Physics Letters B 739 (2014) 133–138

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Dispersive evaluation of the D-term form factor in deeply virtual Compton scattering

B. Pasquini [a,b,*], M.V. Polyakov [c], M. Vanderhaeghen [d,e]

[a] Dipartimento di Fisica, Università degli Studi di Pavia, Pavia, Italy
[b] Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy
[c] Institute of Theoretical Physics II, Ruhr University Bochum, Bochum D44780, Germany
[d] Institut für Kernphysik, Johannes Gutenberg-Universität, Mainz, Germany
[e] PRISMA Cluster of Excellence, Johannes Gutenberg-Universität, Mainz, Germany

ARTICLE INFO

Article history:
Received 7 August 2014
Received in revised form 19 October 2014
Accepted 20 October 2014
Available online 23 October 2014
Editor: J.-P. Blaizot

Keywords:
Dispersion relations
Deeply virtual Compton scattering
Generalized parton distributions

ABSTRACT

We present a dispersive representation of the D-term form factor for hard exclusive reactions, using unsubtracted t-channel dispersion relations. The t-channel unitarity relation is saturated with the contribution of two-pion intermediate states, using the two-pion distributions amplitude for the $\gamma^*\gamma \to \pi\pi$ subprocess and reconstructing the $\pi\pi \to N\bar{N}$ subprocess from available information on pion-nucleon partial-wave helicity amplitudes. Results for the D-term form factor as function of t as well as at $t = 0$ are discussed in comparison with available model predictions and phenomenological parametrizations.

1. Introduction

The D term was originally introduced to complete the parametrization of the generalized parton distributions (GPDs) in hard exclusive reactions in terms of double distributions, and restore the polynomiality property of the singlet moments of unpolarized GPDs [1]. This term turned out to be a crucial contribution in the phenomenological description of deeply virtual Compton scattering (DVCS) observables, where different forms have been assumed with parameters tuned to DVCS data [2,3]. On the theoretical side, the D-term is poorly known, and information is available only from a few models, such as the chiral quark soliton model [4–8], the Skyrme model [9], a Regge-improved diquark model [10], as well as a first moment from lattice simulations [11,12]. Interesting studies have been also performed for the nucleon in nuclear matter [13,14], for Q-ball systems [15,16] and within different variants of chiral perturbation theory [17–23]. Recently, the D-term form factor acquired a new significance in the dispersive representation of DVCS amplitudes [3,24–31]. In particular, it was shown that the DVCS amplitudes satisfy subtracted dispersion relations (DRs) at fixed t with the subtraction function defined by the D-term form factor [26]. In the present Letter we set up dispersion relations in the t channel for this subtraction function. The advantage of this dispersive representation is to provide a microscopic interpretation of the physical content of the D-term form factor in terms of t-channel exchanges with the appropriate quantum numbers. The plan of the Letter is as follows. In Section 2, we review the derivation of the s-channel subtracted dispersion relations for the DVCS amplitudes. In Section 3, we derive t-channel DRs for the D-term form factor. The unitarity relation for the t-channel amplitudes is saturated with two-pion intermediate states, using the two-pion distribution amplitude for the $\gamma^*\gamma \to \pi\pi$ subprocess and reconstructing the $\pi\pi \to N\bar{N}$ subprocess from available information on pion-nucleon partial-wave helicity amplitudes. We then discuss the dispersive predictions for the D-term form factor in Section 4, and we conclude summarizing our results.

2. Subtracted dispersion relations in the s-channel

We consider the DVCS process

$$\gamma^*(q)N(p) \to \gamma(q')N(p'), \tag{1}$$

where the variables in brackets denote the four-momentum of the participating particles. The familiar Mandelstam variables are

$$s = (p+q)^2, \qquad t = (q-q')^2, \qquad u = (q-p')^2, \tag{2}$$

* Corresponding author.
E-mail address: pasquini@pv.infn.it (B. Pasquini).

http://dx.doi.org/10.1016/j.physletb.2014.10.047

and they are constrained by $s+u+t=2M_N^2-Q^2$, with M_N the nucleon mass and $Q^2=-q^2$. We will consider the Bjorken regime, where the photon virtuality Q^2 and s are large, and $-t \ll s, Q^2$.

To calculate the DVCS amplitude, one starts from its definition as a nucleon matrix element of the T-product of two electromagnetic currents:

$$H^{\mu\nu}_{\lambda'_N,\lambda_N} = -i \int \mathrm{d}^4 x\, e^{-i(q\cdot x)} \langle N(p',\lambda'_N) \big| T\big[J^\mu(x) J^\nu(0)\big] \big| N(p,\lambda_N)\rangle, \tag{3}$$

where the four-vector index μ (ν) refers to the virtual (real) photon, and λ_N (λ'_N) is the helicity of the incoming (outgoing) nucleon. The DVCS amplitude is obtained from the DVCS tensor in Eq. (3) by contracting it with the photon polarization vectors as

$$T_{\lambda'_\gamma\lambda'_N,\lambda_\gamma\lambda_N} = \varepsilon_\mu(q,\lambda_\gamma)\varepsilon^*_\nu(q',\lambda'_\gamma) H^{\mu\nu}_{\lambda'_N,\lambda_N}, \tag{4}$$

where λ_γ (λ'_γ) denotes the helicity of virtual (real) photons respectively.

The DVCS amplitude for unpolarized nucleon and at leading order in Q can be parametrized as

$$\begin{aligned} T_{\lambda'_\gamma\lambda'_N,\lambda_\gamma\lambda_N} &= \varepsilon_\mu(q,\lambda_\gamma)\varepsilon^*_\nu(q',\lambda'_\gamma)\frac{(-g_\perp^{\mu\nu})}{2} \\ &\quad \times \Big[\bar u(p',\lambda'_N)\gamma\cdot n u(p,\lambda_N)\sum_q e_q^2 C^q \\ &\quad - \bar u(p',\lambda'_N) u(p,\lambda_N)\frac{1}{M_N}\sum_q e_q^2 F^q\Big], \end{aligned} \tag{5}$$

where we introduced the lightlike vector $n^\mu = 1/(\sqrt{2}P^+)(1,0,0,-1)$, with $P=(p+p')/2$, and the symmetric tensor $g_\perp^{\mu\nu} = g^{\mu\nu} - n^\mu \tilde p^\nu - n^\nu \tilde p^\mu$, with $\tilde p^\mu = P^+/\sqrt{2}(1,0,0,1)$. Furthermore, the light-front component for a generic four-vector a^μ is defined as $(a^0+a^3)/\sqrt{2}$. In Eq. (5), the invariant amplitudes C^q and F^q are given by

$$\begin{aligned} C^q(\xi,t) &= \int_0^1 \mathrm{d}x\big[H^{(+)}(x,\xi,t)+E^{(+)}(x,\xi,t)\big] \\ &\quad \times \left[\frac{1}{x-\xi+i\epsilon}+\frac{1}{x+\xi-i\epsilon}\right] \\ &= \int_{-1}^1 \mathrm{d}x \frac{H^{(+)}(x,\xi,t)+E^{(+)}(x,\xi,t)}{x-\xi+i\epsilon}, \end{aligned} \tag{6}$$

$$\begin{aligned} F^q(\xi,t) &= \int_0^1 \mathrm{d}x E^{(+)}(x,\xi,t)\left[\frac{1}{x-\xi+i\epsilon}+\frac{1}{x+\xi-i\epsilon}\right] \\ &= \int_{-1}^1 \mathrm{d}x \frac{E^{(+)}(x,\xi,t)}{x-\xi+i\epsilon}, \end{aligned} \tag{7}$$

with the skewedness variable defined as $\xi = Q^2/(2s+Q^2)$. $H^{(+)}(x,\xi,t) = H^q(x,\xi,t) - H^q(-x,\xi,t)$ denotes the singlet ($C=+1$) combination of nucleon helicity-conserving GPDs, and analogously for the nucleon helicity-flip GPD $E^{(+)}$. The invariant amplitudes and the GPDs in Eqs. (6) and (7) depend also on the renormalization scale μ^2 which is not explicitly displayed and it is identified with the hard scale of the process Q^2. In the following we will consider the invariant amplitude F^q in the $\nu - t$ plane at fixed Q^2, with $\nu = (s-u)/4M_N = Q^2/4M_N\xi$. In this plane, F^q satisfies the following fixed-t subtracted relation [26,29]

$$F^q(\nu,t) = F^q(0,t) + \frac{\nu^2}{\pi}\int_{\nu_0}^\infty \frac{\mathrm{d}\nu'^2}{\nu'^2}\frac{\operatorname{Im} F^q(\nu',t)}{\nu'^2-\nu^2}, \tag{8}$$

where the lower limit of integration is $\nu_0 = Q^2/4M_N$ and the nucleon pole term residing in this point may be considered separately. Following Refs. [26,28], we can relate the subtraction function $F^q(0,t)$ to the D-term form factor $D^q(t)$ [1] as follows

$$F^q(0,t) = 2\int_{-1}^{+1}\mathrm{d}z \frac{D^q(z,t)}{1-z} = 4D^q(t). \tag{9}$$

The dispersive representation for the D-term form factor $D^q(t)$ of Eq. (9) is obtained by applying unsubtracted DRs, this time in the variable t:

$$F^q(0,t) = \frac{1}{\pi}\int_{4m_\pi^2}^{+\infty}\mathrm{d}t' \frac{\operatorname{Im}_t F^q(0,t')}{t'-t} + \frac{1}{\pi}\int_{-\infty}^{-a}\mathrm{d}t' \frac{\operatorname{Im}_t F^q(0,t')}{t'-t}. \tag{10}$$

The imaginary part in the integral from $4m_\pi^2 \to +\infty$ in Eq. (10) is saturated by the possible intermediate states for the t-channel process, which lead to cuts along the positive-t axis. For low values of t, the t-channel discontinuity is dominated by $\pi\pi$ intermediate states. The second integral in Eq. (10) extends from $-\infty$ to $-a = -2(m_\pi^2 + 2M_N m_\pi) - Q^2$. As we are interested in evaluating Eq. (10) for large Q^2 values and small (negative) values of t ($|t| \ll a$), the integral from $-\infty \to -a$ is suppressed, and will be neglected in this work. Consequently, we shall saturate the integral in Eq. (10) by the contribution of $\pi\pi$ intermediate states, which turns out to be a good approximation for small t.

Using the expansion of the D-term $D(z,t)$ in Eq. (9) in terms of Gegenbauer polynomials C_k^ν for $\nu = 3/2$, the solutions of the leading-order ERBL evolution equations, one obtains the following series for the D-term form factor

$$D^q(t) = \sum_{\substack{n=1\\ \text{odd}}}^\infty d_n^q(t). \tag{11}$$

In the following, we will explicitly evaluate the contribution from the $n=1$ term in (11).

3. t-channel dispersion relations for the D-term form factor

The invariant amplitudes $F^q(\nu,t)$ and $C^q(\nu,t)$ are related to the t-channel helicity amplitude by [32,33]

$$\begin{aligned} T^t_{\lambda_{\bar N}\lambda_N,\lambda_\gamma\lambda'_\gamma} &= \varepsilon_\mu(q_t,\lambda_\gamma)\varepsilon_\nu(q'_t,\lambda'_\gamma) T^{t\mu\nu}_{\lambda_{\bar N}\lambda_N} \\ &= \varepsilon_\mu(q_t,\lambda_\gamma)\varepsilon_\nu(q'_t,\lambda'_\gamma)\frac{(-g_\perp^{\mu\nu})}{2} \\ &\quad \times \Big[\bar u(p_t,\lambda_N)\gamma^+ v(p'_t,\lambda_{\bar N})\frac{1}{\tilde\Delta^+}\sum_q e_q^2 C^q \\ &\quad - \bar u(p_t,\lambda_N) v(p'_t,\lambda_{\bar N})\frac{1}{M_N}\sum_q e_q^2 F^q\Big], \end{aligned} \tag{12}$$

where $\tilde\Delta^+ = \dfrac{p'^+_t - p^+_t}{2}$, and the hadronic tensor $T^{t\mu\nu}_{\lambda_{\bar N}\lambda_N}$ is defined as

$$T^{t\mu\nu}_{\lambda_{\bar N}\lambda_N} = -i\int d^4x e^{-i(q\cdot x)}\langle N(p_t,\lambda_N),\bar N(p'_t,\lambda_{\bar N})| \times T\big[J^\mu(x)J^\nu(0)\big]|0\rangle. \quad (13)$$

In the c.m. system of the t-channel process $\gamma^*\gamma\to N\bar N$ we choose the real photon momentum q'_t (helicity λ'_γ) to point in the z direction and the nucleon momentum p_t in the xz plane at an angle θ_t with respect to the z axis, i.e. $p^\mu_t = (E, p_t\sin\theta_t, 0, p_t\cos\theta_t)$ with $p_t = |\vec p_t| = \sqrt{t/4 - M_N^2}$. In this framework, the t-channel helicity amplitude in Eq. (12) can be written as

$$\begin{aligned} T^t_{\lambda_{\bar N}\lambda_N,\lambda_\gamma\lambda'_\gamma} &= \delta_{\lambda_\gamma\lambda'_\gamma}\delta_{\lambda_N\lambda_{\bar N}}\Bigg[(-1)^{1/2+\lambda_N}\frac{M_N}{\sqrt{2}\tilde\Delta^+}\cos\theta_t\sum_q e_q^2 C^q \\ &\quad + (-1)^{1/2+\lambda_N}\sqrt{\frac{t}{4M_N^2}-1}\sum_q e_q^2 F^q\Bigg] \\ &\quad + \delta_{\lambda_\gamma\lambda'_\gamma}\delta_{-\lambda_N\lambda_{\bar N}}\frac{\sqrt{t}}{2\sqrt{2}\tilde\Delta^+}\sin\theta_t\sum_q e_q^2 C^q. \end{aligned} \quad (14)$$

Since the dispersion integral in Eq. (10) runs along the line $\nu = 0$, we are interested to $\mathrm{Im}_t\, F^q(0,t)$ in Eq. (14). The relation between the scattering angle in the t-channel and the invariant ν and t is $\cos\theta_t = 4M_N\nu/[\beta_N(t+Q^2)]$ with $\beta_N = \sqrt{1-4M_N^2/t}$. Therefore $\nu = 0$ corresponds to 90° scattering for the t-channel process. In this limit, the relations (14) reduce to

$$T^t_{1/2\,1/2,11}\big(t,\theta_t = 90^\circ\big) = -\sqrt{\frac{t}{4M_N^2}-1}\sum_q e_q^2 F^q(0,t), \quad (15)$$

$$T^t_{1/2\,-1/2,11}\big(t,\theta_t = 90^\circ\big) = \frac{\sqrt{t}}{2\sqrt{2}\tilde\Delta^+}\sum_q e_q^2 C^q(0,t). \quad (16)$$

The imaginary part of the t-channel Compton amplitude is determined by using unitarity relation, and taking into account the dominant contribution coming from $\pi\pi$ intermediate states. Following the derivation in Appendix B of Ref. [34], we start by decomposing the t-channel helicity amplitude for $\gamma^*\gamma\to\bar N N$ into a partial wave series,

$$T^t_{\lambda_{\bar N}\lambda_N,\lambda_\gamma\lambda'_\gamma}(\nu,t) = \sum_J \frac{2J+1}{2}T^{J(\gamma^*\gamma\to N\bar N)}_{\lambda_N\lambda_{\bar N},\lambda'_\gamma\lambda_\gamma}(t)\, d^J_{\Lambda_N\Lambda_\gamma}(\theta_t), \quad (17)$$

where $\Lambda_\gamma = \lambda'_\gamma - \lambda_\gamma$, $\Lambda_N = \lambda_N - \lambda_{\bar N}$, and $d^J_{\Lambda_N\Lambda_\gamma}$ are Wigner d-functions. The unitarity relation reads

$$2\,\mathrm{Im}\, T^{\gamma^*\gamma\to N\bar N} = \frac{1}{(4\pi)^2}\frac{p_\pi}{\sqrt{t}}\int d\Omega_\pi\big[T^{\gamma^*\gamma\to\pi\pi}\big]\cdot\big[T^{\pi\pi\to N\bar N}\big]^*, \quad (18)$$

where $p_\pi = |\vec p_\pi| = \sqrt{t/4 - m_\pi^2}$ is the c.m. momentum of the pion. The partial wave expansion for $\gamma^*\gamma\to\pi\pi$ reads

$$T^{\gamma^*\gamma\to\pi\pi}_{\Lambda_\gamma}(t,\theta_{\pi\pi}) = \sum_{\substack{J=0\\ \text{even}}}\frac{2J+1}{2}T^{J(\gamma^*\gamma\to\pi\pi)}_{\Lambda_\gamma}(t) \times\sqrt{\frac{(J-\Lambda_\gamma)!}{(J+\Lambda_\gamma)!}}\cdot P^{\Lambda_\gamma}_J(\cos\theta_{\pi\pi}). \quad (19)$$

Furthermore, the partial wave expansion for $\pi\pi\to N\bar N$ reads

$$T^{\pi\pi\to N\bar N}_{\Lambda_N}(t,\Theta) = \sum_J \frac{2J+1}{2}T^{J(\pi\pi\to N\bar N)}_{\Lambda_N}(t) \times\sqrt{\frac{(J-\Lambda_N)!}{(J+\Lambda_N)!}}\cdot P^{\Lambda_N}_J(\cos\Theta). \quad (20)$$

Combining Eqs. (19) and (20), we can now construct the imaginary parts of the Compton t-channel partial waves,

$$2\,\mathrm{Im}\, T^{J(\gamma^*\gamma\to N\bar N)}_{\lambda_{\bar N}\lambda_N,\lambda_\gamma\lambda'_\gamma}(t) = \frac{1}{(8\pi)}\frac{p_\pi}{\sqrt{t}}\big[T^{J(\gamma^*\gamma\to\pi\pi)}_{\Lambda_\gamma}(t)\big]\big[T^{J(\pi\pi\to N\bar N)}_{\Lambda_N}(t)\big]^*. \quad (21)$$

For the calculation of $\mathrm{Im}\, F^q(0,t)$ from Eq. (15), we should consider Eq. (21) for $\Lambda_\gamma = 0$ and $\Lambda_N = 0$.

The partial wave amplitudes $T^{J(\pi\pi\to N\bar N)}_{\Lambda_N=0}$ of Eq. (20) are related to the amplitudes $f^J_+(t)$ of Frazer and Fulco [35] by the relation

$$T^{J(\pi\pi\to N\bar N)}_{\Lambda_N=0}(t) = \frac{16\pi}{p_t}(p_t p_\pi)^J f^J_+(t).$$

The reaction $\gamma^*\gamma\to\pi\pi$ at large Q^2 and small t can be described in a factorized form [32,36], as the convolution of a short-distance contribution, $\gamma^*\gamma\to q\bar q$, perturbatively calculable, and nonperturbative matrix elements describing the exclusive fragmentation of a $q\bar q$ pair into two-pion. These nonperturbative functions correspond to two-pion generalized distribution amplitudes (GDAs), denoted as $\Phi_q^{\pi\pi}$. For transversely polarized photons, the helicity amplitude for $\gamma^*\gamma\to\pi\pi$ at leading twist reads [32]

$$T^{\gamma^*\gamma\to\pi\pi}_{\Lambda_\gamma=0} = \frac{1}{2}\sum_q e_q^2\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,t), \quad (22)$$

where z is the fraction of light-cone momentum carried by the quark with respect to the pion pair and ζ is the fraction of light-cone momentum carried by the pion with respect to the pion pair, i.e.

$$\zeta = \frac{1+\beta\cos\theta_{\pi\pi}}{2}, \qquad \beta = \sqrt{1-\frac{4m_\pi^2}{t}}. \quad (23)$$

In Eq. (22), we can distinguish the neutral and charged pion channel production. The process $\gamma^*\gamma\to\pi^+\pi^-$ is only sensitive to the C even part of $\Phi_q^{\pi^+\pi^-}$, since the initial two-photon state has positive C-parity. On the other side, the $\pi^0\pi^0$ pair has positive C-parity as well, so that $\Phi_q^{\pi^0\pi^0}$ has no C-odd part at all. Isospin invariance implies that the pion pair is in a state of zero isospin and $\Phi_u^+ = \Phi_d^+$, where the index + denotes the C-even contribution. As a result, we have

$$\Phi_q^{\pi^+\pi^-} = \Phi_q^{\pi^0\pi^0} = \Phi_q^+. \quad (24)$$

The GDAs have the following partial wave expansion [36–38]

$$\Phi_q^+ = 6z(1-z)\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1} B^q_{nl}(t)\, C_n^{(3/2)}(2z-1)\, P_l(2\zeta-1), \quad (25)$$

where $C_n^{(3/2)}$ are Gegenbauer polynomials and the expansion coefficients B^q_{nl} contain a dependence on the factorization scale, which is not shown explicitly. The expansion of the ζ-dependence in Legendre polynomials is directly related to the partial-wave expansion of the two-pion system. As a matter of fact, one can rewrite the polynomials $P_l(2\zeta - 1) = P_l(\beta\cos\theta_{\pi\pi})$ in terms of $P_k(\cos\theta_{\pi\pi})$ with $k \le l$, with the series (25) transforming in

$$\Phi_q^+ = 6z(1-z)\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1}\tilde{B}^q_{nl}(t)C_n^{(3/2)}(2z-1)P_l(\cos\theta_{\pi\pi}), \quad (26)$$

where the coefficients $\tilde{B}_{nl}$ are linear combinations of the form

$$\tilde{B}_{nl} = \beta^l[B_{nl} + c_{l,l+2}B_{n,l+2} + \cdots + c_{l,n+1}B_{n,n+1}], \quad (27)$$

with polynomials $c_{l,l'}$ in β^2.

Inserting Eqs. (22) and (26) in the partial wave expansion of the helicity amplitude in Eq. (19), one finds

$$T^{J(\gamma^*\gamma\to\pi\pi)}_{\Lambda_\gamma=0}(t) = \sum_q e_q^2 T^{J(\gamma^*\gamma\to q\bar{q})}_{\Lambda_\gamma=0}(t) \quad (28)$$

with

$$T^{J(\gamma^*\gamma\to q\bar{q})}_{\Lambda_\gamma=0}(t) = \frac{6}{2J+1}\sum_{\substack{n=\max(1,J-1)\\ \text{odd}}}^{\infty}\int_0^1 dz\,(2z-1)\tilde{B}^q_{nJ}(t)C_n^{(3/2)}(2z-1). \quad (29)$$

Using the partial wave expansion of Eq. (21) and Eq. (15), we can finally express the 2π t-channel contribution to $\mathrm{Im}_t F^q(\nu = 0, t)$ by the partial wave amplitudes for the reactions $\gamma^*\gamma \to \pi\pi$ and $\pi\pi \to N\bar{N}$

$$\mathrm{Im}_t F^{q(\pi\pi)} = -\frac{M_N p_\pi}{\sqrt{t}\,p_t^2}\sum_{\substack{J\\ \text{even}}}\frac{2J+1}{2}(-1)^{J/2}\frac{(J-1)!!}{J!!} \times (p_t p_\pi)^J T^{J(\gamma^*\gamma\to q\bar{q})}_{\Lambda_\gamma=0} f_+^{J*}(t). \quad (30)$$

For the numerical estimate, we restrict ourselves to the S- and D-wave contributions in Eq. (30). The partial-wave amplitudes of the $\pi\pi \to N\bar{N}$ subprocess are taken from the work of Höhler and collaborators [39], in which the lowest $\pi\pi \to N\bar{N}$ partial wave amplitudes were constructed from a partial wave solution of pion-nucleon scattering, by use of the $\pi\pi$ phaseshifts of Ref. [40]. In Ref. [39], the $\pi\pi \to N\bar{N}$ amplitudes are given for t values up to $t \approx 40 \cdot m_\pi^2 \approx 0.78$ GeV^2, which is taken as upper limit of integration in the t-channel dispersion integral (10). The latter value corresponds to the onset of inelasticities in the $\pi\pi$ phase shifts.

The S- and D-wave amplitudes of the $\gamma^*\gamma \to \pi\pi$ subprocess are calculated from Eq. (29), taking into account only the $n = 1$ term. This corresponds to restrict our dispersion evaluation to the $d_1^q(t)$ term in the series (11). The two-pion GDAs are calculated through dispersion relations using the Omnès representation which was first discussed in Ref. [37] and further used in Refs. [38, 41–43]. Following the derivation of Ref. [43], the results for the S- and D-wave coefficients reads

$$\tilde{B}^q_{10}(t) = -B^q_{12}(0)\frac{3C-\beta^2}{2}f_0(t), \quad (31)$$

$$\tilde{B}^q_{12}(t) = \beta^2 B^q_{12}(0)f_2(t), \quad (32)$$

where the Omnès functions $f_{0,2}$ can be related to $\pi\pi$ phase-shifts $\delta^0_{0,2}(t)$ using the Watson theorem and dispersion relations derived in [37]:

$$f_l(t) = \exp\left[\frac{t}{\pi}\int_{4m_\pi^2}^{\infty} dt'\frac{\delta_l^0(t')}{t'(t'-t-i\epsilon)}\right]. \quad (33)$$

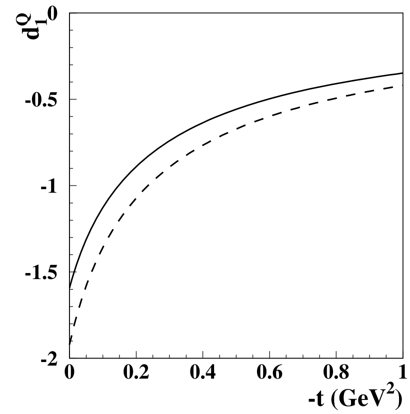

Fig. 1. d_1^Q as function of $-t$, obtained with different inputs for the quark distributions in the pion q_π^f. Solid curve: results with q_π^f from Ref. [45]. Dashed curve: results with q_π^f from Ref. [46]. The results refer to the scale $Q^2 = 4$ GeV^2.

In Eq. (31), the constant C is taken from Ref. [38], using the estimate from the instanton model [44] at low energies, $C = 1 + bm_\pi^2 + \mathcal{O}(m_\pi^4)$ with $b \approx -1.7$ GeV^{-2}, while the coefficient $B_{12}(0)$ is obtained using the crossing relations between the quark 2π DA's and the corresponding parton distributions in the pion, i.e.

$$B^q_{12}(0) = \frac{10}{9}\int dx\,x\frac{1}{N_f}\sum_f\left[q_\pi^f(x) + \bar{q}_\pi^f(x)\right]. \quad (34)$$

As final result, taking into account only the contribution with $J = 0$ and $J = 2$, Eq. (30) simplifies to

$$\mathrm{Im}_t F^{q(\pi\pi)} = \frac{3M_N p_\pi}{2\sqrt{t}p_t^2}B^q_{12}(0) \times \left[\left(3C-\beta^2\right)f_0(t)f_+^{0*}(t) + (p_\pi p_t)^2\beta^2 f_2(t)f_+^{2*}(t)\right]. \quad (35)$$

In Eq. (35), the dependence on the renormalization scale enters only through the coefficient B^q_{12} evaluated at $t = 0$, and therefore is factorized from the t dependence of the amplitude. Furthermore, the coefficients B^q_{12} evolve in the same way as the quark momentum fraction in the pion, in accordance with Eq. (34).

4. Results

In Fig. 1 we present the dispersive predictions for $d_1^Q = \sum_q d_1^q(t)$ as function of t, with the sum over flavors restricted to up and down quarks. The solid and dashed curves are obtained using as input in Eq. (34) the parametrization of the pion distributions at $Q^2 = 4$ GeV^2 from Ref. [45] and [46], respectively. The different inputs for the pion distributions change the results by an overall normalization factor, without affecting the t dependence. As outlined above, the Q^2 dependence enters only through the quark momentum fraction of the pion, which changes only by a few percent in the range of $Q^2 = [1, 10]$ GeV^2. At $t = 0$, we find $d_1^Q(0) = -1.59$ and $d_1^Q(0) = -1.92$ for the solid and dashed curve in Fig. 1, respectively. These values compare with the results obtained, at a low normalization scale, in the χQSM [5], $d_1^Q(0) = -2.35$, in the Skyrme model [9], $d_1^Q(0) = -4.48$, and in a recent calculation with effective light-front wave functions from a Regge-improved diquark model [10], $d_1^q(0) = -2.01$.

Among the form factor in Eq. (11), $d_1^Q(t)$ aroused a particular interest, as it enters in the parametrization of the quark part of the energy momentum tensor of QCD, and provides information

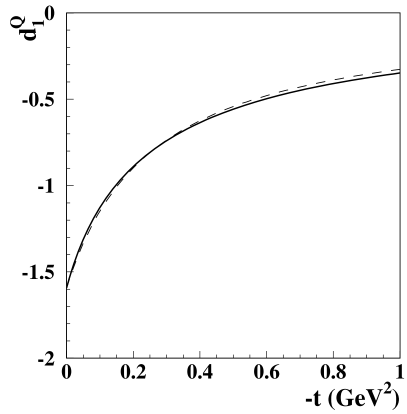

Fig. 2. Model calculation of $d_1^Q(t)$ (solid curve) in comparison with the function in Eq. (38).

on how strong forces are distributed and stabilized in the nucleon [47]. In all theoretical studies so far as well as in the present dispersive calculation, $d_1^Q(t)$ at zero-momentum transfer $t=0$ is found to have a negative sign. The negative values of this constant has a deep relation to the spontaneous breaking of the chiral symmetry in QCD [47,48], and has also an appealing connection with the criterion of stability of the nucleon [5].

Furthermore, d_1 determines the behavior of the D-term form factor in the asymptotic limit $\mu^2\to\infty$. In this limit, all the terms with $n>1$ in the series (11) go to zero, and one has

$$D^{Q,\mathrm{as}}(t)=d(t)\frac{3N_f}{3N_f+16}, \tag{36}$$

where $d(t)=d_1^Q(t)+d_1^G(t)$ is the total, scale-independent, contribution from quark and gluon.

In the dispersive calculation, the asymptotic limit of $D^Q(t)$ can be obtained from the asymptotic limit of $B_{12}(0)$ in Eq. (34), i.e.

$$B_{12}^{Q,\mathrm{as}}(0)=\frac{10}{9}\frac{3N_f}{3N_f+16}. \tag{37}$$

As a result, $d(t)$ has the same t-dependence of $d_1^Q(t)$ shown in Fig. 1, and differs only for the value at $t=0$ which is found $d(0)=-3.32$.

In most of phenomenological studies of DVCS, the t dependence of D-term form factor is parametrized by a dipole function [2]. However, the dispersive results favor a different functional form, as shown in Fig. 2 where we compare the result for d_1^Q as function of t with the following parametrization

$$F_D=\frac{d_1^Q(0)}{[1-t/(\alpha M_D^2)]^\alpha}, \tag{38}$$

with $M_D=0.487$ GeV and $\alpha=0.841$.

In Fig. 3 we show the convergence of the t-channel integral from $4m_\pi^2$ to ∞ in the unsubtracted DR of Eq. (10) for $t=-0.1$ GeV^2. We do so by calculating the dispersion integral as function of the upper integration limit t_{upper} and by showing the ratio to the integral for $t_{\mathrm{upper}}=0.78$ GeV^2. The latter value corresponds to the onset of inelasticities in the $\pi\pi$ phase shifts. One sees from Fig. 3 that the unsubtracted t-channel DR shows only a slow convergence.

In order to improve the convergence of the dispersion integral, we may introduce subtracted DRs, with the subtraction constant at $t=0$:

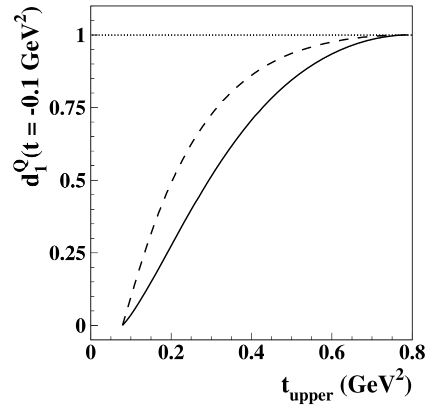

Fig. 3. The results at $t=-0.1$ GeV^2 for the unsubtracted (solid curve) and the subtracted (dashed curve) t-channel dispersion integrals in Eq. (10) and (39), respectively, are shown as function of the upper integration limit t_{upper}. Both results are normalized to their respective values at $t_{\mathrm{upper}}=0.78$ GeV^2.

$$D^q(0,t)=D^q(0)+\frac{t}{4\pi}\int_{4m_\pi^2}^{+\infty}\mathrm{d}t'\frac{\mathrm{Im}_t F^q(0,t')}{t'(t'-t)}, \tag{39}$$

where we omitted the contribution from the negative t-channel cut. In Fig. 3 we see that the subtracted dispersion integral converges faster, reaching its final value around $t\approx 0.6$ GeV^2. The price to pay is the appearance in Eq. (39) of the subtraction constant that has to be fitted to experimental data. To have a rough indication for the contribution expected above the inelastic threshold, we extended the integration up to $t_{\mathrm{upper}}=1.78$ GeV^2, including the inelasticities in the $\pi\pi$ phase shifts and approximating the πN partial-wave amplitudes with the Born contribution. The results of the unsubtracted DRs are affected by $\sim 10\%$, while the subtracted dispersion integrals are quite stable and change just by a few percent.

5. Conclusions

We have presented a dispersive representation for the quark contribution to the D-term form factor in hard exclusive reactions in terms of unsubtracted t-channel dispersion relations. The unitarity relation for the t-channel amplitudes is saturated with two-pion intermediate states, taking into account the contribution from S-and D-wave intermediate states in the numerical estimate. The input for the imaginary part of the dispersion relation is the two-pion GDAs, determined through the first-x moment of the flavor-singlet pion PDFs, the $\pi\pi$ phase shifts up to the inelastic threshold, and the partial waves for the $\pi\pi\to N\bar{N}$ amplitudes obtained from dispersion theory by analytical continuation of πN scattering. We found that the t and Q^2 dependence of the D-term form factor are disjoined. The t-dependence is not trivial and it does not follow a dipole behavior as normally assumed in phenomenological parametrizations. On the other hand, the Q^2 dependence enters only in the normalization point at $t=0$, which is proportional to the first x-moment of the flavor-singlet pion PDFs. The value at $t=0$ is also compatible with estimates in chiral-quark soliton model and a Regge-improved diquark model. In order to improve the convergence of the dispersion integral, we also discussed subtracted dispersion relations, which can be used to determine the t-dependence of the D-term form factor, but leave the value at $t=0$ as free parameter to be fitted to experimental data.

Acknowledgements

The authors are thankful to P. Schweitzer for a careful reading of the manuscript and instructive discussions. B.P. is also grateful to D. Müller, H. Moutarde and O. Teryaev for stimulating comments. This work was supported in part by the European Community Joint Research Activity "Study of Strongly Interacting Matter" (acronym HadronPhysics3, Grant Agreement No. 283286) under the Seventh Framework Programme of the European Community.

References

[1] M.V. Polyakov, C. Weiss, Phys. Rev. D 60 (1999) 114017.
[2] M. Guidal, H. Moutarde, M. Vanderhaeghen, Rep. Prog. Phys. 76 (2013) 066202.
[3] K. Kumericki, D. Mueller, K. Passek-Kumericki, Nucl. Phys. B 794 (2008) 244.
[4] P. Schweitzer, S. Boffi, M. Radici, Phys. Rev. D 66 (2002) 114004.
[5] K. Goeke, J. Grabis, J. Ossmann, M.V. Polyakov, P. Schweitzer, A. Silva, D. Urbano, Phys. Rev. D 75 (2007) 094021.
[6] K. Goeke, J. Grabis, J. Ossmann, P. Schweitzer, A. Silva, D. Urbano, Phys. Rev. C 75 (2007) 055207.
[7] K. Goeke, M.V. Polyakov, M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401.
[8] M. Wakamatsu, Phys. Lett. B 648 (2007) 181.
[9] C. Cebulla, K. Goeke, J. Ossmann, P. Schweitzer, Nucl. Phys. A 794 (2007) 87.
[10] D. Mueller, D.S. Hwang, arXiv:1108.3869 [hep-ph].
[11] M. Gockeler, et al., QCDSF Collaboration, Phys. Rev. Lett. 92 (2004) 042002.
[12] P. Hagler, et al., LHPC Collaboration, Phys. Rev. D 77 (2008) 094502.
[13] H.-C. Kim, P. Schweitzer, U. Yakhshiev, Phys. Lett. B 718 (2012) 625.
[14] J.-H. Jung, U. Yakhshiev, H.-C. Kim, P. Schweitzer, Phys. Rev. D 89 (2014) 114021.
[15] M. Mai, P. Schweitzer, Phys. Rev. D 86 (2012) 076001.
[16] M. Mai, P. Schweitzer, Phys. Rev. D 86 (2012) 096002.
[17] A.V. Belitsky, X. Ji, Phys. Lett. B 538 (2002) 289.
[18] M. Diehl, A. Manashov, A. Schafer, Eur. Phys. J. A 29 (2006) 315.
[19] S.-i. Ando, J.-W. Chen, C.-W. Kao, Phys. Rev. D 74 (2006) 094013.
[20] M. Dorati, T.A. Gail, T.R. Hemmert, Nucl. Phys. A 798 (2008) 96.
[21] P. Wang, A.W. Thomas, Phys. Rev. D 81 (2010) 114015.
[22] A.M. Moiseeva, A.A. Vladimirov, Eur. Phys. J. A 49 (2013) 23.
[23] P. Wein, P.C. Bruns, A. Schäfer, Phys. Rev. D 89 (2014) 116002.
[24] B. Pasquini, M. Gorchtein, D. Drechsel, A. Metz, M. Vanderhaeghen, Eur. Phys. J. A 11 (2001) 185.
[25] D. Drechsel, B. Pasquini, M. Vanderhaeghen, Phys. Rep. 378 (2003) 99.
[26] I.V. Anikin, O.V. Teryaev, Phys. Rev. D 76 (2007) 056007.
[27] O.V. Teryaev, arXiv:hep-ph/0510031.
[28] A.V. Radyushkin, Phys. Rev. D 83 (2011) 076006.
[29] M. Diehl, D.Y. Ivanov, Eur. Phys. J. C 52 (2007) 919.
[30] M.V. Polyakov, M. Vanderhaeghen, arXiv:0803.1271 [hep-ph].
[31] G.R. Goldstein, S. Liuti, Phys. Rev. D 80 (2009) 071501.
[32] M. Diehl, T. Gousset, B. Pire, O. Teryaev, Phys. Rev. Lett. 81 (1998) 1782.
[33] M. Diehl, P. Kroll, C. Vogt, Eur. Phys. J. C 26 (2003) 567.
[34] D. Drechsel, M. Gorchtein, B. Pasquini, M. Vanderhaeghen, Phys. Rev. C 61 (1999) 015204.
[35] W.R. Frazer, J.R. Fulco, J. R. FulcoPhys. Rev. 117 (1960) 1603.
[36] M. Diehl, T. Gousset, B. Pire, Phys. Rev. D 62 (2000) 073014.
[37] M.V. Polyakov, Nucl. Phys. B 555 (1999) 231.
[38] N. Kivel, L. Mankiewicz, M.V. Polyakov, Phys. Lett. B 467 (1999) 263.
[39] G. Höhler, Pion-Nucleon Scattering, H. Schopper (Ed.), Landolt-Börnstein, vol. I/9b2, Springer, Berlin, 1983.
[40] C.D. Froggatt, J.L. Petersen, Nucl. Phys. B 129 (1977) 89.
[41] B. Lehmann-Dronke, P.V. Pobylitsa, M.V. Polyakov, A. Schafer, K. Goeke, Phys. Lett. B 475 (2000) 147.
[42] B. Lehmann-Dronke, A. Schaefer, M.V. Polyakov, K. Goeke, Phys. Rev. D 63 (2001) 114001.
[43] N. Warkentin, M. Diehl, D.Y. Ivanov, A. Schäfer, Eur. Phys. J. A 32 (2007) 273.
[44] M.V. Polyakov, C. Weiss, Phys. Rev. D 59 (1999) 091502.
[45] J.F. Owens, Phys. Rev. D 30 (1984) 943.
[46] M. Gluck, E. Reya, A. Vogt, Z. Phys. C 53 (1992) 651.
[47] M.V. Polyakov, Phys. Lett. B 555 (2003) 57.
[48] M.V. Polyakov, A.G. Shuvaev, arXiv:hep-ph/0207153.