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The νMSM, dark matter and neutrino masses

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Abstract

We investigate an extension of the Minimal Standard Model by right-handed neutrinos (the νMSM) to incorporate neutrino masses consistent with oscillation experiments. Within this theory, the only candidates for dark matter particles are sterile right-handed neutrinos with masses of a few keV. Requiring that these neutrinos explain entirely the (warm) dark matter, we find that their number is at least three. We show that, in the minimal choice of three sterile neutrinos, the mass of the lightest active neutrino is smaller than $\mathcal{O}(10^{-5})$ eV, which excludes the degenerate mass spectra of three active neutrinos and fixes the absolute mass scale of the other two active neutrinos.

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1. Introduction

In the past decade, neutrino experiments have provided convincing evidence for neutrino masses and mixings. The anomaly in atmospheric neutrinos is now understood by $\nu_\mu \to \nu_\tau$ oscillation [1], while the solar neutrino puzzle is solved by the oscillation $\nu_e \to \nu_{\mu,\tau}$ [2,3] incorporating the MSW LMA solution [4]. Current data are consistent with flavor oscillations between three active neutrinos,[1] and show that the mass squared differences are $\Delta m^2_{\rm atm} = [2.2^{+0.6}_{-0.4}] \times 10^{-3}\ {\rm eV}^2$ and $\Delta m^2_{\rm sol} = [8.2^{+0.3}_{-0.3}] \times 10^{-5}\ {\rm eV}^2$ [7]. These phenomena demand physics beyond the minimal standard model (MSM), and various possibilities to incorporate neutrino masses in the theory have been proposed [8]. The simplest one is adding $\mathcal{N}$ right-handed $SU(2) \times U(1)$ singlet neutrinos N_I ($I = 1, \ldots, \mathcal{N}$) with most general gauge-invariant and renormalizable interactions described by the Lagrangian

$$\delta\mathcal{L} = \bar{N}_I i\partial_\mu \gamma^\mu N_I - f^\nu_{I\alpha} \Phi^\dagger \bar{N}_I L_\alpha - \frac{M_I}{2} \bar{N}^c_I N_I + \text{h.c.}, \tag{1}$$

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[1] We do not include here the LSND anomaly [5], which will be tested in the near future [6].

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where Φ and L_α ($\alpha = e, \mu, \tau$) are the Higgs and lepton doublets, respectively, and both Dirac ($M^D = f^\nu \langle \Phi \rangle$) and Majorana ($M_I$) masses for neutrinos are introduced. We have taken a basis in which mass matrices of charged leptons and right-handed neutrinos are real and diagonal. We shall call this model "the ν *Minimal Standard Model* (the νMSM)" (not to be confused with "the new MSM" of [9]). This model satisfies all the principles of quantum field theory which were so successful in the construction of the MSM. It should be thus thoroughly studied as the simplest and experimentally-motivated extension of the MSM.

The νMSM with $\mathcal{N}$ singlet neutrinos contains quite a number of free parameters, i.e., Dirac ($M^D_{I,\alpha}$) and Majorana (M_I) masses. For example, for $\mathcal{N} = 2$ the number of extra real parameters is 11 (2 Majorana masses, 2 Dirac masses, 4 mixing angels and 3 CP-violating phases), whereas for $\mathcal{N} = 3$ this number is 18 (3 Majorana masses, 3 Dirac masses, 6 mixing angels and 6 CP-violating phases). These parameters can be constrained by the observation of neutrino oscillations. The immediate consequence of the existence of two distinct scales $\Delta m^2_{\mathrm{atm}}$ and $\Delta m^2_{\mathrm{sol}}$ is that the number of right-handed neutrinos must be $\mathcal{N} \geqslant 2$. However, we know little about the absolute values of masses for active neutrinos as well as right-handed neutrinos. This is simply because the oscillation experiments tell us only about the mass squared differences of active neutrinos.

On the other hand, cosmology can play an important role to restrict the parameter space of the νMSM. Recently, various cosmological observations have revealed that the universe is almost spatially flat and mainly composed of dark energy ($\Omega_\Lambda = 0.73 \pm 0.04$), dark matter ($\Omega_{\mathrm{dm}} = 0.22 \pm 0.04$) and baryons ($\Omega_b = 0.044 \pm 0.004$) [10]. The νMSM can potentially explain dark matter Ω_{dm} and baryon Ω_b abundances, and can be consistent with the dark energy requirement via the introduction of a small cosmological constant.

To be more precise, the baryon asymmetry of the universe (Ω_b) can be produced via the leptogenesis mechanism [11] or via neutrino oscillations [12] with the use of anomalous electroweak fermion number non-conservation at high temperatures [13]. Furthermore, the νMSM can offer a candidate for dark matter. The present energy density of active neutrinos is severely constrained from the observations of the large scale structure. The recent analysis [14] shows that the sum of active neutrino masses should be smaller than 0.42 eV and $\Omega_\nu h^2 \leqslant 4.5 \times 10^{-3}$, which is far below the observed Ω_{dm}. The unique dark-matter candidate in the νMSM is then a right-handed neutrino which is stable within the age of the universe. Indeed, it has been shown in [15–19] that sterile right-handed neutrinos with masses of $\mathcal{O}(1)$ keV are good candidates for warm dark matter. Note that, in our analysis, we take the very conservative assumption of the validity of the standard Big Bang at temperatures below 1 GeV and disregard the possibilities of extremely low reheating temperatures of inflation as $T_R \lesssim 1$ GeV [20].

In this Letter, we explore the hypothesis that the νMSM is a correct low-energy theory which incorporates dark matter. We demonstrate that the theory with $\mathcal{N} = 2$ fails to do so. We show that for the choice $\mathcal{N} = 3$ the mass of the lightest active neutrino m_1 is constrained from above by the value $\mathcal{O}(10^{-5})$ eV, and therefore, that the masses of other neutrinos are fixed to be $m_2 = \sqrt{\Delta m^2_{\mathrm{sol}}}$ and $m_3 = \sqrt{\Delta m^2_{\mathrm{atm}} + \Delta m^2_{\mathrm{sol}}}$ in the normal or $m_2 = \sqrt{\Delta m^2_{\mathrm{atm}}}$ and $m_3 = \sqrt{\Delta m^2_{\mathrm{atm}} + \Delta m^2_{\mathrm{sol}}}$ in the inverted hierarchy of neutrino masses, respectively. This rejects the possibility that all active neutrinos are degenerate in mass. In other words, for a most natural choice of $\mathcal{N} = 3$, the cosmological observation of dark matter allows one to make a (potentially) testable prediction on the active neutrino masses and on the existence of a sterile neutrino with a mass in the keV range. We stress that these results are valid in spite of a large number of free parameters of the νMSM. Finally, for $\mathcal{N} \geqslant 4$, no model-independent extra constraints on the masses of active neutrino can be derived.

2. Neutrino masses and mixing

Let us first discuss neutrino masses and mixing in the νMSM. We will restrict ourselves to the region in which the Majorana neutrino masses are larger than the Dirac masses, so that the seesaw mechanism [21] can be applied. Note that this does not reduce generality since the latter situation automatically appears when we require the sterile neutrinos to play a role of dark matter, as we shall see. Then, right-handed neutrinos N_I become approximately the mass eigenstates with $M_1 \leqslant M_2 \leqslant \cdots \leqslant M_{\mathcal{N}}$, while other eigenstates can be found by diagonalizing the mass matrix

$$M^\nu = \left(M^D\right)^T M_I^{-1} M^D \tag{2}$$

which we call the seesaw matrix. The mass eigenstates ν_i ($i = 1, 2, 3$) with $m_1 \leqslant m_2 \leqslant m_3$ are found from

$$U^T M^\nu U = M^\nu_{\rm diag} = {\rm diag}(m_1, m_2, m_3), \tag{3}$$

and the mixing in the charged current is expressed by $\nu_\alpha = U_{\alpha i}\nu_i + \Theta_{\alpha I} N_I^c$ where $\Theta_{\alpha I} = (M^D)^\dagger_{\alpha I} M_I^{-1} \ll 1$ under our assumption. This is the reason why right-handed neutrinos N_I are often called "sterile" while ν_i "active".

As explained before, $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\rm sol}$ require the number of sterile neutrinos $\mathcal{N} \geqslant 2$. For the minimal choice $\mathcal{N} = 2$, one of the active neutrinos is exactly massless ($m_1 = 0$). For $\mathcal{N} \geqslant 3$ the smallest mass can be in the range $0 \leqslant m_1 \lesssim \mathcal{O}(0.1)$ eV [14]. In particular, the degenerate mass spectra of active neutrinos are possible when $m_1^2 \gtrsim \Delta m^2_{\rm atm}$. Note also that there are two possible hierarchies in the masses of active neutrinos, i.e., $\Delta m^2_{\rm atm} = m_3^2 - m_2^2$ ($m_2^2 - m_1^2$) and $\Delta m^2_{\rm sol} = m_2^2 - m_1^2$ ($m_3^2 - m_2^2$) in the normal (inverted) hierarchy.

3. Sterile neutrino as warm dark matter

In the νMSM, the only candidates for dark matter are the long-lived sterile neutrinos. Let us discuss here the requirements for this scenario.

A sterile neutrino, say N_1, decays mainly into three active neutrinos in the interesting mass range $M_1 \ll m_e$ (see Eq. (7)) and its lifetime is estimated as [22]

$$\tau_{N_1} = 5 \times 10^{26}\ {\rm s}\left(\frac{M_1}{1\ {\rm keV}}\right)^{-5}\left(\frac{\bar{\Theta}^2}{10^{-8}}\right)^{-1}, \tag{4}$$

where we have taken $|\Theta_{\alpha 1}| = \bar{\Theta}$ for $\alpha = e, \mu, \tau$. We can see that it is stable within the age of the universe $\sim 10^{17}$ s in some region of the parameter space (M_1, Θ).

When the active-sterile neutrino mixing $|\Theta_{\alpha I}|$ is sufficiently small, the sterile neutrino N_I has never been in thermal equilibrium and is produced in non-equilibrium reactions. The production processes include various particle decays and conversions of active into sterile neutrinos (see Ref. [23]). The dominant production mechanism is due to the active-sterile neutrino oscillations [16,18,19], and the energy fraction of the present universe from the sterile neutrino(s) is [18,19]

$$\Omega_N h^2 \sim 0.1 \sum_I \sum_{\alpha=e,\mu,\tau}\left(\frac{|\Theta_{\alpha I}|^2}{10^{-8}}\right)\left(\frac{M_I}{1\ {\rm keV}}\right)^2, \tag{5}$$

where the summation of I is taken over the sterile neutrino N_I being dark matter. The most effective production occurs when the temperature is $T_* \simeq (130\ {\rm MeV})(M_I/1\ {\rm keV})^{1/3}$ [16,24]. Here we assumed for simplicity the flavor universality among leptons in the hot plasma, which is actually broken since $T_* \leqslant m_\tau$. However, its effect does not alter our final results. Further, we have taken the lepton asymmetry at the production time to be small ($\sim 10^{-10}$), which is a most conservative assumption. In this case there is no resonant production of sterile neutrinos coming from large lepton asymmetries [17,19]. We therefore find from the definition of Θ that the correct dark-matter density is obtained if

$$\sum_I \sum_{\alpha=e,\mu,\tau} \left|M^D_{I\alpha}\right|^2 = m_0^2, \tag{6}$$

where $m_0 = \mathcal{O}(0.1)$ eV. Notice that this constraint on dark-matter sterile neutrinos is independent of their masses, at least for M_I in the range discussed below.

The sterile neutrino, being warm dark matter, further receives constraints from various cosmological observations and the possible mass range is very restricted as

$$2 \lesssim M_I \lesssim 5\ {\rm keV}, \tag{7}$$

where the lower bound comes from the cosmic microwave background and the matter power spectrum inferred from Lyman-α forest data [25], while the upper bound is given by the radiative decays of sterile neutrinos in dark matter halos limited by X-ray observations [26]. (See also Ref. [27].) These constraints are somewhat stronger than the one coming from Eq. (4).

4. Consequence of sterile neutrino dark matter

We have found that the hypothesis of sterile neutrinos being warm dark matter is realized in the νMSM when the two constraints (6) and (7) are satisfied. We shall now see that they put important bounds on the number of sterile neutrinos and on the masses of the active ones. To find them, let us first rewrite the diagonalized seesaw mass matrix (3) in the form

$$M^\nu_{\rm diag} = S_1 + \cdots + S_{\mathcal{N}}, \tag{8}$$

where S_I denotes a contribution from each sterile neutrino and is given by $(S_I)_{ij} = X_{Ii}X_{Ij}$ with $X_{Ii} = (M^D U)_{Ii}/\sqrt{M_I}$. Note that each matrix satisfies the relation $\det S_I = \det(S_I + S_J) = 0$ from its construction. The condition (6) is then written as

$$\sum_I \sum_{i=1}^{3} \frac{M_I}{M_1}|X_{Ii}|^2 = \frac{m_0^2}{M_1} \equiv m_\nu^{\rm dm}, \tag{9}$$

and the mass range in Eq. (7) gives

$$m_\nu^{\rm dm} = \mathcal{O}\bigl(10^{-5}\bigr)\ {\rm eV}. \tag{10}$$

First of all, let us show that the minimal possibility $\mathcal{N} = 2$ cannot satisfy the dark-matter constraints and the oscillation data simultaneously. In this case, the lightest active neutrino becomes massless ($m_1 = 0$). By taking the trace of both sides in Eq. (8), we find that

$$m_2 + m_3 = \sum_{i=1}^{3}\bigl(X_{1i}^2 + X_{2i}^2\bigr). \tag{11}$$

This equation must hold for both real and imaginary parts. When both sterile neutrinos N_1 and N_2 are assumed to be dark matter, the condition (9) together with M_1 and M_2 in Eq. (7) leads to

$$m_2 + m_3 \leqslant \sum_{i=1}^{3}\bigl(|X_{1i}|^2 + |X_{2i}|^2\bigr) \leqslant m_\nu^{\rm dm}. \tag{12}$$

This inequality cannot be satisfied since $m_\nu^{\rm dm} = \mathcal{O}(10^{-5})$ eV and $m_3 = \sqrt{\Delta m^2_{\rm atm} + \Delta m^2_{\rm sol}} \simeq 5 \times 10^{-2}$ eV from neutrino oscillations.

Further, when only one of two sterile neutrinos, say N_1, is assumed to be dark matter, its Dirac Yukawa couplings are restricted as shown in Eq. (9). Although the couplings of N_2 can be taken freely, they are not important for our discussion. What we shall use here is the simple fact that the determinant of the matrix S_2 in Eq. (8) is zero. Then, the equation $\det(S_2) = \det(M^\nu_{\rm diag} - S_1) = 0$ induces $X_{11}^2 m_2 m_3 = 0$, which is satisfied only if $X_{11} = 0$ since $m_{2,3} \neq 0$ from the oscillation data. This means that the first row and column of S_1 vanish, and the matrix S_2 should have the same structure ($X_{21} = 0$) because $M_\nu^{\rm diag}$ is diagonal and $m_1 = 0$. Then, Eq. (8) is reduced to that for 2×2 matrices:

$$\mathrm{diag}(m_2, m_3) = X_{1i}X_{1j} + X_{2i}X_{2j} \quad (i, j = 2, 3). \tag{13}$$

The vanishing determinant of the second matrix on the right-hand side leads to

$$m_2 = X_{12}^2 + \frac{m_2}{m_3}X_{13}^2. \tag{14}$$

By taking into account the dark matter constraint $|X_{12}|^2 + |X_{13}|^2 = m_\nu^{\rm dm}$, we obtain the upper bound on m_2:

$$m_2 \leqslant m_\nu^{\rm dm}. \tag{15}$$

This inequality is inconsistent with $m_\nu^{\rm dm}$ in Eq. (10) and $m_2 = \sqrt{\Delta m^2_{\rm sol}} \simeq 9 \times 10^{-3}$ eV or $\sqrt{\Delta m^2_{\rm atm}}$ for the normal or inverted hierarchy cases, respectively. The same discussion can be applied to the case when only the heavier sterile neutrino N_2 is dark matter. Therefore, we have shown that in the $\mathcal{N} = 2$ νMSM the requirements on dark matter conflict with the oscillation data.

We then turn to discuss the case $\mathcal{N} = 3$. First, when all three sterile neutrinos play a role of dark matter simultaneously, the real part of the trace of Eq. (8) gives

$$m_1 + m_2 + m_3 \leqslant \sum_{I=1}^{3}\sum_{i=1}^{3}|X_{Ii}|^2 \leqslant m_\nu^{\rm dm}, \tag{16}$$

where the final inequality comes from the dark matter constraint (9) as in the previous case. Although we do not know the overall scale of m_i from the oscillation data, the heaviest one m_3 should be larger than $\sqrt{\Delta m^2_{\rm atm}}$ in any case. Then, this inequality cannot be satisfied by $m_\nu^{\rm dm}$ in Eq. (10) and this situation is excluded.

Next, we consider the case when two of the three sterile neutrinos, say N_1 and N_2, are dark matter. In this case, from the real part of the trace of Eq. (8), we find that

$$m_1 + m_2 + m_3 \leqslant m_\nu^{\rm dm} + \sum_{i=1}^{3}\mathrm{Re}\,X_{3i}^2, \tag{17}$$

and thus $\sum \mathrm{Re}\, X_{3i}^2 > m_3$ since $m_\nu^{\mathrm{dm}} \ll \sqrt{\Delta m_{\mathrm{sol}}^2} \leqslant m_2$. On the other hand, it is found from $\det(S_1 + S_2) = \det(M_{\mathrm{diag}}^\nu - S_3) = 0$ that, if $m_1 \neq 0$,

$$1 = \frac{X_{31}^2}{m_1} + \frac{X_{32}^2}{m_2} + \frac{X_{33}^2}{m_3}. \tag{18}$$

However, this equation cannot be satisfied, since the real part of the right-hand side is bounded from below as

$$\frac{\mathrm{Re}\, X_{31}^2}{m_1} + \frac{\mathrm{Re}\, X_{32}^2}{m_2} + \frac{\mathrm{Re}\, X_{33}^2}{m_3} > \frac{\sum \mathrm{Re}\, X_{3i}^2}{m_3} > 1. \tag{19}$$

If $m_1 = 0$, $\det(M_{\mathrm{diag}}^\nu - S_3) = 0$ gives us $X_{31} = 0$. This results in that M_{diag}^ν and S_3 as well as $(S_1 + S_2)$ are reduced to 2×2 matrices, which verify $\det S_3 = \det(M_{\mathrm{diag}}^\nu - S_1 - S_2) = 0$, i.e.,

$$\begin{aligned}&\left(m_2 - X_{12}^2 - X_{22}^2\right)\left(m_3 - X_{13}^2 - X_{23}^2\right) \\ &\quad = (X_{12}X_{13} + X_{22}X_{23})^2.\end{aligned} \tag{20}$$

This equation cannot be satisfied by X_{Ii} restricted by the dark matter constraint (9). Thus, this case is also excluded in either $m_1 = 0$ or $m_1 \neq 0$ situations.

Finally, let us consider the remaining possibility, i.e., assume that only one sterile neutrino (e.g., N_1) becomes a dark matter particle. In this case, we also note that $\det(S_2 + S_3) = \det(M_{\mathrm{diag}}^\nu - S_1) = 0$, which induces

$$m_1 = X_{11}^2 + \frac{m_1}{m_2} X_{12}^2 + \frac{m_1}{m_3} X_{13}^3. \tag{21}$$

Now, the dark matter constraint (9) takes the form: $\sum_{i=1}^3 |X_{1i}|^2 = m_\nu^{\mathrm{dm}}$. It is then found that the lightest active neutrino should verify

$$m_1 \leqslant m_\nu^{\mathrm{dm}}. \tag{22}$$

This shows that, when $\mathcal{N} = 3$, there exists a region in the parameter space of the νMSM consistent with the observation of neutrino oscillations and in which one of sterile neutrinos becomes the warm dark matter of the universe. Finally, we should stress here that the above argument holds independently of the mixing angles of neutrinos in U.

If the number of sterile neutrinos is greater than the number of fermionic generations, no general constraints on the masses of active neutrinos can be derived, since extra sterile neutrinos may be almost decoupled from the active neutrinos and thus do not contribute to the seesaw formula. At the same time, they can easily satisfy the dark matter constraint.

5. Conclusions

Let us summarize the obtained results. First, we have shown that the νMSM can explain the dark matter in the universe only provided $\mathcal{N} \geqslant 3$, although the neutrino oscillation experiments allow $\mathcal{N} = 2$. Interestingly, in this successful and minimal scenario with $\mathcal{N} = 3$, the number of sterile neutrinos is the same as the number of families of quarks and leptons. Second, in the $\mathcal{N} = 3$ case, the mass of the lightest active neutrino should lie in the range $m_1 \leqslant m_\nu^{\mathrm{dm}} = \mathcal{O}(10^{-5})$ eV, which is much smaller than $\sqrt{\Delta m_{\mathrm{sol}}^2}$. This clearly excludes the possibility that three active neutrinos are degenerate in mass and fixes their masses to be $m_3 = [4.8^{+0.6}_{-0.5}] \times 10^{-2}$ eV and $m_2 = [9.05^{+0.2}_{-0.1}] \times 10^{-3}$ eV ($[4.7^{+0.6}_{-0.5}] \times 10^{-2}$ eV) in the normal (inverted) hierarchy. An experimental test of the $\mathcal{N} = 3$ νMSM origin of dark matter would be the discovery of a keV sterile neutrino by the X-ray observatories [26] and the finding of the active neutrino masses in the predicted range.

Finally, we should mention that the sterile neutrinos irrelevant to dark matter can be responsible for the baryon asymmetry of the universe through leptogenesis [11] or neutrino oscillations [12]. These considerations would restrict further the parameter space of the νMSM. For example, the conventional thermal scenario [28] works when the lightest among them is about 10^{10} GeV. The other scenario using neutrino oscillations requires masses of 100 GeV $\gg M_I \geqslant 1$ GeV [12].

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