Positive solutions of second-order \( m \)-point boundary value problems with changing sign singular nonlinearity

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Abstract

In this work, some new existence results of positive solutions for a class of singular \( m \)-point boundary value problems with changing sign nonlinearity are obtained, which improve on many known results.

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1. Introduction

In this work, we consider the following second-order \( m \)-point boundary value problem:

\[
\begin{align*}
(p(t)x'(t))' - q(t)x(t) + \lambda f(t, x(t)) &= 0, \quad t \in (0, 1), \\
ax(0) - bp(0)x'(0) &= \sum_{i=1}^{m-2} \alpha_i x(\xi_i), \quad cx(1) + dp(1)x'(1) = \sum_{i=1}^{m-2} \beta_i x(\xi_i),
\end{align*}
\]

(1.1)

where \( \lambda > 0 \) is a parameter, \( a, c \in [0, \infty) \), \( b, d \in (0, \infty) \), \( \xi_i \in (0, 1) \), \( \alpha_i, \beta_i \in [0, \infty) \) for \( i \in \{1, 2, \ldots, m-2\} \) are given constants, \( p \in C^1([0, 1], (0, \infty)), q \in C([0, 1], (0, \infty)) \) and \( f \in C((0, 1) \times [0, \infty), (-\infty, +\infty)) \) satisfies

\[ -r(t) \leq f(t, x) \leq z(t)h(x) \]

with \( r, z \in C((0, 1), (0, \infty)), h \in C([0, \infty), (0, \infty)) \).

The form of (1.1) for differential equations arises from many fields of applied mathematics and physics, and can describe a great deal of nonlinear problems; see [1,2]. If \( p \equiv 1, q \equiv 0, \lambda = 1, \alpha_i, \beta_i = 0, (\text{for } i = 1, 2, \ldots, m-2) \), \( m \)-point boundary value problem (1.1) reduces to the two-point boundary value problem

\[
\begin{align*}
x''(t) + f(t, x(t)) &= 0, \quad t \in (0, 1), \\
ax(0) - bx'(0) &= cx(1) + dx'(1) = 0.
\end{align*}
\]

(1.2)

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In the case where \( f \) is nonnegative, i.e. the positone case, (1.2) has been intensively studied; see [3–7]. Recently, Ma [8] and Ma and Thompson [9] obtained many excellent results about the existence of positive solutions and eigenvalue intervals for the more general \( m \)-point boundary value problem (1.1), but they only considered the case of the nonlinearity taking on nonnegative values and being nonsingular. Moreover their method cannot deal with the nonlinearity taking on negative values. In this work, we consider the existence of positive solutions for the BVP (1.1) when the nonlinearity \( f \) may change sign and \( f(t,x) \) may be singular at \( t = 0,1 \). Such problems where the nonlinearity is restricted to nonnegative values are known as semipositone problems in the literature and they arise naturally in chemical reactor theory [10]. The constant \( \lambda \) is usually called the Thiele modulus, and in applications one is interested in showing the existence of positive solutions for \( \lambda > 0 \) small. However, because the semipositone problem is different from the positone problem, one can easily find that a former prior bound estimation method is not applicable for (1.1). So we will use a different method of proof. In particular, we will use Krasnoselskii’s fixed point theorem to prove our main result. For the convenience of the reader, we now state Krasnoselskii’s fixed point theorem for cones.

**Lemma 1.1** (11). Let \( X \) be a real Banach space, \( Q \subset X \) be a cone. Assume \( \Omega_1, \Omega_2 \) are two bounded open subsets of \( X \) with \( x \in \Omega_1, \overline{\Omega}_1 \subset \Omega_2 \), and let \( T : Q \cap (\overline{\Omega}_2 \setminus \Omega_1) \rightarrow Q \) be a completely continuous operator such that either

1. \( \|Tu\| \leq \|u\|, u \in Q \cap \partial \Omega_1 \) and \( \|Tu\| \geq \|u\|, u \in Q \cap \partial \Omega_2 \), or
2. \( \|Tu\| \geq \|u\|, u \in Q \cap \partial \Omega_1 \) and \( \|Tu\| \leq \|u\|, u \in Q \cap \partial \Omega_2 \).

Then \( T \) has a fixed point in \( Q \cap (\overline{\Omega}_2 \setminus \Omega_1) \).

This work is organized as follows. In Section 2, we present some lemmas that are used to prove our main result. In Section 3, by using Lemma 1.1, we will complete the proof of our main result.

2. Preliminaries and some lemmas

In the rest of the work, we adopt the following assumptions:

(H1) \( p \in C^1([0,1], (0, \infty)) \), \( q \in C([0,1], (0, \infty)) \).

(H2) \( a, c \in [0, \infty), b, d \in (0, \infty) \) with \( ac + ad + bc > 0, \alpha_i, \beta_i \in [0, \infty) \) for \( i \in \{1, \ldots, m-2\} \).

(H3) For any \( (t,x) \in (0,1) \times (0, \infty) \), \( f(t,x) \) satisfies

\[
-r(t) \leq f(t,x) \leq z(t)h(x)
\]

with \( r, z \in C((0,1), (0, \infty)), h \in C([0, \infty), (0, \infty)) \).

The following lemma, which can be found in paper [8] or [9], plays an important role in proving our main result.

**Lemma 2.1.** Let (H1) and (H2) hold. Let \( \psi \) and \( \phi \) be the solutions of the linear problems

\[
\begin{align*}
(p(t)\psi'(t))' - q(t)\psi(t) &= 0, & t \in (0,1), \\
\psi(0) &= b, & p(0)\psi'(0) = a,
\end{align*}
\]

and

\[
\begin{align*}
(p(t)\phi'(t))' - q(t)\phi(t) &= 0, & t \in (0,1), \\
\phi(1) &= d, & p(1)\phi'(1) = -c,
\end{align*}
\]

respectively. Then

(i) \( \psi \) is strictly increasing on \( [0,1] \), and \( \psi(t) > 0 \) on \( [0,1] \).

(ii) \( \phi \) is strictly decreasing on \( [0,1] \), and \( \phi(t) > 0 \) on \( [0,1] \).

As in [9], set

\[
\Gamma = \left| \begin{array}{cc}
- \sum_{i=1}^{m-2} \alpha_i \psi(\xi_i) & \rho - \sum_{i=1}^{m-2} \alpha_i \phi(\xi_i) \\
\rho - \sum_{i=1}^{m-2} \beta_i \psi(\xi_i) & - \sum_{i=1}^{m-2} \beta_i \phi(\xi_i)
\end{array} \right|, \quad \rho = p(t) \left| \begin{array}{cc}
\phi(t) & - \psi(t) \\
\phi'(t) & \psi'(t)
\end{array} \right|.
\]
Then, by Liouville’s formula, we have
\[ \rho = p(0) \frac{\phi(0)}{\phi'(0)} \frac{\psi(0)}{\psi'(0)} = \text{constant}. \]

Now we define
\[ G(t, s) = \frac{1}{\rho} \begin{cases} \phi(t)\psi(s), & 0 \leq s \leq t \leq 1, \\ \phi(s)\psi(t), & 0 \leq t \leq s \leq 1. \end{cases} \] (2.1)

It is clear that
\[ 0 \leq G(t, s) \leq G(s, s), \quad 0 \leq s, t \leq 1. \] (2.2)

**Lemma 2.2.** For any \( t \in [0, 1] \), we have \( G(t, s) \geq \theta G(s, s) \), where
\[ \theta = \min \left\{ \frac{d}{\phi(0)}, \frac{b}{\psi(1)} \right\}. \] (2.3)

**Proof.** By (2.1) and Lemma 2.1, for any \( t \in [0, 1] \), we have
\[ \frac{G(t, s)}{G(s, s)} = \begin{cases} \frac{\phi(t)}{\phi(s)}, & 0 \leq s \leq t \leq 1, \\ \frac{\psi(t)}{\psi(s)}, & 0 \leq t \leq s \leq 1, \end{cases} \geq \begin{cases} \frac{d}{\phi(0)}, & 0 \leq s \leq t \leq 1, \\ \frac{b}{\psi(1)}, & 0 \leq t \leq s \leq 1. \end{cases} \]

Let \( \theta = \min \left\{ \frac{d}{\phi(0)}, \frac{b}{\psi(1)} \right\} \). Then \( G(t, s) \geq \theta G(s, s) \). \( \square \)

**Lemma 2.3.** Let \( (H_1) \) and \( (H_2) \) hold. Assume that \( \Gamma \neq 0 \) and \( \int_0^1 G(s, s)r(s)ds < +\infty \). Then the problem
\[ \begin{cases} (p(t)x'(t))' - q(t)x(t) + \lambda r(t) = 0, & t \in (0, 1), \\ ax(0) - bp(0)x'(0) = \sum_{i=1}^{m} \alpha_i x(\xi_i), \quad cx(1) + dp(1)x'(1) = \sum_{i=1}^{m} \beta_i x(\xi_i), \end{cases} \] (2.4)

has a unique solution
\[ w(t) = \lambda \left\{ \int_0^1 G(t, s)r(s)ds + A(r)\psi(t) + B(r)\phi(t) \right\}, \] (2.5)

where
\[ A(r) = \frac{1}{\Gamma} \begin{vmatrix} \alpha_i & \int_0^1 G(\xi_i, s)r(s)ds \end{vmatrix} \frac{\rho - \sum_{i=1}^{m} \alpha_i \phi(\xi_i)}{m-2} \]
\[ \sum_{i=1}^{m-2} \beta_i \int_0^1 G(\xi_i, s)r(s)ds - \sum_{i=1}^{m-2} \beta_i \phi(\xi_i) \] (2.6)

and
\[ B(r) = \frac{1}{\Gamma} \begin{vmatrix} \alpha_i \psi(\xi_i) & \int_0^1 G(\xi_i, s)r(s)ds \end{vmatrix} \frac{\rho - \sum_{i=1}^{m-2} \beta_i \psi(\xi_i)}{m-2} \]
\[ \sum_{i=1}^{m-2} \beta_i \int_0^1 G(\xi_i, s)r(s)ds \] (2.7)

with \( w(t) \leq \lambda \eta \), where \( \eta \) is defined by (2.3),
\[ \eta = \frac{1 + A\psi(1) + B\phi(0)}{\theta} \int_0^1 G(s, s)r(s)ds \] (2.8)
and

\[ A = \frac{1}{T} \begin{vmatrix} \sum_{i=1}^{m-2} \alpha_i & \rho - \sum_{i=1}^{m-2} \alpha_i \phi(\xi_i) \\ \sum_{i=1}^{m-2} \beta_i & - \sum_{i=1}^{m-2} \beta_i \phi(\xi_i) \end{vmatrix}, \quad B = \frac{1}{T} \begin{vmatrix} - \sum_{i=1}^{m-2} \alpha_i \psi(\xi_i) & \sum_{i=1}^{m-2} \alpha_i \\ \rho - \sum_{i=1}^{m-2} \beta_i \psi(\xi_i) & \sum_{i=1}^{m-2} \beta_i \end{vmatrix}. \]

**Proof.** Using \( \int_0^1 G(s, s)r(s)ds < +\infty \), according to Lemma 2.2 in [8] or [9], it is easy to prove that (2.5) holds. By Lemma 2.1 and (2.2), (2.6) and (2.7), we have

\[ w(t) \leq \lambda \left( \int_0^1 G(s, s)r(s)ds + A\psi(t) \int_0^1 G(s, s)r(s)ds + B\phi(t) \int_0^1 G(s, s)r(s)ds \right) \]
\[ \leq \lambda (1 + A\psi(1) + B\phi(0)) \int_0^1 G(s, s)r(s)ds \leq \lambda \eta \theta. \]

Now we shall denote by \( X = C[0, 1] \) the space of all continuous functions \( u: [0, 1] \rightarrow \mathbb{R} \). This is a real Banach space when it is endowed with the usual maximal norm \( \| x \| = \max_{t \in [0, 1]} |x(t)| \) for \( x \in C[0, 1] \). Let \( C^+[0, 1] = \{ x \mid x \in X, x(t) \geq 0 \} \), \( P = \{ x \mid x \in C^+[0, 1], x(t) \geq \theta \| x \| \text{ for } t \in [0, 1] \} \). Clearly \( P \) is a cone of \( C[0, 1] \).

Next we consider the following approximately singular nonlinear boundary value problem:

\[
\begin{cases}
(p(t)x'(t))' - q(t)x(t) + [f(t, [x(t)]^*) + r(t)] = 0, & t \in (0, 1), \\
ax(0) - bp(0)x'(0) = \sum_{i=1}^{m-2} \alpha_i x(\xi_i), & cx(1) + dp(1)x'(1) = \sum_{i=1}^{m-2} \beta_i x(\xi_i),
\end{cases}
\]

where

\[ [x(t)]^* = \max\{x(t) - w(t), 0\} \]
and \( w(t) \) is given by (2.5) which is the solution of the BVP (2.4).

In this work, we will also use the following conditions.

(H4') \( \Gamma < 0, \quad \rho - \sum_{i=1}^{m-2} \alpha_i \phi(\xi_i) > 0, \quad \rho - \sum_{i=1}^{m-2} \beta_i \psi(\xi_i) > 0. \)

(H5) \( \int_0^1 G(s, s)[r(s) + z(s)]ds < +\infty. \)

(H6) \( \lim_{x \to +\infty} \frac{f(u, x)}{x} = +\infty \) for \( t \) uniformly on \([\xi_1, \xi_1] \).

**Remark 2.1.** If \( \lambda > 0 \) and (H4') hold, then \( w \) given in (2.5) satisfies \( w(t) \geq 0 \) for any \( t \in [0, 1] \). In fact, this is a direct conclusion of Lemma 2.3 of [9].

For any fixed \( x \in \mathcal{C}^+[0, 1] \), set \( L = \max_{t \in [0, 1]} x(t) \) and \( f^*(t, x) = f(t, [x(t)]^*) + r(t) \). Applying (H2), (H3) and noticing that

\[ [x(t)]^* \leq x(t) \leq L, \]
we have

\[
\int_0^1 G(t, s)f^*(s, x)ds \leq \int_0^1 G(s, s)\left(z(s)h([x(s)]^*) + r(s)\right)ds
\]
\[
\leq \left( \max_{0 \leq \tau \leq L} h(\tau) + 1 \right) \int_0^1 G(s, s)(r(s) + z(s))ds < +\infty.
\] (2.9)
In addition, by (2.2), for \( i = 1, \ldots, m - 2 \), we know
\[
\int_0^1 G(\xi_i, s)(r(s) + z(s))ds \leq \int_0^1 G(s, s)(r(s) + z(s))ds.
\]
Thus,
\[
A(r + z) = \frac{1}{\Gamma} \left| \sum_{i=1}^{m-2} \alpha_i \int_0^1 G(\xi_i, s)(r(s) + z(s))ds - \sum_{i=1}^{m-2} \alpha_i \phi(\xi_i) \right| \\
\geq \frac{1}{\Gamma} \left| \sum_{i=1}^{m-2} \alpha_i \rho - \sum_{i=1}^{m-2} \alpha_i \phi(\xi_i) \right| \int_0^1 G(s, s)(r(s) + z(s))ds
\]
\[
< +\infty,
\]
and
\[
B(r + z) = \frac{1}{\Gamma} \left| \sum_{i=1}^{m-2} \beta_i \int_0^1 G(\xi_i, s)(r(s) + z(s))ds - \sum_{i=1}^{m-2} \beta_i \phi(\xi_i) \right| \\
\leq \frac{1}{\Gamma} \left| \sum_{i=1}^{m-2} \beta_i \rho - \sum_{i=1}^{m-2} \beta_i \phi(\xi_i) \right| \int_0^1 G(s, s)(r(s) + z(s))ds
\]
\[
< +\infty.
\]
Therefore, by (2.9)–(2.11), we can define an operator \( T: C^+[0, 1] \to C^+[0, 1] \) by
\[
T x(t) = \lambda \left\{ \int_0^1 G(t, s) f^*(s, x)ds + A(f^*) \psi(t) + B(f^*) \phi(t) \right\},
\]
where
\[
A(f^*) = \frac{1}{\Gamma} \left| \sum_{i=1}^{m-2} \alpha_i \int_0^1 G(\xi_i, s) f^*(s, x(s))ds - \sum_{i=1}^{m-2} \alpha_i \phi(\xi_i) \right| \\
\]
and
\[
B(f^*) = \frac{1}{\Gamma} \left| \sum_{i=1}^{m-2} \beta_i \phi(\xi_i) \right| \int_0^1 G(\xi_i, s) f^*(s, x(s))ds.
\]
Lemma 2.4. Suppose that \((H_1)-(H_5)\) hold. Then \(T: P \to P\) is a completely continuous operator.

Proof. For any \(x \in P\), (2.2) and Lemma 2.1 imply that

\[
Tx(t) \leq \lambda \left\{ \int_0^1 G(s, s) f^*(s, x) ds + A(f^*) \psi(1) + B(f^*) \phi(0) \right\}.
\]  

(2.12)

On the other hand, from Lemma 2.2 and the monotonicity of \(\psi\) and \(\phi\), we have

\[
Tx(t) \geq \lambda \theta \left\{ \int_0^1 G(s, s) f^*(s, x) ds + \lambda \left( \frac{b}{\psi(1)} + \frac{d}{\phi(0)} \right) \right\}
\geq \lambda \theta \left\{ \int_0^1 G(s, s) f^*(s, x) ds + A(f^*) \psi(1) + B(f^*) \phi(0) \right\}.
\]  

(2.13)

Then, (2.12) and (2.13) yield that

\[
Tx(t) \geq \theta \| Tx \|
\]

which implies that \(T: P \to P\). According to the Ascoli–Arzela theorem, we can easily check that \(T: P \to P\) is a completely continuous operator. \(\square\)

3. Main result

Theorem. Suppose that \((H_1)-(H_6)\) hold. Then there exists \(\lambda^* > 0\) such that the m-point boundary value problem (1.1) has at least one positive solution for any \(\lambda \in (0, \lambda^*)\).

Proof. By Lemma 2.4, we know \(T\) is a completely continuous operator; let \(\Omega_1 = \{x \in C[0, 1]: \|x\| < \eta\}\), where \(\eta\) is given in (2.8). Choose

\[
\lambda^* = \min \left\{ 1, \frac{\int_0^1 G(s, s) r(s) ds}{\theta \left( \max_{0 \leq t \leq \eta} h(t) + 1 \right) \int_0^1 G(s, s) [z(s) + r(s)] ds} \right\},
\]

where \(\theta\) is defined by (2.3). Then for any \(x \in P \cap \partial \Omega_1\), notice that \(0 \leq [x(s)]^* \leq x(s) \leq \|x\| = \eta\); we have

\[
Tx(t) \leq \lambda \int_0^1 G(s, s) \left[ z(s) h([x(s)]^*) + r(s) \right] ds
+ \lambda \left( A \psi(1) + B \phi(0) \right) \int_0^1 G(s, s) \left[ z(s) h([x(s)]^*) + r(s) \right] ds
\leq \lambda \left( 1 + A \psi(1) + B \phi(0) \right) \left( \max_{0 \leq t \leq \eta} h(t) + 1 \right) \int_0^1 G(s, s) [z(s) + r(s)] ds
\leq \eta = \|x\|.
\]

Therefore, \(\|Tx\| \leq \|x\|\), \(x \in P \cap \partial \Omega_1\).

Now choose a real number \(K > 0\) such that

\[
1 \leq \frac{\lambda K \theta}{\eta + 1} \min_{\xi_1 \leq t \leq \xi_1} \int_{\frac{t}{2}}^{\xi_1} G(t, s) ds.
\]

By \((H_6)\), for any \(t \in [\frac{1}{2} \xi_1, \xi_1]\), there exists a constant \(N > 0\) such that

\[
\frac{f(t, x)}{x} > K, \quad x > N.
\]  

(3.1)
Choose \( R = \max \left\{ \lambda (\eta + 1), \eta + 1, \frac{N(\eta + 1)}{\theta} \right\} \). Let \( \Omega_2 = \{ x \in C[0, 1]; \| x \| < R \} \). Then for any \( x \in P \cap \partial \Omega_2 \), we have

\[
 x(t) - w(t) \geq x(t) - \lambda \eta \theta \geq x(t) - \frac{\lambda \eta}{R} x(t) \geq \left( 1 - \frac{\lambda \eta}{R} \right) x(t) \\
 \geq \left( 1 - \frac{\lambda \eta}{\lambda (\eta + 1)} \right) x(t) \geq \frac{1}{\eta + 1} x(t) \geq 0, \quad t \in [0, 1].
\]

And so,

\[
 \min_{t \in [\frac{1}{2}\xi_1, \xi_1]} (x(t) - w(t)) \geq \min_{t \in [\frac{1}{2}\xi_1, \xi_1]} \frac{1}{\eta + 1} x(t) \geq \frac{\theta R}{\eta + 1} \geq N.
\] (3.2)

Therefore from (3.1) and (3.2), for any \( x \in P \cap \partial \Omega_2 \), we have

\[
 \min_{\frac{1}{2}\xi_1 \leq t \leq \xi_1} T x(t) = \min_{\frac{1}{2}\xi_1 \leq t \leq \xi_1} \lambda \left\{ \int_0^1 G(t, s) f^*(s, x) ds + A(f^*) \psi(t) + B(f^*) \phi(t) \right\} \\
 \geq \lambda \min_{\frac{1}{2}\xi_1 \leq t \leq \xi_1} \int_0^1 G(t, s) f^*(s, x) ds \geq \lambda \min_{\frac{1}{2}\xi_1 \leq t \leq \xi_1} \int_0^{\xi_1} G(t, s)[f(s, [x(s)])^* + r(s)] ds \\
 \geq \lambda K \min_{\frac{1}{2}\xi_1 \leq t \leq \xi_1} \int_0^{\xi_1} G(t, s) x(s) - w(s) ds \geq \frac{\lambda K \theta}{\eta + 1} \min_{\frac{1}{2}\xi_1 \leq t \leq \xi_1} \int_0^{\xi_1} G(t, s) ds R \\
 \geq \frac{\lambda K \theta}{\eta + 1} \min_{\frac{1}{2}\xi_1 \leq t \leq \xi_1} \int_0^{\xi_1} G(t, s) \| x \| \geq \| x \|. 
\]

Thus \( \| Tx \| \geq \| x \| \), \( x \in P \cap \partial \Omega_2 \). By Lemma 1.1, \( T \) has a fixed point \( x \) such that \( \eta \leq \| x \| \leq R \).

Since \( \| x \| \geq \eta \), we have \( x(t) - w(t) \geq \eta \theta - \lambda \eta \theta \geq (1 - \lambda) \eta \theta \geq 0 \). (3.3)

By direct computation and combining (3.3), we have

\[
 \begin{align*}
 (p(t)x'(t))' - q(t)x(t) + \lambda \left[ f(t, x(t) - w(t)) + r(t) \right] &= 0, \quad t \in (0, 1), \\
 ax(0) - bp(0)x'(0) = \sum_{i=1}^{m-2} \alpha_i x(\xi_i), & \quad cx(1) + dp(1)x'(1) = \sum_{i=1}^{m-2} \beta_i x(\xi_i).
\end{align*}
\]

Let \( u(t) = x(t) - w(t) \). Then \( (p(t)x'(t))' = (p(t)u'(t))' + (p(t)w'(t))' \) and \( x'(t) = u'(t) - w'(t) \). Notice that \( w \) is a solution of (2.4); then

\[
 \begin{align*}
 (p(t)u'(t))' - q(t)u(t) + \lambda f(t, u(t)) &= 0, \quad t \in (0, 1), \\
 au(0) - bp(0)u'(0) = \sum_{i=1}^{m-2} \alpha_i u(\xi_i), & \quad cu(1) + dp(1)u'(1) = \sum_{i=1}^{m-2} \beta_i u(\xi_i).
\end{align*}
\]

Thus \( u(t) \) is a positive solution of the singular semipositone \( m \)-point boundary value problem (1.1). The proof is completed. \( \square \)

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