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Transverse magnetic field on Jeffery–Hamel problem with Cu–water nanofluid between two non parallel plane walls by using collocation method

I. Rahimi Petroudi [a], D.D. Ganji [b,*], M. Khazayi Nejad [a], J. Rahimi [c], E. Rahimi [d], A. Rahimifar [b]

[a] Young Researchers Club, Sari Branch, Islamic Azad University, Sari, Iran
[b] Department of Mechanical Engineering, Babol University of Technology, Babol, Iran
[c] Department of Electrical and Computer Engineering, Babol University of Technology, Babol, Iran
[d] Department of Mechanical Engineering, Science and Research Branch, Islamic Azad University, Arak, Iran

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ABSTRACT

An analysis has been performed to study the problem of magneto-hydrodynamic (MHD) Jeffery–Hamel flow with nanoparticles. The governing equations for this problem are reduced to an ordinary form and is solved using collocation method (CM) and numerically by fourth order Runge–Kutta technique. Also, Velocity fields have been computed and shown graphically for various values of physical parameters. The objective of the present work is to investigate the effect of the semi angles between the plates, Reynolds number, magnetic field strength and nanoparticles volume fraction on the velocity field. As an important outcome, Increasing Reynolds numbers leads to reduce velocity and excluded backflow in convergent channel.

1. Introduction

During last few years the incompressible viscous fluid flow through convergent -divergent channels is one of the most applicable cases in fluid mechanics, civil, environmental, mechanical and bio-mechanical engineering. The mathematical investigations of this problem were pioneered by Jeffery [1] and Hamel [2] and so, it is known as Jeffery–Hamel problem, too. One of the most significant examples of Jeffery Hamel problems are those subjected to an applied magnetic field. The MHD Jeffery–Hamel problem have been extensively studied by several authors and discussed in some textbooks and articles: [3–7] etc. The term nanofluid was envisioned to describe a fluid in which nanometer-sized particles were suspended in conventional heat transfer basic fluids. Nanotechnology aims to manipulate the structure of the matter at the molecular level with the goal for innovation in virtually every industry and public endeavor including biological sciences, physical sciences, electronics cooling, transportation, the environment and national security [8,9].

As everyone knows, most scientific problems such as Jeffery–Hamel flows and other fluid mechanic problems are inherently nonlinear. In most cases, these problems do not admit analytical solution, so these equations should be solved

* Corresponding author. Tel.: +98 111 32 34 501; fax: +98 111 32 34 201.
E-mail address: ddg_davood@yahoo.com (D.D. Ganji).

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Nomenclature

Re	Reynolds number
H	Hartman number
A^*	particle parameter
$R(x)$	residual function
W_i	weighted function
c_i	constant
CM	collocation method
NUM	numerical method
A^*	particle parameter
P	pressure
r	cylindrical coordinates
f	non-dimensional velocity
u	velocity component in radial-direction
$\tilde{u}$	trial function
r	cylindrical coordinates
η	non-dimensional angle

Greek symbols

ρ	density of the fluid
υ	kinematic viscosity
ϕ	nanoparticle volume fraction
α	angle between two plates
θ	cylindrical coordinates

using special techniques. In recent decades, much attention has been devoted to the newly developed methods to construct an analytic solution of equation; such as Perturbation techniques which are too strongly dependent upon the so-called "small parameters" [10]. Many other different methods have introduced to solve nonlinear equation such as the Adomian's decomposition method [11], homotopy perturbation method (HPM) [12–15], variational iteration method (VIM) [16], differential transformation method [17], homotopy asymptotic method (HAM) [18–21] and optimal homotopy asymptotic method (OHAM) [22] and collocation method (CM) [23,24].

Many advantages of CM compared to other analytical and numerical methods make it more valuable and motivate researchers to use it for solving problems. It has the following benefits: First, unlike all previous analytic techniques, It solves the equations directly and no simplifications needs. Second, unlike all previous analytic techniques, It does not need to any perturbation, linearization or small parameter versus homotopy perturbation method (HPM) and parameter perturbation method (PPM). Third, unlike homotopy asymptotic method, it does not need to determine the auxiliary function and parameter versus HAM.

The main purpose of this study is to apply collocation method to find approximate solutions of the velocity profile on MHD Jeffery–Hamel flow with nanoparticles. A clear conclusion can be drawn from the numerical method's (*NUM*) results that the collocation method provides highly accurate solutions for nonlinear differential equations.

2. Problem statement and mathematical formulation

We consider the boundary layer flow of an electrically conducting viscous fluid with nanoparticle. A magnetic field $B(x)$ acts transversely to the flow. As can be seen in Fig. 1.

The steady two-dimensional flow of an incompressible conducting viscous fluid from a source or sink at the intersection between two non parallel plane walls is considered. We assume that the velocity is purely radial and depends on r and θ only. The governing equations in polar coordinates are [11,17]

$$\frac{\rho_{nf}}{r}\frac{\partial}{\partial r}(r\,u(r,\theta))=0 \tag{1}$$

$$u(r,\theta)\frac{\partial u(r,\theta)}{\partial r}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial r}+\upsilon_{nf}\left[\frac{\partial^2 u(r,\theta)}{\partial r^2}+\frac{1}{r}\frac{\partial u(r,\theta)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u(r,\theta)}{\partial \theta^2}-\frac{u(r,\theta)}{r^2}\right]-\frac{\sigma B_0^2}{\rho_{nf}\,r^2}u(r,\theta) \tag{2}$$

$$\frac{1}{\rho_{nf}\,r}\frac{\partial p}{\partial \theta}-\frac{2\upsilon_{nf}}{r^2}\frac{\partial u(r,\theta)}{\partial \theta}=0 \tag{3}$$

here B_0 is the electromagnetic induction, $u(r)$ is the velocity along radial direction, P is the fluid pressure, σ is the conductivity of the fluid, ρ_{nf} is the density of fluid and υ_{nf} is the coefficient of kinematic viscosity. By introducing φ as a solid volume fraction, fluid density, dynamic viscosity and the kinematic viscosity of nanofluid can be written as follows:

$$\rho_{nf}=\rho_f(1-\phi)+\rho_s\phi_s \tag{4}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}} \tag{5}$$

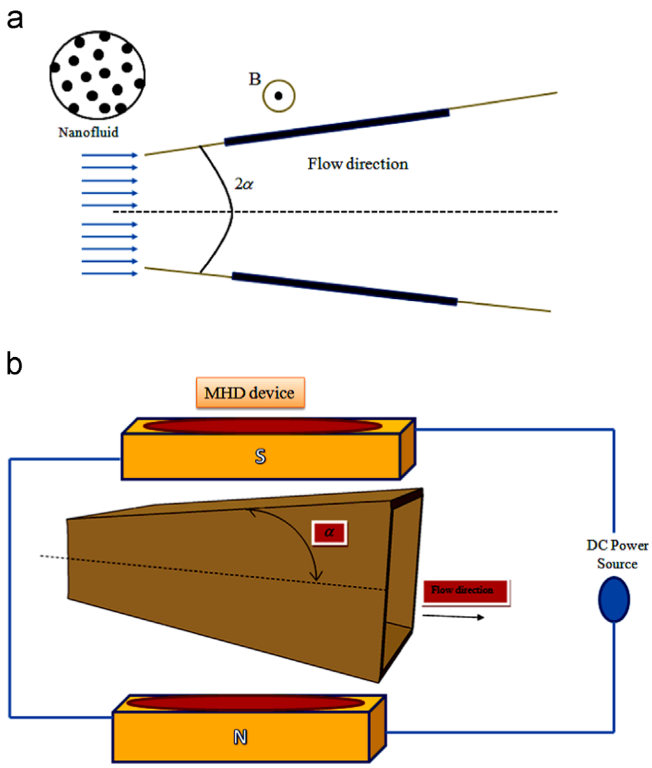

Fig. 1. Geometry of the MHD Jeffery–Hamel flow in divergent channel (a) 2D view and (b) schematic setup of problem.

$$\upsilon_{nf} = \frac{\mu_f}{\rho_{nf}} \tag{6}$$

From Eq. (1) and using dimensionless parameters we get

$$f(\theta) = r\, u(r, \theta) \tag{7}$$

$$f(\eta) = \frac{f(\theta)}{f_{\max}}, \quad \eta = \frac{\theta}{\alpha}. \tag{8}$$

Substituting Eq. (5) into Eqs. (2) and (3) and eliminating P, we obtain an ordinary differential equation for the normalized function profile $f(\eta)$:

$$f'''(\eta) + 2\,\alpha\; ReA^{*}\,(1-\phi)^{2.5} f(\eta) f'(\eta) + \left(4 - (1-\phi)^{1.25} H\right)\alpha^2 f'(\eta) = 0. \tag{9}$$

The relevant boundary conditions are

$$f(0) = 1, \quad f'(0) = 0, \quad f(1) = 0. \tag{10}$$

The Reynolds number is

$$Re = \frac{f_{\max}\,\alpha}{\upsilon} = \frac{U_{\max}\, r\alpha}{\upsilon}\left(\begin{array}{c}\text{divergevt} - \text{channel}: \alpha \succ 0, f_{\max} \succ 0 \\ \text{convergevt} - \text{channel}: \alpha \prec 0, f_{\max} \prec 0\end{array}\right). \tag{11}$$

Table 1
Thermophysical properties of nanofluids and nanoparticles.

Material	Density (kg/m^3)	Heat capacity $(J/kg\ K)$	Thermal conductivity $(W/m\ K)$
Al_2O_3	3970	765	40
Cu	8933	385	401
TiO_2	4250	686.2	8.9538
Fluid phase (water)	997.1	4179	0.613

The Hartmann number is

$$H=\sqrt{\frac{\sigma B_0^2}{\rho\,\upsilon}}. \tag{12}$$

The particle parameter is

$$A^*=(1-\phi)+\frac{\rho_s}{\rho_f}\phi \tag{13}$$

Eq. (9) is valid for spherical particles and its values for different materials are listed in Table 1.

3. Principles of collocation method

Suppose we have a differential operator D acting on a function u to produce a function p [24,25]:

$$D(u(x))=p(x) \tag{14}$$

We wish to approximate u by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is:

$$u\cong\tilde{u}=\sum_{i=1}^{n}C_i\varphi_i \tag{15}$$

Now, when substituted into the differential operator,D, the result of the operations is not, in general, $p(x)$. Hence an error or residual will exist:

$$E(x)=R(x)=D(\tilde{u}(x))-p(x)\neq 0 \tag{16}$$

The notion in the collocation is to force the residual to zero in some average sense over the domain. That is

$$\int_x R(x)\,W_i(x)=0 \quad i=1,2,\ldots,n \tag{17}$$

where the number of weight functions W_i are exactly equal the number of unknown constants C_i in $\tilde{u}$. The result is a set of n algebraic equations for the unknown constants C_i. For collocation method, the weighting functions are taken from the family of Dirac δ functions in the domain. That is, $W_i(x)=\delta(x-x_i)$. The Dirac δ function has the property that

$$\delta(x-x_i)=\begin{cases}1 & \text{if } x=x_i\\ 0 & \text{Otherwise}\end{cases} \tag{18}$$

And residual function in Eq. (16) must force to be zero at specific points.

3.1. Implementation of analytical method

Consider the trial function as:

$$f(\eta)=1+c_1\eta^2+c_2\eta^3+c_3\eta^4+c_4\eta^5+c_5\eta^6+c_6\eta^7+c_7\eta^8 \tag{19}$$

Witch satisfies the boundary condition in Eq. (10) and set it into Eq. (16), residual function, $R(c_1,c_2,c_3,c_4,c_5,c_6,c_7,\eta)$, is found as

$$R(c_1,c_2,c_3,c_4,c_5,c_6,c_7,\eta)=\alpha ReA^*(1-\phi)^{2.5}\left[12c_1\eta^5c_3+24c_4\eta^{11}c_6+16c_7\eta^7+4c_1\eta\right.$$

Table 2
Value of unknown constants C_i at various Re, H, and α when $\phi=0.05$.

α		−5		−5		+5		+5	
H	Re	0	100	500	100	50	10	50	200
c_1		−0.18980343050		−0.1170250892		−0.843677670500		0.1850545794	
c_2		−0.51213080310		−0.6790852135		0.0008002290346		−2.2725634490	
c_3		1.71378465000		2.44448599500		−0.177065267200		9.3061472200	
c_4		−5.23821147300		−6.8505244380		0.007455837047		−22.640263420	
c_5		7.13282146600		9.22953945400		0.001303268316		30.302274640	
c_6		−5.78821382800		−7.2538790160		0.006672129808		−22.161738310	
c_7		1.88175341900		2.22648830800		0.004511473517		6.2810887350	

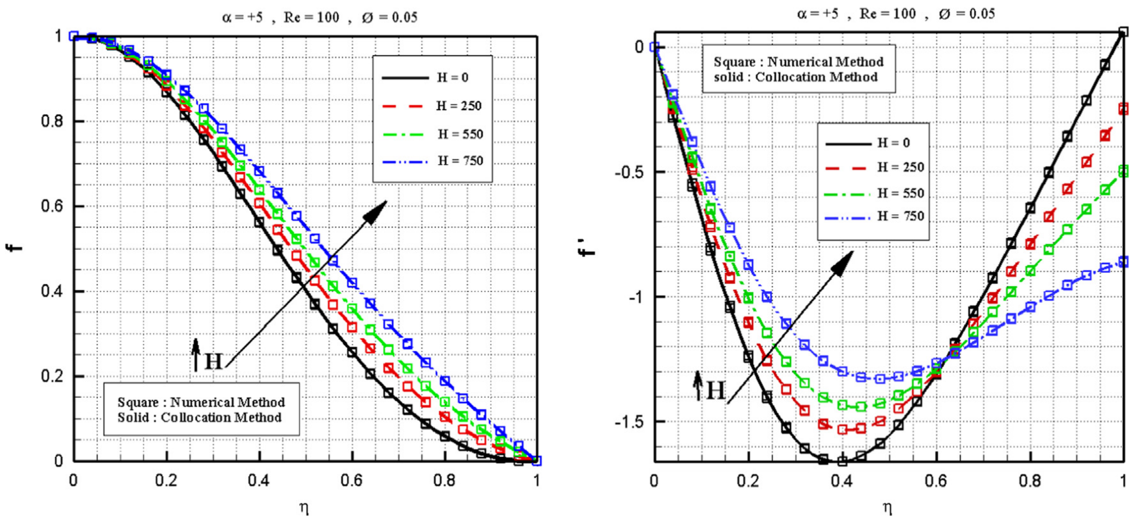

Fig. 2. Effect of Hartmann number (H) on velocity profiles $f(\eta)$ and $f'(\eta)$ when $\alpha=+5$, $Re=100$ and $\phi=0.05$.

$$\begin{aligned}
&+24c_3\eta^{11}c_7+26c_5\eta^{12}c_6+16c_1\eta^7c_5+28c_5\eta^{13}c_7+22c_2\eta^{10}c_7+6\,c_2+30\,c_6\eta^{14}c_7+4\,c_1^2\eta^3\\
&+14c_1\eta^6c_4+18c_3\eta^8c_4+10c_1\eta^4c_2+20c_3\eta^9c_5+18c_1\eta^8c_6+18\,c_2\,\eta^8\,c_5+6\,c_2^2\,\eta^5+14\,c_6^2\eta^{13}\\
&+16c_2\eta^7c_4+26c_4\eta^{12}c_7+14c_2\eta^6c_3+20c_2\eta^9c_6+20c_1\eta^9\,c_7+12c_5\,\eta^5+14c_6\,\eta^6+8\,c_3^2\,\eta^7\\
&+22c_4\eta^{10}c_5+22c_3\eta^{10}c_6+6c_2\eta^2+24c_3\eta+8c_3\eta^3+10c_4\eta^4+10c_4^2\eta^9+12c_5^2\eta^{11}+16c_7^2\eta^{15}]\\
&-5\alpha^2(1-\phi)^{1.25}H\,c_4\eta^4+28\;\alpha^2c_6\eta^6-6\;\alpha^2(1-\phi)^{1.25}H\,c_5\,\eta^5-2\,\alpha^2(1-\phi)^{1.25}H\,c_1\,\eta\\
&-7\alpha^2(1-\phi)^{1.25}H\,c_6\,\eta^6+8\;\alpha^2\,c_1\,\eta+16\,\alpha^2\,c_3\,\eta^3+32\alpha^2c_7\,\eta^7-8\,\alpha^2(1-\phi)^{1.25}H\,c_7\eta^7\\
&+12\,\alpha^2\,c_2\,\eta^2+20\,\alpha^2\,c_4\,\eta^4+60c_4\eta^2+210c_6\,\eta^4-3\,\alpha^2(1-\phi)^{1.25}H\,c_2\,\eta^2+24\alpha^2\,c_5\,\eta^5\\
&-4\alpha^2(1-\phi)^{1.25}Hc_3\eta^3+120c_5\eta^3+336c_7\eta^5=0
\end{aligned} \tag{20}$$

On the other hands, the residual function must be close to zero. For reaching this importance, two specific points in the domain $t\in[0,1]$ should be chosen. These points are

$$\eta_1=\frac{1}{7},\quad \eta_2=\frac{2}{7},\quad \eta_3=\frac{3}{7},\quad \eta_4=\frac{4}{7}\quad \eta_5=\frac{5}{7},\quad \eta_6=\frac{6}{7} \tag{21}$$

Finally by substitutions these points into the residual function, $R(c_n,\eta)$, a set of four equations and seven unknown coefficients are obtained.

In the following, we will obtain the approximate solution of the third order for $\alpha=-5$, $Re=50$, $H=100$ and $\phi=0.05$. In this particular case, the constants $(c_1,c_2,c_3,c_4,c_5,c_6,c_7)$ are

$$\left\{\begin{array}{l} c_1=-0.4573621116, c_2=-0.07130345721, c_3=-0.1390392833\\ , c_4=-0.7602969598, c_5=1.003347820, c_6=-0.9535254918\\ , c_7=0.3781794840\end{array}\right\} \tag{22}$$

Table 3
The results of CM and Numerical methods for $f(\eta)$ and $f'(\eta)$ for $\alpha=-5$, $H=750$, $Re=10$ and $\phi=0.05$.

η	$f(\eta)$			$f'(\eta)$		
	CM	NUM	Error	CM	NUM	Error
0.00	1.000000000	1.000000000	0.00000000	0.000000000	0.000000000	0.000000000
0.10	0.994278317	0.994271793	6.52460E−06	−0.115195704	−0.115276195	8.04910E−05
0.20	0.976670165	0.976657171	1.29939E−05	−0.239148945	−0.239198571	4.96260E−05
0.30	0.945855446	0.945838378	1.70688E−05	−0.380935195	−0.380968632	3.34366E−05
0.40	0.899546175	0.899526435	1.97393E−05	−0.550885223	−0.550904481	1.92576E−05
0.50	0.834341990	0.834321113	2.08771E−05	−0.761001698	−0.761005495	3.7965E−06
0.60	0.745536091	0.745515460	2.06311E−05	−1.025500935	−1.025492459	8.4760E−06
0.65	0.690394661	0.690374623	2.00382E−05	−1.183311622	−1.183296074	1.5548E−05
0.70	0.626869073	0.626850033	1.90395E−05	−1.361279710	−1.361254959	2.4751E−05
0.75	0.553888111	0.553870569	1.75420E−05	−1.561968173	−1.561933069	3.5104E−05
0.80	0.470248162	0.470232596	1.55660E−05	−1.788107079	−1.788063880	4.3199E−05
0.85	0.374605389	0.374592074	1.33156E−05	−2.042570470	−2.042524533	4.5937E−05
0.90	0.265469361	0.265458387	1.09740E−05	−2.328341147	−2.328291696	4.9451E−05
0.95	0.141198775	0.141190959	7.81570E−06	−2.648461763	−2.648373505	8.8258E−05
1.00	0.000000000	0.000000000	0.00000000	−3.005970697	−3.005710902	0.000259795

Table 4
The results of CM and Numerical methods for $f(\eta)$ and $f'(\eta)$ for $\alpha=+5$, $H=600, Re=30$ and $\phi=0.02$.

η	$f(\eta)$			$f'(\eta)$		
	CM	NUM	Error	CM	NUM	Error
0.00	1.000000000	1.0000000000	0.000000000	0.000000000	0.000000000	0.000000000
0.10	0.989831339	0.989838538	7.19830E−06	−0.203096385	−0.203005287	9.10975E−05
0.20	0.959469984	0.959484798	1.48137E−05	−0.403520657	−0.403459692	6.09645E−05
0.30	0.909273894	0.909293747	1.98530E−05	−0.599588715	−0.599547629	4.10865E−05
0.40	0.839713174	0.839736189	2.30155E−05	−0.790897888	−0.790876528	2.13606E−05
0.50	0.751200975	0.751225047	2.40718E−05	−0.979094450	−0.979094189	2.61100E−07
0.60	0.643859464	0.643882692	2.32278E−05	−1.168441426	−1.168457912	1.64860E−05
0.65	0.583016261	0.583038458	2.21973E−05	−1.265734476	−1.265759349	2.48730E−05
0.70	0.517229880	0.517250595	2.07152E−05	−1.366405148	−1.366439696	3.45480E−05
0.75	0.446293494	0.446312228	1.87343E−05	−1.472021770	−1.472066266	4.44960E−05
0.80	0.369913902	0.369930217	1.63148E−05	−1.584479055	−1.584530475	5.14200E−05
0.85	0.287693905	0.287707601	1.36964E−05	−1.706052195	−1.706104499	5.23040E−05
0.90	0.199111677	0.199122776	1.10986E−05	−1.839463171	−1.839516365	5.31940E−05
0.95	0.103496485	0.103504315	7.83060E−06	−1.987961031	−1.988049984	8.89530E−05
1.00	0.000000000	0.000000000	0.000000000	−2.155417798	−2.155678915	0.000261117

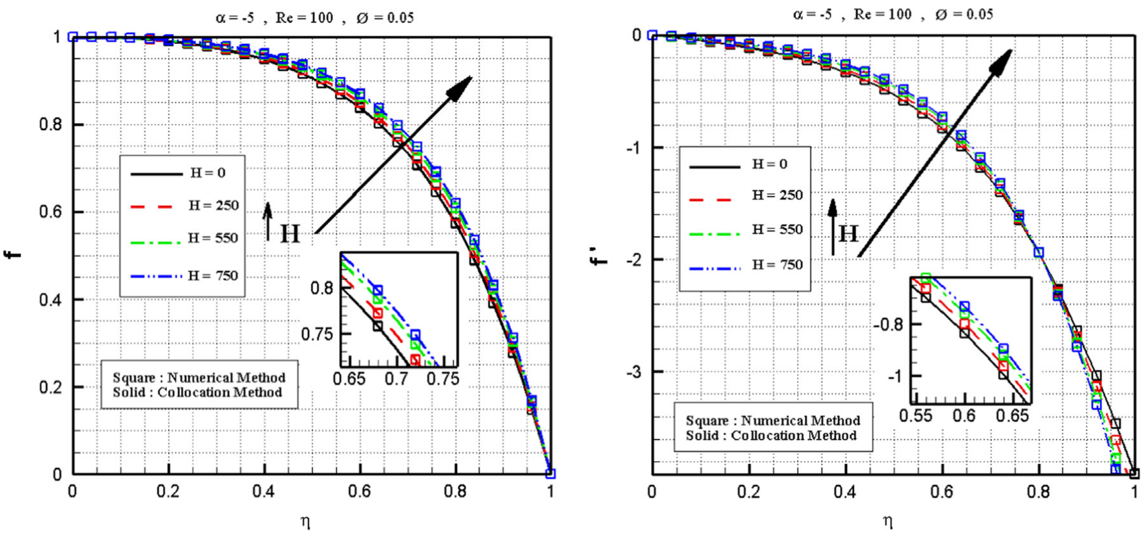

Fig. 3. Effect of Hartmann number (H) on velocity profiles $f(\eta)$ and $f'(\eta)$ when $\alpha=-5$, $Re=100$ and $\phi=0.05$.

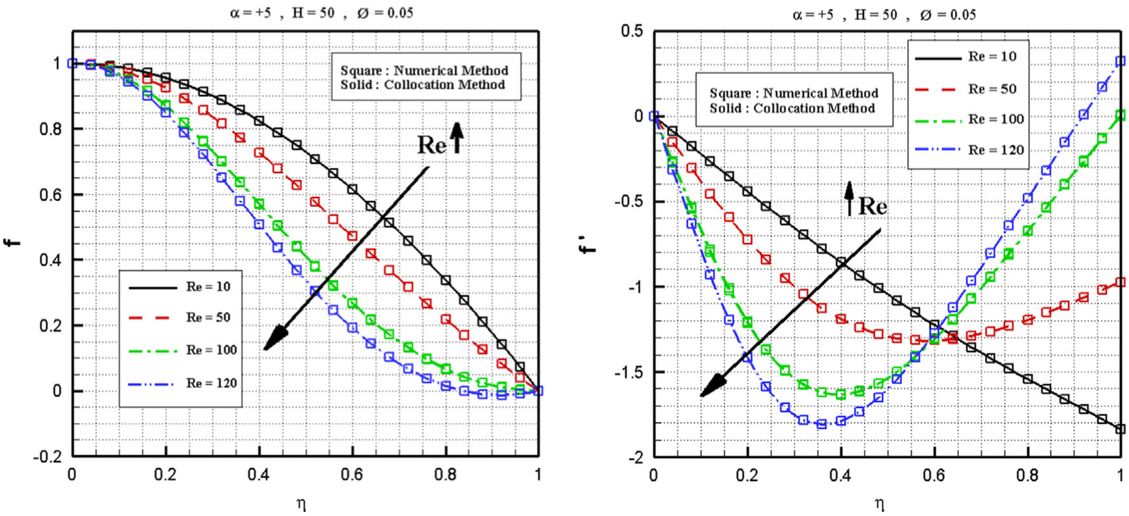

Fig. 4. Effect of Reynolds number (Re) on velocity profiles $f(\eta)$ and $f'(\eta)$ when $\alpha=+5$, $H=50$ and $\phi=0.05$.

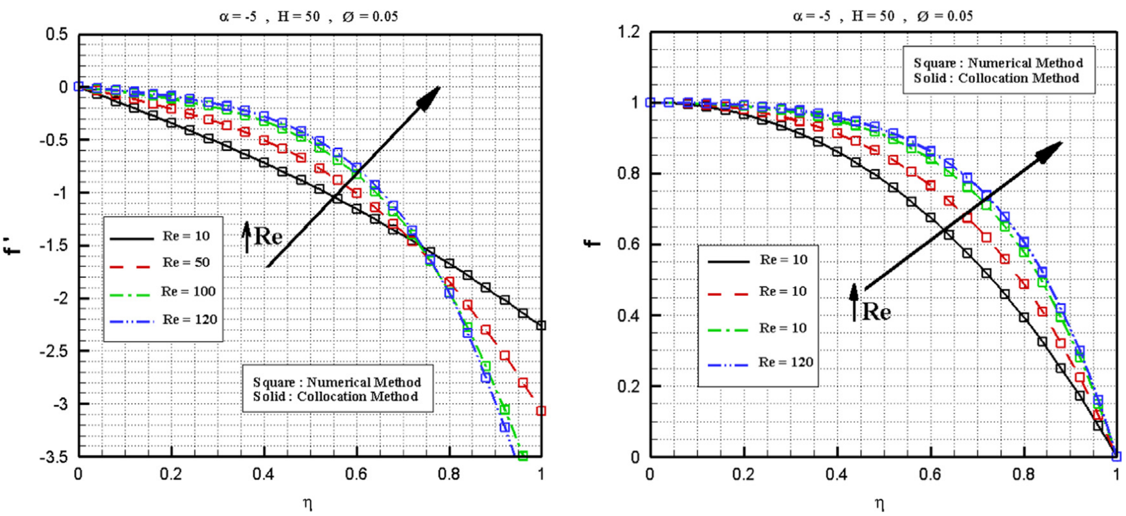

Fig. 5. Effect of Reynolds number (Re) on velocity profiles $f(\eta)$ and $f'(\eta)$ when $\alpha=-5$, $H=50$ and $\phi=0.05$.

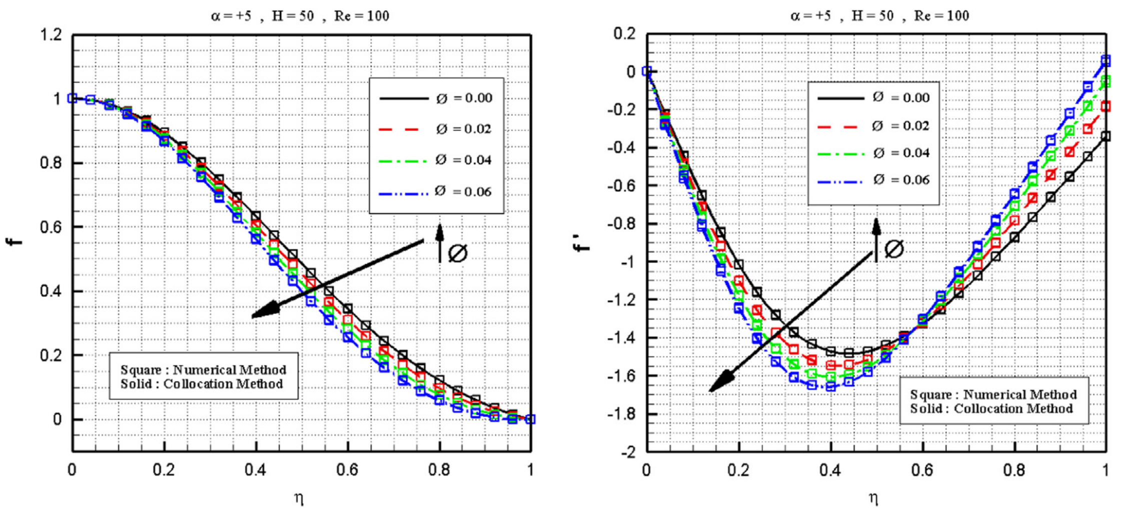

Fig. 6. Effect of nanoparticle volume fraction ϕ on velocity profiles $f(\eta)$ and $f'(\eta)$ when $\alpha=+5$, $H=50$ and $Re=100$.

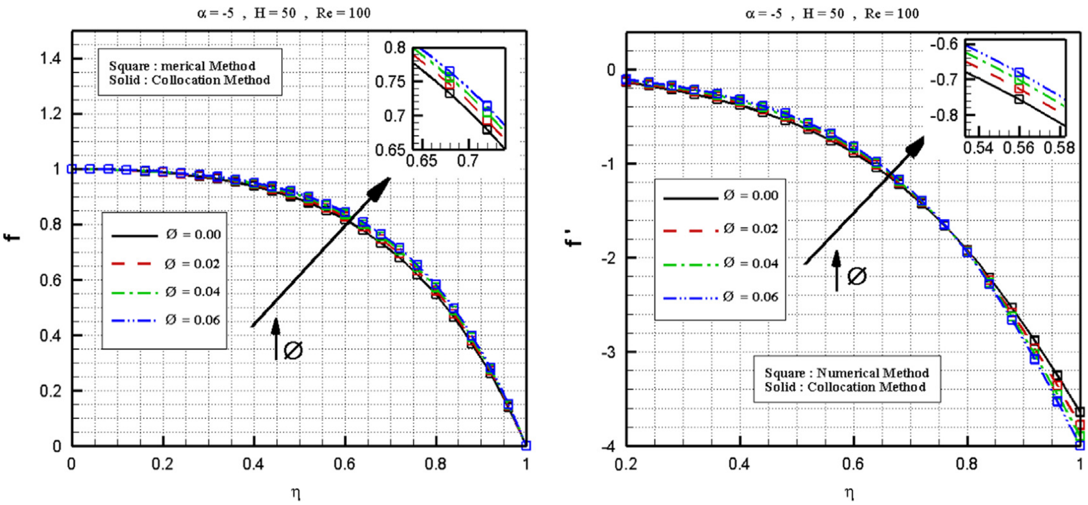

Fig. 7. Effect of nanoparticle volume fraction ϕ on velocity profiles $f(\eta)$ and $f'(\eta)$ when $\alpha=-5$, $H=50$ and $Re=100$.

After solving these unknown parameters $(c_1, c_2, c_3, c_4, c_5, c_6, c_7)$, the velocity equation will be determined. Using collocation method, the velocity formulation is as follows:

$$f(\eta)=1-0.4573621116\eta^2-0.07130345721\eta^3-0.1390392833\eta^4$$
$$-0.7602969598\eta^5+1.003347820\eta^6-0.9535254918\eta^7+0.3781794840\eta^8 \quad (23)$$

In a similar manner, we will obtain other solutions for different cases of α , H and Re that shows in Table 2.

4. Result and discussions

In the present study CM method is applied to obtain an explicit analytic solution of the MHD) Jeffery–Hamel flow with nanoparticles (Fig. 1). First, a comparison between the applied methods, obtained by the numerical method and CM for different values of active parameters is shown in Fig. 2 till Fig. 7.

The numerical solution is performed using the algebra package Maple 15.0, to solve the present case. The package uses a second-order difference scheme combined with an order bootstrap technique with mesh-refinement strategies: the difference scheme is based on either the trapezoid or midpoint rules; the order improvement/accuracy enhancement is either Richardson extrapolation or a method of deferred corrections [26]. Validity of CM is shown in Tables 2 and 3. In these tables, the %Error is defined as

$$\%\ \text{Error}=|f(\eta)_{\text{NUM}}-f(\eta)_{\text{HAM}}| \quad (24)$$

The results are proved to be precise and accurate in solving a wide range of mathematical and engineering problems especially Fluid mechanic cases. This accuracy gives high confidence to us about validity of this problem and reveals an excellent agreement of engineering accuracy. This investigation is completed by depicting the effects of some important parameters to evaluate how these parameters influence on this fluid (Table 4).

From a physical point of view, Figs. 2 –7 are prepared in order to see the effects of the semi angles between the plates, Reynolds number, magnetic field strength and nanoparticles volume fraction on the velocity distribution. Figs. 2 and 3 shows the magnetic field effect on the velocity profiles for divergent and convergent channels when $\alpha=\pm 5, Re=100,\ \phi=0.05$. The first result worth pointing out is that, under magnetic field the Lorentz force effect is in opposite of the momentum's direction that stabilizes the velocity profile. So it can be see, by increasing Hartmann number, the velocity profiles of convergent channel is moderate increases $\alpha<0$. In addition, the result shows that Backflow is occurred in divergent channels $\alpha>0$.

In addition, the effect of Reynolds number on the velocity profile for different values of α is shown in Figs. 4 and 5. The graph shows that the fluid velocity decreases with Reynolds numbers in the case of divergent channels but increases with Re in the case of convergent channels. Also, as it clears, by increasing Reynolds number, the thickness of boundary layer decreases when $\alpha<0$ and so, fluid in most parts of channels has maximum velocity.

Most noticeably of all, by increasing of nanofluid volume fraction, the fluid velocity decreases in the case of divergent channels but increases with ϕ in the case of convergent channels as depicted in Figs. 6 and 7.

5. Conclusion

In this investigation, the analytical approach called collocation method (CM) has been successfully applied to find the most accurate analytical solution for the velocity distributions of MHD Jeffery Hamel problem with nanoparticles. The governing equations, continuity and momentum for this problem are reduced to an ordinary single third form by using a similarity transformation. Furthermore, the obtained solutions by proposed methods have been compared with the direct numerical solutions generated by the symbolic algebra package Maple 15. The following main points can be concluded from the present study:

- Collocation technique is a powerful approach for solving MHD Jeffery–Hamel flow in high magnetic field with nanoparticles. It does not needs to any perturbation, linearization or small parameter versus Homotopy Perturbation Method (HPM) and Parameter Perturbation Method (PPM). Also it does not needs to determining the auxiliary parameter and auxiliary function versus Homotopy Analysis Method (HAM).
- Increasing Reynolds numbers leads to reduce velocity and excluded backflow in convergent channel.
- In greater angles, increasing Hartmann number will lead to backflow increases.
- When $\alpha > 0$ and steep of the channel is divergent, Increasing of nanofluid volume fraction will lead to the fluid velocity decreases.

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