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Decay of Affleck–Dine condensates with application to Q-balls

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Abstract

Analytical and numerical estimates show that a charged Affleck–Dine condensate will fracture into Q-balls only when the Hubble time is significantly larger than the inverse soft-breaking mass of the field in question. This would generally imply that the decay of the field into light fermions will compete with Q-ball formation. We will show that for typical flat directions the large field value will significantly suppress decays of the condensate to fermions even if no baryon charge asymmetry exists. We will consider the details of the decay process for a condensate that does carry charge, and show that it is qualitatively different from that of an uncharged condensate. Finally, we will consider the possibility of resonant production of heavy bosons. We will show that this can have a strong effect on the condensate. Contrary to intuition, however, our results indicate that boson production would actually assist Q-ball formation in condensates with significant charge.

1. Introduction

A Q-ball is a non-perturbative solution of the equation of motion for a scalar field which is charged under a continuous U(1) symmetry [1]. The MSSM requires several such scalars, and so it is expected that Q-balls could be formed in a supersymmetric universe [2]. In fact, Q-balls seem inevitable in the context of Affleck–Dine [3] (AD) baryogenesis. Here, a flat direction composed of several squark fields gains a large expectation value and is set into coherent rotation by the action of a phase-dependent term in the potential. Such a charged scalar condensate has been shown to fracture into Q-balls in analytical treatments [4–6] and in numerical simulations [7].

Both numerical and analytical estimates agree that in gravity-mediated SUSY breaking models, Q-balls will only form when the age of the universe has reached a value of order $\sim 10^3 m_\phi^{-1}$ where m_ϕ is the soft-breaking mass of the AD condensate field (assumed to be of order 1 TeV). Naively speaking, however, we would estimate the decay width of the scalar field into light fermions to be of order:

$$\Gamma(\phi \to \psi\psi) \approx \frac{g^2 m_\phi}{8\pi}, \tag{1}$$

where we have substituted the gauge coupling g for the usual Yukawa coupling because the squark fields making up ϕ are coupled to gluino/chargino plus quark through gauge interactions.

If we do not invent a suppression for this decay by making the fermions heavy then we see that the time for our condensate to decay can be as short

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as order $100m_\phi^{-1}$. Thus, we expect a competition between decay of the condensate into light fermions and fracturing of the condensate into Q-balls. As we will explore in this Letter, however, there is not only a suppression to the decays, but there is also an important difference between the decay of a partially charge-asymmetric condensate and a neutral one. Taken together, these details will ensure that Q-ball formation is uninterrupted (and perhaps even aided) by decay of the condensate.

2. Decay into fermions

2.1. Necessity of non-perturbative approach

The physics leading to the suppression of decay into fermions is familiar. Any fermions coupling directly to the ϕ field will gain a mass of order $g|\phi|$ where g is the relevant coupling and $|\phi|$ is the magnitude of the complex scalar field. In modern formulations of the AD scenario [8], when the Hubble constant H reaches m_ϕ the AD field will have a magnitude of order:

$$|\phi| \sim \left(m_\phi M^{n-3}\right)^{1/(n-2)}, \tag{2}$$

where M is a large mass scale (order M_{GUT} or M_{pl}) and n describes the flatness of the flat direction (two standard choices are $n=4$ or $n=6$). We will take $n=6$ as our canonical value because it minimizes thermal concerns [9]. For typical numbers, then, we expect $|\phi| > 10^{12} m_\phi$ (or $10^9 m_\phi$ for $n=4$) so that all fermions have effective masses much larger than the mass of the ϕ field.

At first glance, this solves our problem completely since the ϕ field is stable. However, it is important that when $H \leqslant m_\phi$ the ϕ field will begin to execute harmonic oscillations about $\phi = 0$ [8]. This will result in a sinusoidally varying mass for the fields coupled to the ϕ field. Such a situation has already been studied for real (non-complex) scalar condensates in the context of post-inflation reheating, and has been shown to lead to decay of the condensate [10,11]. We wish to expand this analysis to the case of an oscillating complex ϕ field.

Explicitly, we anticipate that after a few oscillations the ϕ field will see the effective potential:

$$U(|\phi|) \approx m_\phi^2 |\phi|^2, \tag{3}$$

where higher order terms can be neglected due to the small size of $|\phi|$ [8]. We can now make the analogy between $|\phi|$ and the radial position r of a particle in an r^2 potential. We know from basic classical mechanics that the angular momentum in such a system will be conserved (in this case, "angular momentum" is equivalent to baryon number), and further that the particle will follow closed orbits [13]. Numerical integration shows that this approximation is very nearly exact even in the presence of small corrections due to non-renormalizable terms and log running of the mass parameter.

All of this amounts to the fact that we lose no generality in parameterizing the final solution for the ϕ field in the form of an ellipse centered on the origin:

$$\phi = a\sin(m_\phi t) - ib\cos(m_\phi t), \tag{4}$$

where we have assumed that $a \geqslant b$.

Using the Noether current expression for the global U(1) yields the expression for net baryon number density of the condensate:

$$n_B = i\beta\left(\dot{\phi}^*\phi - \dot{\phi}\phi^*\right) = 2\beta m_\phi ab, \tag{5}$$

where β is the baryon charge per ϕ particle (usually $1/3$). Then, by assuming that each scalar particle associated with the ϕ field has an energy of approximately m_ϕ, it is simple to show that the ratio of net baryons to total scalars is:

$$\frac{n_B}{n_\phi} = \frac{2\beta ba}{a^2+b^2}. \tag{6}$$

From this expression, we can see that the limit $b=0$ corresponds to a completely uncharged scalar field. The limit $b=a$ indicates a total charge asymmetry (the condensate is made up entirely of baryons or entirely of antibaryons). Intermediate values of b indicate a partially charge-asymmetric condensate.

Using the parameterization (4), we also find that:

$$|\phi| = \sqrt{b^2 + (a^2-b^2)\sin^2(m_\phi t)}. \tag{7}$$

Thus, we do anticipate an oscillating mass for any fields coupled to the AD scalar.

2.2. Method of imaginary time

The method of imaginary time is convenient for the calculation of non-perturbative production of fermions. We will use the results of [10] essentially verbatim. For an introduction to the imaginary time formalism, see the review [12] and the references therein.

The method relies on finding the branch points in the complex-time plane of the fermion Hamiltonian. For our fermions, we expect:

$$\mathcal{H} = \left(p^2 + g^2|\phi|^2\right)^{1/2} = \left(p^2 + g^2b^2 + g^2\left(a^2 - b^2\right)\sin^2(m_\phi t)\right)^{1/2}. \quad (8)$$

Fortunately, exactly this form was treated in [10].

Two limits are analytically approachable using their results. First, note that by Eq. (2) we always expect $m_\phi \ll a$. Now, if we also assume $b \ll a$ (equivalent to $m_0 \ll m_1$ in the conventions of [10]) we can take an analytical limit:

$$\Gamma_\phi \approx \frac{e^{-\pi/2} m_\phi^{3/2}}{8\pi^2 (ga)^{1/2}} \quad (9)$$

(note that e here is the base of the natural log). In this approximation we have basically followed [12] exactly, except that we have added the assumption that $\log(4a/b)$ is of the order π. Note that here we have assumed b is non-zero, and larger than the momentum p. If b were to approach zero, the results of [10,12] apply exactly (there is no divergence). Putting in typical numbers will tell us that Γ is of order $10^{-6} m_\phi$ at the largest ($10^{-5} m_\phi$ for $n = 4$). This suppression is enough to keep our condensate intact until Q-balls can form, even for the less-favorable $n = 4$ case.

Next let us take the limit $b \approx a$. Here again we can analytically approximate the decay width (this is equivalent to $m_0 \gg m_1$ in [10]):

$$\Gamma_\phi \approx \frac{m_\phi^{3/2}}{16\pi^2 (ga)^{1/2}} \exp\left(-2\frac{ga}{m_\phi} \ln\left[\frac{16a^2}{a^2 - b^2}\right]\right), \quad (10)$$

which, for our typical numbers yields a decay width that has been exponentially suppressed to the extent that it is effectively zero.

In each of these limits for the ratio of b to a, order of magnitude estimates show that the decay rate is highly suppressed. It is logical that the rate does not peak between, but rather goes smoothly from one to the other. To be safe, however, we can numerically evaluate the complete elliptical functions outside the range of validity of our approximations to estimate the exponential part of the suppression for arbitrary values of b/a. The curve shown in Fig. 1 computes only the exponential suppression of the decay rate, but it is enough to show that this assumption is correct. Thus, the large field values suppress the decay into fermions sufficiently to allow Q-ball formation to proceed.

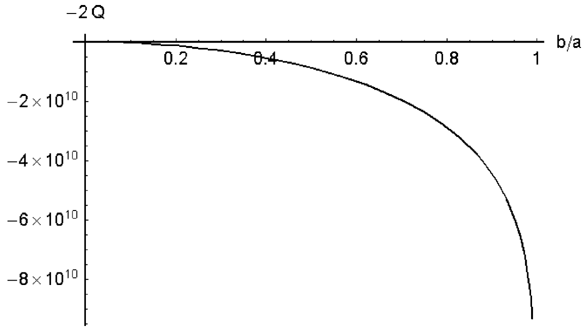

Fig. 1. Exponential damping constant in the $\phi \to \psi\psi$ decay width for various values of b/a. $\Gamma_\phi \sim e^{-2Q}$. We have assumed $ga = 10^{10} m_\phi$.

It is important to notice in Eq. (10) that at $b = a$ the decay rate goes to zero. This is no accident. A completely charge asymmetric scalar condensate is stable against decay into fermions—even non-perturbative decay [14]. This is an important component of Q-ball stability.

3. Parametric resonance

Finally, we must consider the decay of our field into bosons. This decay will be suppressed by the same physics that affected the decay into fermions. It is well known, however, that in the case of decay to bosons enhancement from particle statistics can offset this suppression. This phenomenon was mentioned in [10,11]. The first modern treatment, however, was [15]. We will follow the expanded version presented in [16].

We expect the ϕ field to have several couplings of the form $g^2|\phi|^2|\chi|^2$ where g is the strong gauge coupling and χ is a scalar (a linear combination of squarks that is orthogonal to the combination that makes up the ϕ direction).

This contribution to the Lagrangian will result in an oscillating mass term for the χ field when the ϕ field is undergoing elliptical oscillation. The scalar χ will have the following equation of motion (after transforming to momentum space):

$$\ddot{\chi} + 3H\dot{\chi} + \left[\frac{k^2}{R^2} + g^2 b^2 + g^2\left(a^2 - b^2\right)\sin^2(m_\phi t)\right]\chi = 0, \tag{11}$$

where R is the scale factor of the universe ($H = \dot{R}/R$). We can simplify this by using the substitution $\mathrm{X} \equiv R^{3/2}\chi$. This gives:

$$\ddot{\mathrm{X}} + \left[\frac{k^2}{R^2} + g^2 b^2 + g^2\left(a^2 - b^2\right)\sin^2(m_\phi t) - \frac{3}{4}H^2 - \frac{3}{2}\frac{\ddot{R}}{R}\right]\mathrm{X} = 0. \tag{12}$$

Noting that the typical scale for H is $m_\phi \ll a, b$ at the time ϕ is coherently oscillating, we assume we can rewrite this:

$$\ddot{\mathrm{X}} + \left[\frac{k^2}{R^2} + g^2 b^2 + g^2\left(a^2 - b^2\right)\sin^2(m_\phi t)\right]\mathrm{X} = 0. \tag{13}$$

This is almost exactly the case of stochastic resonance in the expanding universe treated in [16]. The only issue to be careful of is the size of b. We will discuss that shortly. In the meantime, we will adapt Section IX of [16] to our purposes.

The fundamental procedure of the parametric resonance approach is the rewriting of our equation of motion in the form of the Mathieu equation:

$$x'' + \left(A - 2q\cos(2z)\right)x = 0, \tag{14}$$

where prime denotes differentiation with respect to z (please note that q in this context is an unfortunate choice for us—it has nothing to do with quarks). The appropriate substitutions here are:

$$A = \frac{k^2}{R^2 m_\phi^2} + \frac{g^2 b^2}{m_\phi^2} + 2q, \tag{15}$$

$$q = \frac{g^2(a^2 - b^2)}{4m_\phi^2}, \tag{16}$$

and

$$z = mt. \tag{17}$$

The work of [16] shows that it is q which determines the efficiency of the condensate decay. For $q > 10^4$, we expect the condensate to retain a considerable q value even after completion of the "first stage" of preheating (with low density of decay products). If q remains greater than order one after this phase is completed we expect the decay to continue and the condensate will give up a significant fraction of its energy to decay products.

We are interested, therefore, in estimating q for a typical Q-ball scenario. At the beginning of oscillations we expect $a^2 - b^2$ to be of order $10^{20} m_\phi$ which gives a q factor of:

$$q \sim 10^{20} g^2. \tag{18}$$

It is pointed out in [16] that the $q = 10^4$ cutoff value for strong preheating is weakly model dependent. Given that we expect $g^2 \sim 0.1$ for strong interactions, however, it seems safe to assume that we are well into the strong preheating regime (this is also true for the $n = 4$ flat directions, where we expect $q \sim 10^{14} g^2$).

Now let us consider, as we did for fermion production, what it would mean for this decay to go forward. Once again, we see that the decay efficiency is proportional to the difference $a^2 - b^2$ so that it vanishes as we approach a completely asymmetric condensate. In fact, in the case of resonant production of bosons, we can have a second suppression since b is functioning as an effective bare mass in the formulas above. Thus, as mentioned in [16] we might expect preheating to become inefficient if $2b^2 > a^2 - b^2$. It appears, then, that our decays take away the neutral condensate but could leave a charged remnant.

The physics behind this result is straightforward. The four point coupling $g^2|\phi|^2|\chi|^2$ that we have considered here should only mediate annihilation, not true decays. This would certainly respect any baryon asymmetry present in the condensate.

This leaves us with an important possibility. If decay into bosons is strong enough, it could be that generic AD condensates will damp much of their ellipticity. This process could actually aid Q-ball formation. In fact, even the time scales estimated in [16] for the decay process are right for Q-ball formation. They expect the first stage of resonant production to end at a time scale of order $100m_\phi^{-1}$, just before Q-balls would form according to numerical simulations [7].

There are two important issues to address, however, before we can say with confidence that annihilations help produce Q-balls. First, it is important to note that we have not included a study of rescattering of the produced χ particles. Such back-reaction could have a negative impact on Q-ball formation, and should be examined. Second, annihilation would almost certainly have a negative impact on Q-ball production in a very weakly charged condensate, since it could lead to significant decay of the condensate before Q-ball formation would occur.

4. Conclusion

In summary, decay of a partly charge-asymmetric Affleck–Dine condensate into fermions will be strongly suppressed. This suppression will give more than enough time for the condensate to fracture into Q-balls. Annihilation of the neutral part of the condensate, however, can be enhanced by non-perturbative effects completely analogous to preheating in inflation. These effects must be studied further, as they could have important consequences for Q-ball formation. In particular, it seems that they should make Q-ball formation in strongly charged condensates more likely, while suppressing formation in weakly charged condensates.

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