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Robust synchronization of fractional-order unified chaotic systems via linear control

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ABSTRACT

A new scheme for accomplishing synchronization between two fractional-order unified chaotic systems is proposed in this paper. The scheme does not require that the nonlinear dynamics of the synchronization error system must be eliminated. Moreover, the parameter of the systems does not have to be known. A controller is a linear feedback controller, which is simple in implementation. It is designed based on an LMI condition. The LMI condition guarantees that the synchronization between the slave system and the master system is achieved. Numerical simulations are performed to demonstrate the effectiveness of the proposed scheme.

1. Introduction

Fractional calculus is a 300-year-old mathematical topic [1,2]. However, its practical applications have just recently been investigated. Many physics and engineering systems have been found that they display fractional-order dynamics, such as electromagnetic waves [3], electrode–electrolyte polarization [4], dielectric polarization [5], and viscoelastic systems [6]. More recently, the chaotic behavior of fractional-order systems has been investigated [7–15]. Some examples of the systems include the fractional-order Duffing system [8,9], the fractional-order Lorenz system [10], the fractional-order Chen system [11], the fractional-order Lü system [12], the fractional-order unified chaotic system [13,14] and the fractional-order Liu system [15].

Nowadays, whereas chaos synchronization of conventional integer-order chaotic systems has been extensively studied (see, e.g. [16–24]), chaos synchronization of fractional-order chaotic systems is still considered as a challenging research topic. Since theories for analyzing the dynamics of fractional-order systems are still very limited, many recent schemes for chaos synchronization of fractional-order chaotic systems fall back on an active control scheme or its equivalent.

The active control scheme renders the synchronization error system linear by requiring that the nonlinear parts of the error system are eliminated by a controller. Thus, a stability criterion for linear fractional-order systems can be used (see, e.g. [25–32]). However, the scheme results the nonlinear controller that might not be suited for implementation. Moreover, most of them are theoretically valid only when the parameters of the systems are known. However, in many practical situations, the uncertainties of the parameters are not avoidable. The chaos synchronization can be destroyed with the effects of these uncertainties. In order to overcome the problem, adaptive control and robust control are the two candidates that are widely recognized. The main idea of the adaptive control is to utilize an adaptive mechanism to compensate the effects of the uncertainties whereas the robust control results a controller that works independently from the uncertainties. Many adaptive control and robust control strategies have been successfully demonstrated in chaos synchronization of

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conventional integer-order chaotic systems (see, e.g. [19–24]). Sliding-mode control is the robust control scheme that is mostly found in chaos synchronization.

Recently, applications of adaptive control and sliding-mode control in chaos synchronization of fractional-order chaotic systems have been investigated [33–37]. The adaptive control typically utilizes the adaptive law that is also fractional and the sliding-model control usually makes use of the sliding manifold containing fractional integral. In practice, however, fractional-order adaptive controllers and fractional-order sliding-model controllers might be too complicated for implementation.

This paper proposes a new scheme for chaos synchronization of two fractional-order unified chaotic systems. An LMI condition is utilized to obtain a linear controller. The proposed scheme does not require that the nonlinear dynamics of the synchronization error system are directly eliminated by the controller and that the system's parameter must be known. The rest of the paper is organized as follows. In Section 2, some preliminaries are introduced. In Section 3, the fractional-order unified chaotic system and its synchronization are presented. Numerical simulations are given in Section 4 to illustrate the effectiveness of the presented scheme. In Section 5 some conclusions are drawn.

2. Preliminaries

In the first part of this section some definitions are briefly reviewed. The reader is referred to [1,2] for more details. It then follows with an LMI condition theorem, which will be utilized later to design a robust linear control law.

The frequently used definitions for fractional derivatives are Riemann–Liouville, Grünwald–Letnikov, and Caputo definitions. The Riemann–Liouville definition is given as

$$D^q f(t) = \frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-q)}\int_0^t \frac{f(\tau)}{(t-\tau)^{q-m+1}}d\tau\right] \tag{1}$$

where m is the integer that $m-1 < q < m$ and $\Gamma(\cdot)$ is the Gamma function. The Grünwald–Letnikov definition can be written as

$$D^q f(t)|_{t=kh} = \lim_{h\to 0}\frac{1}{h^q}\sum_{j=0}^{k}(-1)^j\binom{q}{j} f(kh-jh) \tag{2}$$

where $\binom{q}{j} = \frac{q(q-1)(q-2)\cdots(q-j+1)}{j!}$ is the usual notation for the binomial coefficients. The Caputo definition is described by

$$D^q f(t) = \frac{1}{\Gamma(m-q)}\int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{q-m+1}}d\tau \tag{3}$$

where m is the integer that $m-1 < \alpha < m$ and $\Gamma(\cdot)$ is the Gamma function. These three definitions are equivalent for a wide class of functions.

Fractional-order systems can be considered as a generalization of integer-order systems. They can be described as

$$\begin{aligned}
&D^{q_1}x_1(t) = f_1(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), \ldots, u_m(t))\\
&D^{q_2}x_2(t) = f_2(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), \ldots, u_m(t))\\
&\vdots\\
&D^{q_n}x_n(t) = f_n(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), \ldots, u_m(t))
\end{aligned} \tag{4}$$

where $x_1, x_2, \ldots, x_n$ are the state variables, $u_1, u_2, \ldots, u_m$ are the input variables, and $q_1, q_2, \ldots, q_n$ are fractional orders. The system is called a commensurate-order system if $q_1 = q_2 = \cdots = q_n = q$. The vector representation of the commensurate-order system (4) can be expressed as

$$D^q x = f(x, u), \tag{5}$$

where $x = [x_1, x_2, \ldots, x_n]^T$ and $u = [u_1, u_2, \ldots, u_m]^T$ are the state vector and the input vector, respectively, and q is the fractional commensurate order. The linear time-invariant (LTI) version of the system (5) is written as

$$D^q x = Ax + Bu. \tag{6}$$

The system (6) with $u = Kx$ is asymptotically stable if and only if $|\arg(\lambda)| > q\frac{\pi}{2}$ for all eigenvalues λ of $(A+BK)$ [38–40]. A fractional-order LTI interval system is described as [41]

$$D^q x = \breve{A}x + \breve{B}u \tag{7}$$

where the system matrices $\breve{A}$ and $\breve{B}$ are interval uncertain satisfying

$$\breve{A} \in [A^l, A^u] = \{[a_{ij}] : a^l_{ij} \le a_{ij} \le a^u_{ij},\ 1 \le i,j \le n\}$$

$$\breve{B} \in [B^l, B^u] = \{[b_{ij}] : b^l_{ij} \le b_{ij} \le b^u_{ij},\ 1 \le i,j \le m\}.$$

The following notations are used in the following theorem:

$$A_0 = \frac{1}{2}(A^l + A^u)$$
$$\Delta A = \frac{1}{2}(A^l - A^u) = [\gamma_{ij}]$$
$$B_0 = \frac{1}{2}(B^l + B^u)$$
$$\Delta B = \frac{1}{2}(B^l - B^u) = [\beta_{ij}]^T$$
$$D_A = \left[\sqrt{\gamma_{11}}e_1^n \cdots \sqrt{\gamma_{1n}}e_1^n \cdots \sqrt{\gamma_{n1}}e_n^n \cdots \sqrt{\gamma_{nn}}e_n^n\right]$$
$$E_A = \left[\sqrt{\gamma_{11}}e_1^n \cdots \sqrt{\gamma_{1n}}e_n^n \cdots \sqrt{\gamma_{n1}}e_1^n \cdots \sqrt{\gamma_{nn}}e_n^n\right]^T$$
$$D_B = \left[\sqrt{\beta_{11}}e_1^n \cdots \sqrt{\beta_{1m}}e_1^n \cdots \sqrt{\beta_{n1}}e_n^n \cdots \sqrt{\beta_{nm}}e_n^n\right]$$
$$E_B = \left[\sqrt{\beta_{11}}e_1^m \cdots \sqrt{\beta_{1m}}e_m^m \cdots \sqrt{\beta_{n1}}e_1^m \cdots \sqrt{\beta_{nm}}e_m^m\right]^T$$

where e_i^p is the p-column vector with the ith element being 1 and all the other being 0.

Theorem 1 *([41]). The interval system (7) with input $u = Kx$ and $0 < q < 1$ is asymptotically stabilizable if there are a $m \times n$ real matrix X, a $n \times n$ symmetric positive-definite real matrix Q, and four real positive scalars $\alpha_1, \alpha_2, \beta_1, \beta_2$ such that*

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^T & \Gamma_{22} \end{bmatrix} < 0 \tag{8}$$

where

$$\Gamma_{11} = \sum_{i=1}^{2} Sym\{\Theta_{i1} \otimes (A_0Q + B_0X)\} + \sum_{i=1}^{2} \alpha_i(I_2 \otimes D_AD_A^T) + \sum_{i=1}^{2} \beta_i(I_2 \otimes D_BD_B^T)$$
$$\Gamma_{12} = [I_2 \otimes (E_AQ)^T \quad I_2 \otimes (E_AQ)^T \quad I_2 \otimes (E_BX)^T \quad I_2 \otimes (E_BX)^T]$$
$$\Gamma_{22} = -\mathrm{diag}(\alpha_1, \alpha_2, \beta_1, \beta_2) \otimes I_{2n}$$
$$\Theta_{11} = \begin{bmatrix} \sin(\theta) & -\cos(\theta) \\ \cos(\theta) & \sin(\theta) \end{bmatrix}, \quad \theta = q\frac{\pi}{2}$$
$$\Theta_{12} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$
$$\Theta_{21} = \begin{bmatrix} \sin(\theta) & \cos(\theta) \\ -\cos(\theta) & \sin(\theta) \end{bmatrix}$$
$$\Theta_{22} = \begin{bmatrix} -\cos(\theta) & \sin(\theta) \\ -\sin(\theta) & -\cos(\theta) \end{bmatrix}$$
$$\mathrm{Sym}\{Z\} = Z^T + Z$$

and $\otimes$ is the Kronecker product.

Moreover, a stabilization feedback gain matrix is given by

$$K = XQ^{-1}. \tag{9}$$

The reader is referred to [41] for the proof of Theorem 1. Note that the condition (8) given in the theorem is an LMI in $X, Q, \alpha_1, \alpha_2, \beta_1, \beta_2$ and it can be easily solved by various LMI solvers such as MATLAB's LMI Control Toolbox.

3. Fractional-order unified chaotic system and its synchronization

Similar to the classical unified chaotic system, the fractional-order unified system could be considered as the system that bridges the gap among the fractional-order Lorenz system, the fractional-order Lü system, and the fractional-order Chen system. The fractional-order unified chaotic system [13,14] is described by

$$\begin{aligned} D^qx &= (25\alpha + 10)(y - x), \\ D^qy &= (28 - 35\alpha)x + (29\alpha - 1)y - xz, \\ D^qz &= xy - \left(\frac{\alpha + 8}{3}\right)z, \end{aligned} \tag{10}$$

where $\alpha \in [0, 1]$ is the parameter of the system and $0 < q < 1$ is the fractional commensurate order. Some examples of the chaotic attractor of the system (10) with $q = 0.95$ are shown in Fig. 1.

Two systems in synchronization are called the master system and the slave system respectively. From (10), the master system and the slave system can be expressed, respectively, as

$$\begin{aligned} D^q x_m &= (25\alpha + 10)(y_m - x_m), \\ D^q y_m &= (28 - 35\alpha)x_m + (29\alpha - 1)y_m - x_m z_m, \\ D^q z_m &= x_m y_m - \left(\frac{\alpha + 8}{3}\right) z_m, \end{aligned} \tag{11}$$

and

$$\begin{aligned} D^q x_s &= (25\alpha + 10)(y_s - x_s) + u_1, \\ D^q y_s &= (28 - 35\alpha)x_s + (29\alpha - 1)y_s - x_s z_s + u_2, \\ D^q z_s &= x_s y_s - \left(\frac{\alpha + 8}{3}\right) z_s + u_3, \end{aligned} \tag{12}$$

where the subscripts m and s stand for the master and the slave, respectively, and u_1, u_2, u_3 are the control signals used to drive the slave system to follow the master system.

By defining the synchronization errors as $e_1 = x_s - x_m$, $e_2 = y_s - y_m$ and $e_3 = z_s - z_m$ and using Eqs. (11) and (12), the synchronization error system can be written as

$$\begin{aligned} D^q e_1 &= (25\alpha + 10)(e_2 - e_1) + u_1, \\ D^q e_2 &= (28 - 35\alpha)e_1 + (29\alpha - 1)e_2 - z_m e_1 - x_s e_3 + u_2, \\ D^q e_3 &= y_m e_1 + x_s e_2 - \left(\frac{\alpha + 8}{3}\right) e_3 + u_3 \end{aligned} \tag{13a}$$

or

$$D^q \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{13b}$$

where $a_{11} = -25\alpha - 10$, $a_{12} = 25\alpha + 10$, $a_{21} = 28 - 35\alpha - z_m$, $a_{22} = 29\alpha - 1$, $a_{23} = -x_s$, $a_{31} = y_m$, $a_{32} = x_s$, and $a_{33} = -\left(\frac{\alpha+8}{3}\right)$. Due to the attractiveness of the attractors of the fractional-order unified chaotic system, there exist X, Y and Z such that $|x_m| \le X$, $|y_m| \le Y$ and $|z_m| \le Z$. It also assumes that $|x_s| \le X$, $|y_s| \le Y$ and $|z_s| \le Z$. Since $0 \le \alpha \le 1$, it results $-35 \le a_{11} \le -10$, $10 \le a_{12} \le 35$, $-7 - Z \le a_{21} \le 28 + Z$, $-1 \le a_{22} \le 28$, $-X \le a_{23} \le X$, $-Y \le a_{31} \le Y$, $-X \le a_{32} \le X$, and $-3 \le a_{33} \le -8/3$. Thus, the error system (13a) can be written in the linear interval system form (7) as

$$D^q x = \breve{A} x + \breve{B} u \tag{14}$$

where $x = [e_1\ e_2\ e_3]^T$, $u = [u_1\ u_2\ u_3]^T$ and the lower/upper boundaries of $\breve{A}$ and $\breve{B}$ are

$$\begin{aligned} A^l &= \begin{bmatrix} -35 & 10 & 0 \\ -7 - Z & -1 & -X \\ -Y & -X & -3 \end{bmatrix}, \\ A^u &= \begin{bmatrix} -10 & 35 & 0 \\ 28 + Z & 28 & X \\ Y & X & -8/3 \end{bmatrix}, \\ B^l = B^u &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned}$$

By assuming the LMI stabilization condition in Theorem 1 is fulfilled, the asymptotic convergence of the synchronization errors is guaranteed. Therefore, the slave system is asymptotically synchronized with the master system. Note that the parameter α does not appear in (14). Therefore, the value of the parameter α does not have to be known.

4. Numerical simulations

In this section, computer simulations are utilized to demonstrate the effectiveness of the proposed LMI-based scheme. The order of the systems is selected as $q = 0.95$. The values of X, Y and Z estimated through simulations are found to be 30, 30, and 50, respectively. By solving the LMI (8) of the system (14), the following solution is obtained:

$$X = \begin{bmatrix} -694.5 & -36.4 & 0 \\ -36.4 & -3247.7 & 0 \\ 0 & 0 & -1724.7 \end{bmatrix}$$

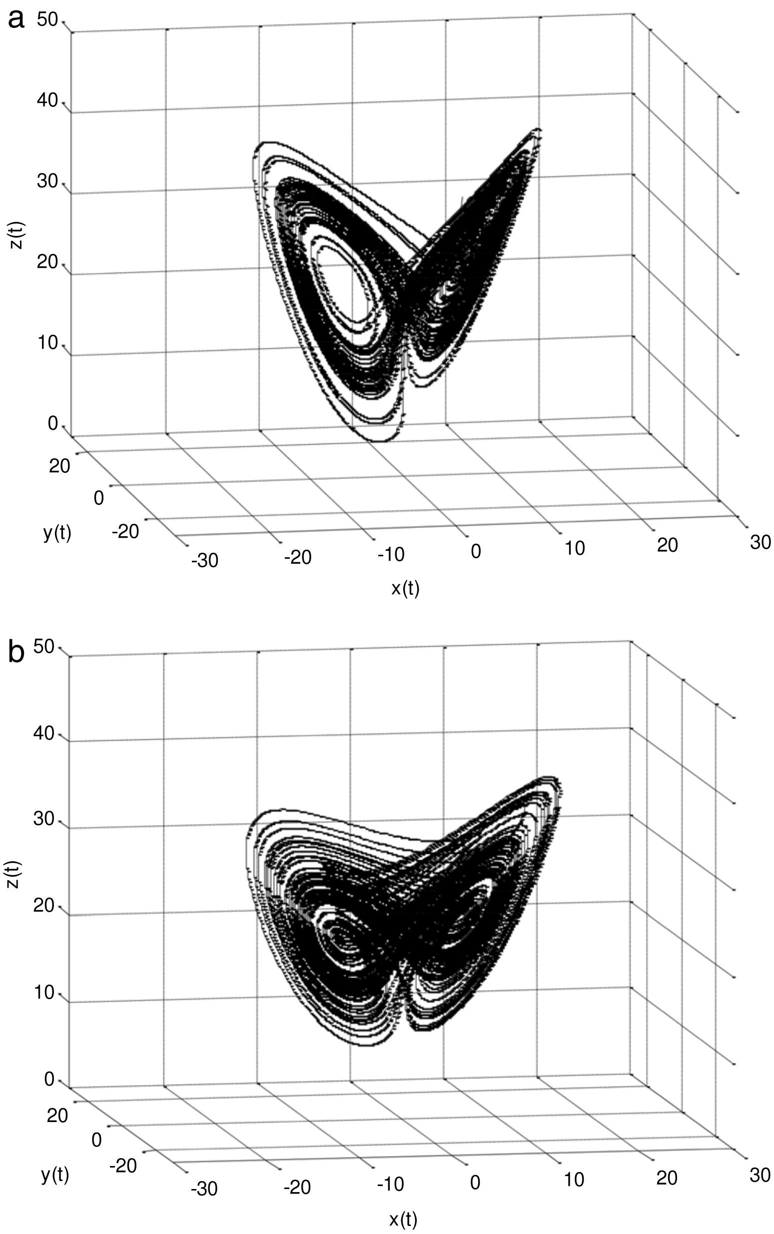

Fig. 1. The fractional-order unified chaotic system with $q = 0.95$: (a) $\alpha = 0.2$, (a) $\alpha = 1.0$.

$$Q = \begin{bmatrix} 1.5546 & 0 & 0 \\ 0 & 2.5058 & 0 \\ 0 & 0 & 3.5630 \end{bmatrix}$$

$$\alpha_1 = 56.91, \qquad \alpha_2 = 56.91, \qquad \beta_1 = 55.40, \qquad \beta_2 = 55.40$$

which yields

$$K = \begin{bmatrix} -446.8 & -14.5 & 0 \\ -23.4 & -1296.1 & 0 \\ 0 & 0 & -484.1 \end{bmatrix}.$$

Therefore, the control law

$$\begin{aligned} u_1 &= -446.8e_1 - 14.5e_2 \\ u_2 &= -23.4e_1 - 1296.1e_2 \\ u_3 &= -484.1e_3 \end{aligned} \tag{15}$$

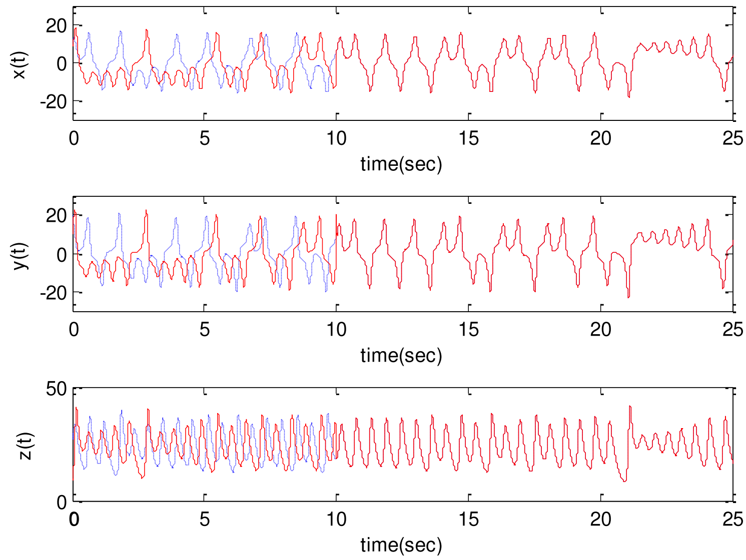

Fig. 2. Synchronization of the fractional-order unified chaotic system with $\alpha = 0.2$ (red solid line represents slave system, blue dotted line represents master system). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

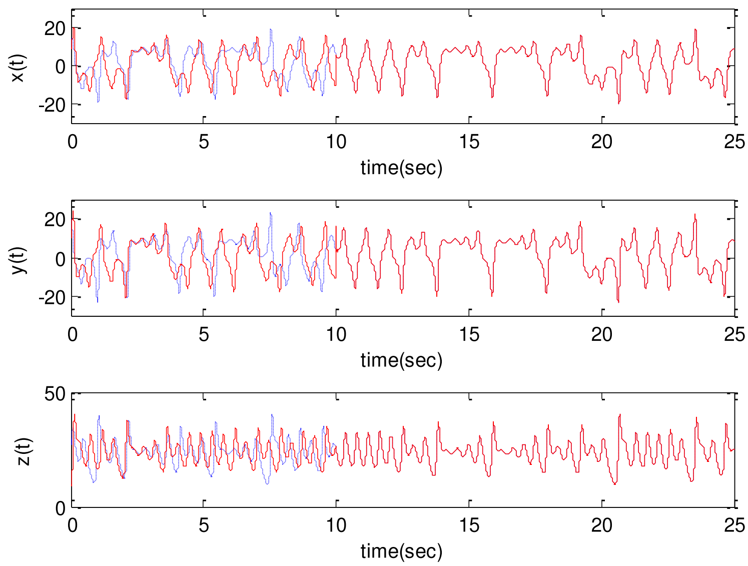

Fig. 3. Synchronization of the fractional-order unified chaotic system with $\alpha = 0.5$ (red solid line represents slave system, blue dotted line represents master system). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

guarantees the asymptotic convergence of the synchronization errors between the master system (11) and the slave system (12). The state responses of the systems with the control law (15) for the various values of the parameter α are shown in Figs. 2–5. It is found that the controller drives the slave system to follow the master system as desired for all cases. Note that the controller in all cases is activated at time $=$ 10 s.

Remark. In comparison to the sliding-mode control law proposed in [36] and the adaptive control law proposed in [34], the control law (15) is much simpler and easier for implementation. Moreover, the control law (15) is independent from the parameter α.

5. Conclusions

In this paper, an alternative solution for robust synchronization of the fractional-order unified chaotic systems has been proposed. The solution is achieved by utilizing the recently developed LMI stabilization condition introduced in [41]. In comparison to existing schemes, the proposed scheme is very simple and does not require that nonlinear parts of

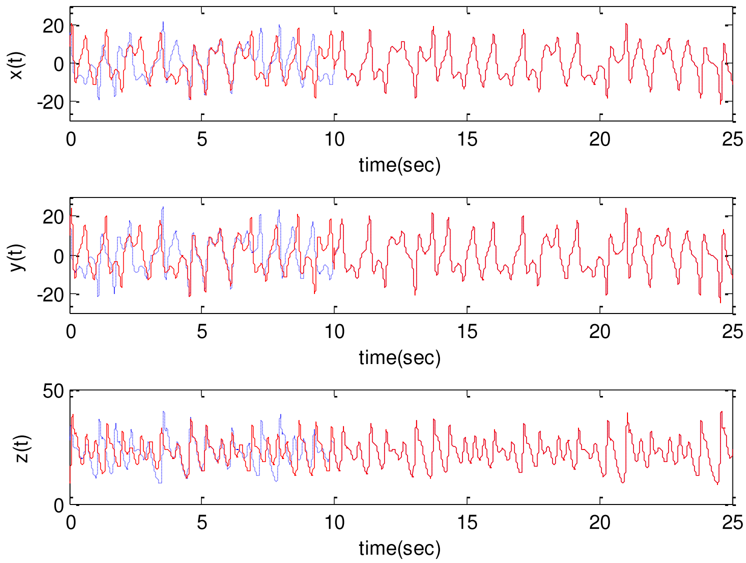

Fig. 4. Synchronization of the fractional-order unified chaotic system with $\alpha = 0.8$ (red solid line represents slave system, blue dotted line represents master system). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

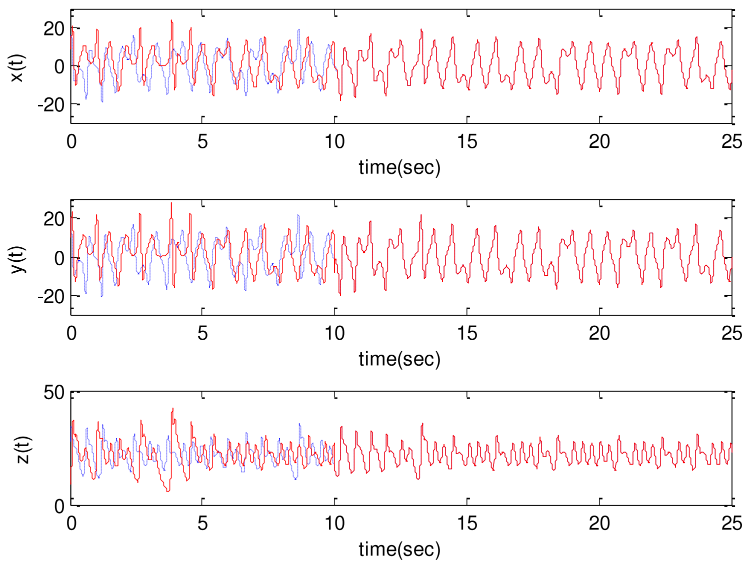

Fig. 5. Synchronization of the fractional-order unified chaotic system with $\alpha = 1.0$ (red solid line represents slave system, blue dotted line represents master system). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

synchronization error dynamics are directly eliminated by the controller and that the parameter α of the systems must be known. Moreover, the controller is linear with constant gains, which is simple in implementation. Computer simulations for various values of the parameter α show that the proposed scheme is effective. For all cases, the controller is able to drive the states of the slave system to asymptotically synchronize the states of the master system as desired.

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