi-elliptical surface cracks to account for the stress concentration effect in welded connections. Yamada and Hirt model is one of these methods. Comparison between Yamada & Hirt SIF equations and Newman & Raju SIF equations are applied to SIF of semi-elliptical surface cracks. The results suggest that Yamada & Hirt SIF equations are valid when the crack aspect ratio is less than 0.6. New empirical SIF equations are developed based on Yamada & Hirt SIF equations. The newly derived SIF equations are verified by comparing with experimental data.

© 2011 Published by Elsevier Ltd. Open access under [CC BY-NC-ND license](https://creativecommons.org/licenses/by-nc-nd/4.0/).**Keywords:** Stress intensity factor, Surface crack, Crack propagation, Fatigue, Fracture mechanics

1. Introduction

Prediction of fatigue lives is important for road and railway infrastructures, offshore platforms and supporting frames of wind turbines, where high cycle fatigue is a major concern. Weld is a convenient and efficient solution for connecting components in these structures. There are several approaches for

* Corresponding author: Email: tonglw@tongji.edu.cn

†Presenter: Email: py5544@yahoo.com.cn

fatigue analysis of welded structures: stress-fatigue life (S-N) approach, linear elastic fracture mechanics (LEFM) approach, hot-spot stress method, notch stress method and one-millimetre method (Zhao et al. 2001; Xiao and Yamada 2004; Pang et al. 2006; Radaj et al. 2009). All of these methods have a trend towards accurate description of stress field around weld seam. However there are some disadvantages with the stress-fatigue life method. One of these is that it is invalid for the assessment of the remaining fatigue lives of cracked welded components under service. LEFM approach does not have such a disadvantage. LEFM approach provides basis for prediction of fatigue lives, materials selection and tolerance of acceptable weld imperfections. It is also used during the operational stage of a structure to make important decisions on inspection periods and repair plans.

Some factors should be considered when engineers conduct fatigue analysis of welded structural components by fracture mechanics approach. Two key factors are weld defects and severe variation of shape near the welds, which causes high stress concentration at weld toes or weld roots. These two factors reduce fatigue lives and increase cost of maintenance.

The defects are normally assumed as semi-elliptical surface cracks in fracture mechanics approach. For the stress concentration, Xiao and Yamada (2004) demonstrated that stress concentration can be separated into two parts: geometric stress due to structural geometry change and local peak stress (or called weld notch stress) due to weld profile. For the analysis of the stress concentration, there are two methods-Mk factor method (Bowness and Lee 2000) and FG factor method (Albrecht and Yamada 1977). All of these use a magnification factor to multiply the SIF solution for a semi-elliptical surface cracks in a flat plate (e.g. Newman and Raju, 1981).

It is important to accurately predict the SIF value of the surface crack in a flat plate. This paper presents an investigation into the SIF values for semi-elliptical surface cracks. Comparisons are made between Yamada & Hirt SIF equations and Newman & Raju SIF equations. The results suggest that Yamada & Hirt SIF equations are valid when the crack aspect ratio is less than 0.6. New empirical SIF equations are developed based on Yamada & Hirt SIF equations. The newly derived SIF equations are verified by comparing with experimental data.

2. Empirical models for fatigue crack propagation of welded joints

Maddox defined a factor called magnification factor M_k which could be expressed as (Bowness and Lee 2000) :

$$M_k = \frac{K_{(\text{in plate with attachment})}}{K_{(\text{in same plate but without attachment})}} \quad (1)$$

where K is stress intensity factor. It indicates the local peak stress at weld toes due to attachment and weld profile. Parametric numerical analysis by finite element method and regression analysis are standard procedure to obtain M_k (Bowness and Lee 2000). From this definition, the SIF at a weld toe can be obtained by multiplying the SIF of flat plate obtained from the equations of Newman and Raju (1981) by M_k . The SIF values at weld toe can be used to demonstrate variation of crack shape during fatigue crack growth (FCG) with the ‘two-point plus semi-ellipse’ method (Lin and Smith 1999) (Fig. 1) based on Paris law.

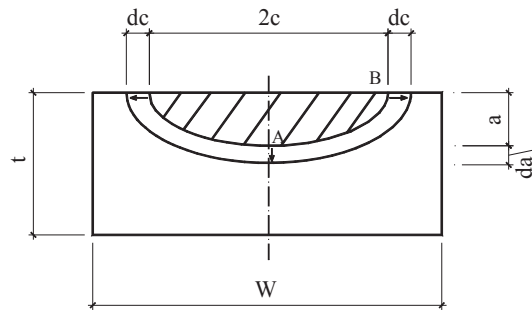


Fig. 1 Schematic figure of two points plus semi-ellipse method and geometrical parameters

Albert and Yamada (1977) defined a factor F_G similar to M_k . They deduced an analytical equation of F_G from the SIF equation of a central crack embedded in an infinite plate under two equal pairs of splitting forces. Yamada and Hirt (1982) combined F_G with the SIF of a flat plate in an independent iterative procedure at point A and point B to demonstrate the variation of crack shape during FCG (Fig. 1).

These two models consider different stress behaviour which related to stress classification. Newman-Raju-Maddox model separates stress field around weld seam into three parts: membrane stress, shell bending stress and local peak stress. Yamada and Hirt model separates stress field around weld seam into two parts: uniform stress and non-uniform stress (including bending stress and local peak stress). Yamada and Hirt model uses F_G to demonstrate whole non-uniform stress distribution.

3. Comparison between these two models with experimental data

The two models are used to calculate the variation of crack shape during FCG. The FCG is described by Paris law at point A and point B in both two models, which can be expressed as:

$$\frac{da}{dN} = C \Delta K_A^m \tag{2}$$

$$\frac{dc}{dN} = C \Delta K_B^m \tag{3}$$

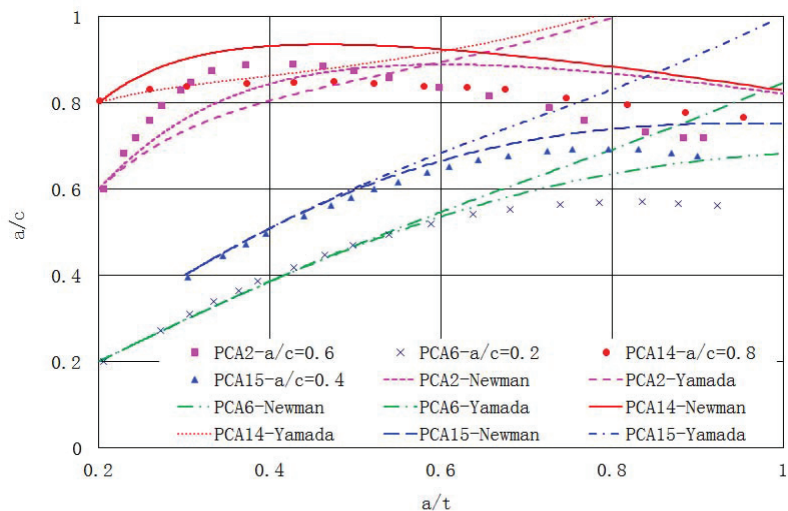
where N is the number of cycles, a is the crack depth, c is the half of crack length, C and m are the material constants, ΔK_A is the range of SIF at point A, ΔK_B is the range of SIF at point B. Results from these calculations are compared to experimental data from literatures.

The data published by Putra & Schijve (1992) and McFadyen et al.(1990) are selected. Putra and Schijve reported a fatigue test for 7075-T6 aluminium alloy plates under tension. The data of crack aspect ratios are at 0.2, 0.4, 0.6 and 0.8. They were represented by specimens PCA6, PCA15, PCA2 and PCA 14. Constants of Paris equation, C and m, were $2.29 \times 10^{-4} \mu m / cycle \cdot (MPa\sqrt{m})^{-2.88}$ and 2.88, respectively. McFadyen et al. reported a fatigue experiment of four-point bending. The data of crack aspect ratios are at 0.12, 0.13, 0.9 and 0.22. They were represented by specimen number 4, 5, 6 and 9. Constants of Paris equation, C and m, were $2.9 \times 10^{-12} m / cycle \cdot (MPa\sqrt{m})^{-3}$ and 3, respectively. The comparison is shown in Figure 2.

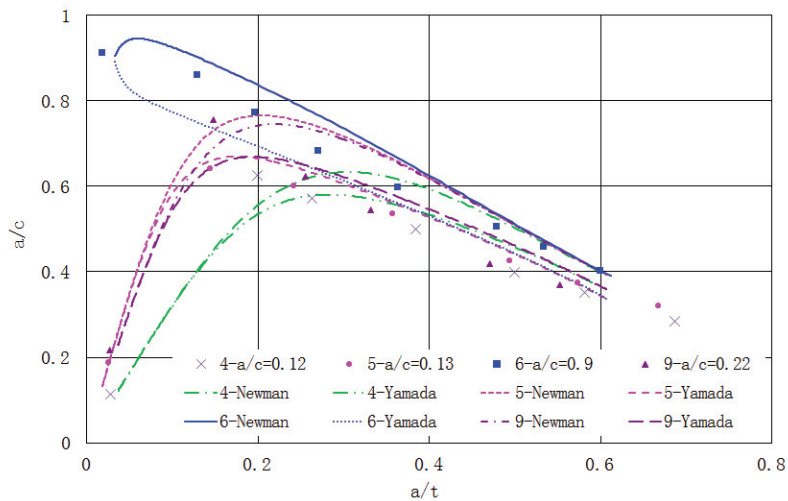
It can be seen from Figure 2 (a) that Yamada and Hirt model predicts almost the same value as that by Newman-Raju-Maddox model and that they are close to experimental results at small relative depth ratio ($a/t = 0.2$ to 0.6) and low crack aspect ratio ($a/c = 0.2$ to 0.6) under tensile loading case. Yamada and Hirt

prediction linearly increases when relative depth ratio is higher than 0.6. There is no convergence of crack aspect ratio for Yamada and Hirt model, which is proved by experiments and FEA simulation (Putra and Schijve 1992;Lin and Smith 1999).

Fig. 2(b) shows the comparison of calculated results and experimental data under the bending load case. Yamada & Hirt model and Newman-Raju-Maddox model show the lower and upper bound of experimental data. At crack aspect ratio 0.9, predicted values of Newman-Raju-Maddox model tend to shift upward and that of Yamada and Hirt model tend to shift downward when relative depth ratio is less than 0.2.



(a) Experimental data under tensile loading



(b) Experimental data under bending loading

Fig.2 Comparison of variation of crack shape during FCG between experimental data and two methods under tensile and bending loads

Figure 3 shows the comparison of the boundary correction factor from Yamada and Hirt SIF equations and that from Newman and Raju SIF equations under tensile loading case. The boundary correction factor is defined as the ratio of the SIF obtained from the SIF equations for the semi-elliptical surface cracks to Irwin’s SIF equation for an elliptical crack in an infinite solid (Newman, 1979). It is obvious that the boundary correction factor of Yamada and Hirt SIF equations at point B (see Figure 1) does not change with relative depth ratio. When crack aspect ratio is 1, Newman and Raju SIF equations indicate that SIF value at point B is higher than that of SIF value at point A. Yamada and Hirt SIF equations cannot demonstrate this phenomenon due to unchanging boundary correction factors for point B.

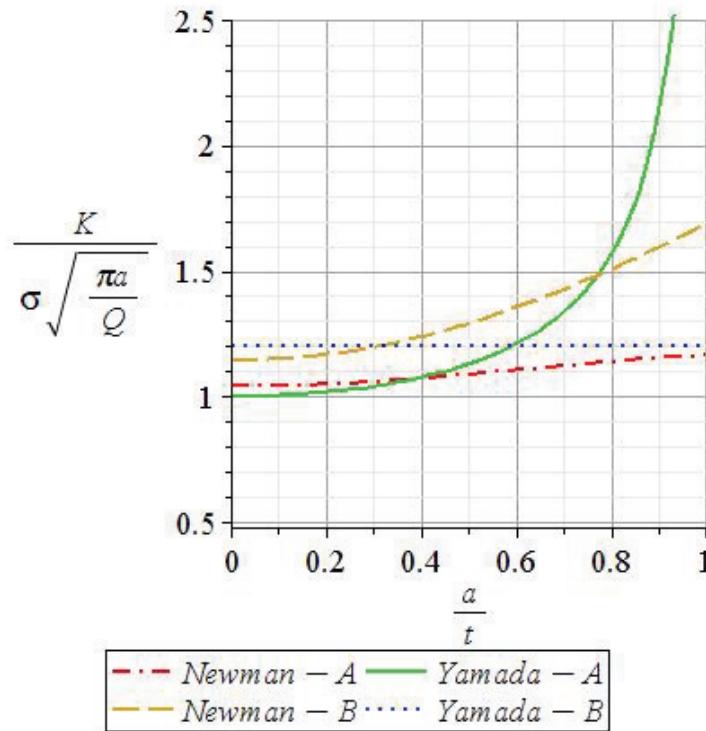


Fig. 3 Comparison of boundary correction factors from Yamada & Hirt SIF equations with Newman & Raju SIF equations under tensile loading (a/c=1).

4. New empirical SIF equations and its verification

It seems that Yamada and Hirt SIF equations are not perfect to indicate the finite width correction and finite thickness correction. The modified equation is expressed as:

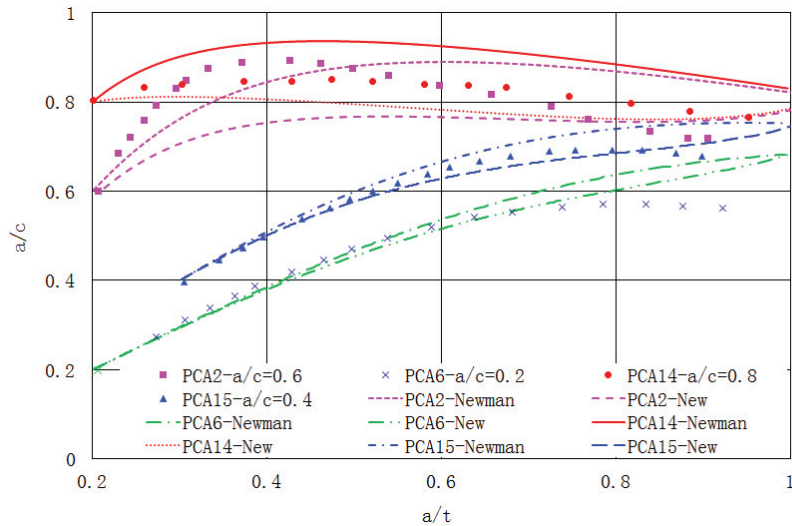
$$K_A = [1.12 - 0.12(\frac{a}{c})] F_G \left(\sqrt{\frac{2t}{\pi a} \tan \frac{\pi a}{2t}} \right)^{0.6 - 0.5(\frac{a}{c})} \sqrt{\sec \left(\frac{\pi c}{W} \left(\frac{a}{t} \right)^{0.5} \right)} \sigma \sqrt{\frac{\pi a}{Q}} \tag{4}$$

$$K_B = [1.12 + 0.08(\frac{a}{c})]F_G \frac{a}{c} \left[1 + \left(1.4 - \frac{a}{c} \right)^{\frac{c}{a}} \left(\frac{a}{t} \right)^2 \right] \sqrt{\sec\left(\frac{\pi c}{W} \left(\frac{a}{t} \right)^{0.5}\right)} \sigma \sqrt{\frac{\pi c}{Q}} \tag{5}$$

where a is crack depth, c is crack length, b is plate width, F_G is correction factor for uneven stress field before crack tip, σ is remote nominal stress, Q is shape factor for elliptical crack (Newman, 1979).

Newman’s finite width correction factor (Newman, 1979) is used for point A and point B. An exponent term is combined with finite thickness correction factor of Yamada and Hirt SIF equations at point A. It ensures that finite correction factor dose not increase too quickly. The finite thickness correction factor for SIF equation at point B includes crack aspect ratio and relative crack depth. A quadratic term of relative crack depth is used in finite thickness correction factor at point B (Eq. 3). It is enough to indicate variation of SIF values due to change of crack depth. The inverse of crack aspect ratio is arranged as exponent term in finite thickness correction factor at point B (Eq. 3). It reduces the speed of increase of SIF values at point B.

The new empirical SIF equations are combined with Yamada and Hirt iterative procedure of FCG. This model is used to calculate data from Putra & Schijve (1992) and McFadyen et al. (1990) experiments. Results from new empirical SIF equation were compared with experimental data and results of Newman and Raju model (1981) (see Fig. 4). The results from new empirical SIF equations agree well with experimental results when initial crack aspect ratio is low for both loading cases (tension or bending). The results from new empirical SIF equations converge quickly. For high crack aspect ratio, the results of new empirical SIF equation or that of Newman & Raju model deviate the experimental results. It may be due to scatter of experimental results.



(a) Experimental data under tensile loading

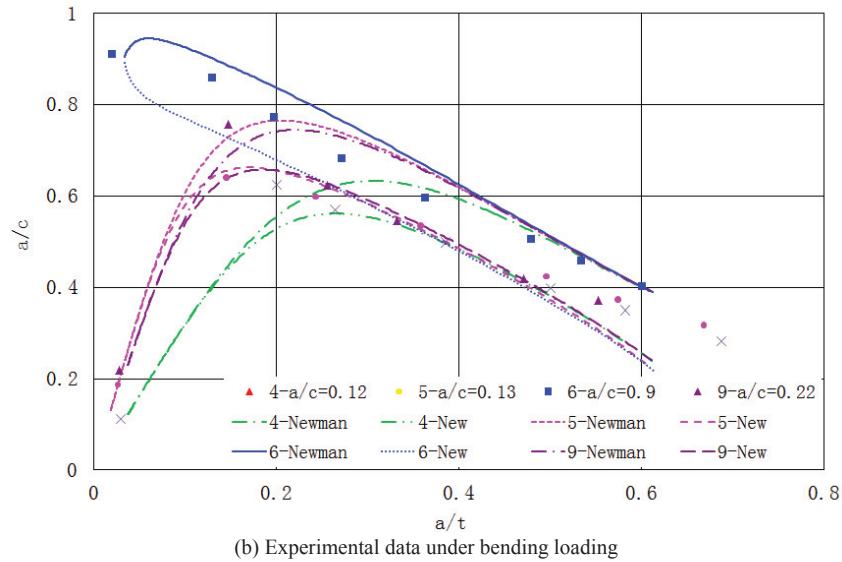


Fig.4 Comparison of variation of crack shape during FCG between experimental data with new empirical SIF equations and Newman & Raju SIF equations under tension and bending load

5. Conclusions

Yamada and Hirt model is one of the commonly used models to predict fatigue life of welded joints. It was found that this model needs improvement at high initial crack aspect ratio or high relative depth ratio. New empirical SIF equations obtained from mathematical judgment were proposed here in this paper for the SIF of a semi-elliptical surface crack within a finite width and thickness in a flat plate. These equations can be applied for crack aspect ratio ranging from 0.2 to 1.0 and relative crack depth less than 0.8. Combining with Yamada and Hirt iterative procedure, the new SIF equations can accurately predict the crack shape change during FCG. It was proved by comparing with experimental data under tension and bending loading. The new SIF equations should be useful for predicting fatigue lives of welded joints.

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