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Light-front projection of spin-1 electromagnetic current and zero-modes

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ABSTRACT

The issue of the contribution of zero-modes to the light-front projection of the electromagnetic current of phenomenological models of vector particles vertices is addressed in the Drell–Yan frame. Our analytical model of the Bethe–Salpeter amplitude of a spin-1 fermion–antifermion composite state gives a physically motivated light-front wave function symmetric by the exchange of the fermion and antifermion, as in the ρ-meson case. We found that among the four independent matrix elements of the plus component in the light-front helicity basis only the $0 \to 0$ one carries zero-mode contributions. Our derivation generalizes to symmetric models, important for applications, the above conclusion found for a simplified non-symmetrical form of the spin-1 Bethe–Salpeter amplitude with photon–fermion point-like coupling and also for a smeared fermion–photon vertex model.

The electromagnetic from factors of composite spin-1 particles as the ρ-meson, have been addressed with increasing interest in the last years (see e.g. [1–12]), being an instrument to investigate the hadronic structure in terms of their basic constituents. Particularly Light-Front (LF) models are useful to describe the composite structure of hadrons in terms of constituent quarks degrees of freedom, which despite the inherent simplicity implement correctly kinematical boost properties of the corresponding amplitudes in exclusive processes [13,14].

However, the light-front description of a physical state in a truncated Fock-space basis breaks the rotational symmetry, as the associated transformation corresponds to a dynamical boost [2,15–17]. It is a formidable task to study the transformation properties of the Fock-space wave function under dynamical boosts in light-front quantization [14]. An analysis starting with covariant and analytical models of the Bethe–Salpeter (BS) amplitude is helpful to pin down the missing features in respect to boosts transformations in a truncated LF Fock-space description of the composite system. The projection onto the LF of a field theoretical model BS equation can be performed by integrating the relevant loop-integrals over the LF energy $k^- = k^0 - k^3$. It can be done systematically trough a quasi-potential technique [18] allowing to define concomitantly the relevant operators to be used with the valence wave function [19] (see also [20,21]).

The lack of contributions of higher Fock-components beyond the valence wave function for spin-1 composite systems, breaks the rotational symmetry relations between the matrix elements of the plus component of the electromagnetic current ($J^+ = J^0 + J^3$) in the Drell–Yan frame (momentum transfer $q^+ = q^0 + q^3 = 0$). For example, this was verified by starting with a covariant model of the ρ-meson as a $q\bar{q}$ bound state [2,15]. It was shown that, if pair-term (Z-diagram) contributions are ignored in the evaluation of the matrix elements of the electromagnetic current, the covariance of the form factors is lost [2,3,5,15,17]. Pair terms appearing in the matrix elements of current operator are associated with non-valence contributions (see e.g. [19]).

Within a field theoretic framework, the nonvalence terms in the Mandelstam formula or impulse approximation are due to the coupling between the valence and higher Fock sectors in the LF hamiltonian. In this respect, the nonvalence terms can be translated to two-body current operators acting on the valence sector (see e.g. [19,22]). The covariance of the form factors is broken if nonvalence contributions to the current are disregarded and only valence matrix elements are computed. In the Drell–Yan frame, the surviving pair diagrams are associated with zero-modes in the limit of $q^+ \to 0$. It remains an open question, if the singular behavior at the end points of naive analytical covariant vertex models, which originates zero-modes in the computation of exclusive process in the Drell–Yan frame, is indeed brought by light-front QCD, or by hadronic models with meson exchange currents projected onto the light-front [19].

The problem of the missing covariant properties of the form factors computed with the LF valence wave function, motivated

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different approaches to obtain the associated observables. This is the case of the explicitly covariant light-front dynamics [23] or the introduction of other kinematical conditions together with a Poincaré Covariant form of the current operator as done by Lev, Pace and Salmè (LPS) [24]. The form factors in the LPS framework are computed with momentum transfer along the z-direction in the Breit-frame. Noteworthy to observe that, the Z-diagram gives an important part of the form factor of strongly bound systems, as the pion, with the choice of momentum transfer according to LPS (see e.g. [25,26]).

For spin-1 particles, the missing rotational properties of the microscopic matrix elements of J^+ computed only with valence states in the Breit-frame with $q^+=0$, imply in an ambiguity in the extraction of the form factors [27–29]. In this case J^+ has four independent matrix elements, although only three form factors exist due to the constraints of covariance and current conservation. The matrix elements are related by an identity, namely, the angular condition (see e.g. [27]), which is violated by computing matrix elements of one-body current operators only with the valence component of the LF wave function. In this regard, several extraction schemes for evaluating spin-1 form factors were proposed [27,30–32].

The different extraction schemes were tested through the calculation of the ρ-meson electromagnetic form factors, with a covariant model of the quark–antiquark–ρ vertex, both in a covariant form and using the LF projection to the valence sector [2]. The prescription proposed by Grach and Kondratyuk (GK) [27] provides form factors in agreement with the fully covariant calculation. Later on, Ref. [3] demonstrated that the above prescription eliminates the nonvalence zero-mode contributions to the form factors computed with a smeared photon vertex model. In view of the interest in exclusive processes involving composite spin-1 particles, like the ρ-meson or the deuteron, it remains the question how this analytical conclusion generalizes to different models of the spin-1 BS amplitude, which have an LF valence wave function with a structure beyond the asymptotic one.

Here, we study an analytical BS amplitude model of the spin-1 fermion–antifermion composite state, which has a physically motivated valence wave function, symmetric by the exchange of the fermion and antifermion as required by phenomenological ρ-meson LF models (see e.g. [4]). We show analytically that zero-mode contributions appearing in the matrix elements of the spin-1 electromagnetic current are canceled in the computation of form factors with the prescription suggested by Ref. [27].

The present results go beyond previous findings and establish for symmetric vertex models, appealing for phenomenological applications, that the matrix elements of J^+ computed in the LF helicity basis are free from zero-modes apart the $0\to 0$ one, as has been previously found for a simplified non-symmetrical form of the ρ-meson vertex in [2] and for a smeared photon vertex model in [3]. Our analysis starts with the instant-form (IF) polarization basis in cartesian representation, because the angular condition has an intuitive form, relating the diagonal matrix elements of J^+ with polarization states transverse to the momentum transfer [32]. The cartesian basis permits to isolate in a transparent form the zero-modes in all matrix elements, adding further insight to the calculations with the helicity basis in the Breit-frame for $q^+=0$.

We should add that, the pion form factor obtained from J^+ does not have contributions from pair terms in the Breit-frame and $q^+=0$, as found for an analytical and covariant pion vertex model with a γ^5 coupling to the quarks [26,33]. It is enough to evaluate the valence part of the matrix element of the current to reproduce the covariant result. However, a zero-mode contributes to the matrix element of the current $J^-=J^0-J^3$ as shown in [33]. Another example of the contribution of light-front zero-modes to the electromagnetic form factors and their cancellation was given in [34] in the case of a spin 1/2 fermion.

Symmetrical light-front model for vector particles. The adopted spin and momentum structure of the ρ–$q\bar{q}$ vertex, as a prototype of spin-1 composite particle, is given by a symmetrical expression with constituent fermions of mass m. It comes as a generalization of the spin-one composite particle vertex proposed in [2] and the one given in [26] used for the evaluation of the pion form factor. Our model for the vector particle vertex reads:

$$\Lambda^\mu(k,p)=\gamma^\mu\left[D(k')\right]^{-2}-\frac{m_v}{2}\left(k^\mu+k'^\mu\right)\left[D_v(k)D^2(k')\right]^{-1}+\left[k\leftrightarrow -k'\right], \tag{1}$$

where m_v is the mass of the vector particle, $D(k)=(k^2-m_R^2+\imath\epsilon)$, $D_v(k)=(p.k+m_v m-\imath\epsilon)$ and $k'=k-p$. The regularization function is enough to render finite the photo-absorption amplitude. The regularization parameter is m_R.

The valence component of the light-front wave function is the projection at $x^+=0$ of the Bethe–Salpeter amplitude with the instantaneous terms in the external fermionic legs separated out [18]. Only the LF time propagating part of the Dirac propagator is left in the external legs. The resulting valence wave function with the vector particle polarized along ϵ_i is given by:

$$\Phi^{(i)}_{LF}\left(k^+,\vec{k}_\perp;p\right)=\int\frac{dk^-}{2\pi\imath}\frac{(\not{k}_{on}+m)}{(k^2-m^2+\imath\epsilon)}\epsilon_i\cdot\Lambda(k,p)\frac{(\not{k}'_{on}+m)}{(k'^2-m^2+\imath\epsilon)}, \tag{2}$$

where $k'=k-p$. After integration over k^-, we get:

$$\Phi^{(i)}_{LF}\left(k^+,\vec{k}_\perp;p\right)=\frac{\not{k}_{on}+m}{k^+}\frac{\Lambda^{(i)}_{LF}(k^+,\vec{k}_\perp;p)}{p^--p^-_0}\frac{\not{k}'_{on}+m}{p^+-k^+}, \tag{3}$$

where the LF momenta are $k^\pm=k^0\pm k^3$ and $k_\perp=(k_x,k_y)$. The support of the LF wave function is $0<k^+<p^+$ and p_0^- is the minus component of the free momentum of the quark–antiquark system. The minus-on-shell momentum k_{on} and k'_{on} have minus components $k^-_{on}=(\vec{k}^2_\perp+m^2)/k^+$ and $k'^-_{on}=((\vec{p}-\vec{k})^2_\perp+m^2)/(k^+-p^+)$, respectively. The momentum part of the LF vertex is

$$\Lambda^{(i)}_{LF}\left(k^+,\vec{k}_\perp;p\right)=\frac{1}{((p-k_{on})^2-m_R^2)^2}\left\{\not{\epsilon}_i-\frac{m_v}{2}\frac{\epsilon_i\cdot(2k_{on}-p)}{(p.k_{on}+m_v m)}\right\}+\left[k\leftrightarrow -k'\right], \tag{4}$$

which is symmetrical under the exchange between the quark and antiquark.

Electromagnetic current in impulse approximation and notation. The electromagnetic current for spin-1 particles has the following general form (see e.g. [35]):

$$J^\mu_{\alpha\beta}=\left[F_1\left(q^2\right)g_{\alpha\beta}-F_2\left(q^2\right)\frac{q_\alpha q_\beta}{2m_v^2}\right]\left(p^\mu+p'^\mu\right)-F_3\left(q^2\right)\left(q_\alpha g^\mu_\beta-q_\beta g^\mu_\alpha\right), \tag{5}$$

where q^μ is the momentum transfer, p^μ and p'^μ are on-shell initial and final momenta respectively. From the covariant form factors F_1, F_2 and F_3, one can obtain the charge (G_0), magnetic (G_1) and quadrupole (G_2) form factors (see e.g. [2]).

The matrix elements of the electromagnetic current $\mathcal{J}_{ji}=\epsilon'^{\alpha}_j\epsilon^\beta_i J^\mu_{\alpha\beta}$ in the impulse approximation are written as [2]:

$$\mathcal{J}^{\mu}_{ji} = \int [d^4k] \times \frac{Tr[(\not k + m)\Lambda_\alpha(k,p')\epsilon'^{\alpha}_j(\not k - \not p' + m)\gamma^\mu(\not k - \not p + m)\Lambda_\beta(k,p)\epsilon^\beta_i]}{(k^2 - m + \imath\epsilon)((p-k)^2 - m + \imath\epsilon)((p'-k)^2 - m + \imath\epsilon)}, \quad (6)$$

where $[d^4k] = d^4k/(2\pi)^4$, ϵ'_j and ϵ_i are the polarization four-vectors of the final and initial states, respectively.

The electromagnetic form factors are calculated in the Breit-frame with the Drell–Yan condition, which gives the momentum transfer $q^\mu = (0, q_x, 0, 0)$. The particle initial momentum is $p^\mu = (p^0, -q_x/2, 0, 0)$ and the final one is $p'^\mu = (p^0, q_x/2, 0, 0)$. We use $\eta = -q^2/4m_v^2$ and $p^0 = m_v\sqrt{1+\eta}$. The polarization four-vectors in the instant-form basis are given by

$$\epsilon^\mu_x = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \qquad \epsilon^\mu_y = (0,0,1,0),$$
$$\epsilon^\mu_z = (0,0,0,1), \quad (7)$$

for the initial state and by

$$\epsilon'^\mu_x = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \qquad \epsilon'^\mu_y = \epsilon^\mu_y, \qquad \epsilon'^\mu_z = \epsilon^\mu_z, \quad (8)$$

for the final state.

LF spin basis matrix elements and the angular condition. The matrix elements of J^+ in the instant-form spin basis, are related to the matrix elements in the LF spin basis, with the unitary transformation between these spin bases given by the Melosh rotation [36]. For notational convenience, we use I^+, to express the matrix elements in the LF spin basis. The relations between the current matrix elements in the two spin bases are (see also [32]):

$$I^+_{11} = \frac{J^+_{xx} + (1+\eta)J^+_{yy} - \eta J^+_{zz} - 2\sqrt{\eta}J^+_{zx}}{2(1+\eta)},$$
$$I^+_{10} = \frac{\sqrt{\eta}J^+_{xx} + \sqrt{\eta}J^+_{zz} - (\eta-1)J^+_{zx}}{\sqrt{2}(1+\eta)},$$
$$I^+_{1-1} = \frac{(1+\eta)J^+_{yy} - J^+_{xx} + \eta J^+_{zz} + 2\sqrt{\eta}J^+_{zx}}{2(1+\eta)},$$
$$I^+_{00} = \frac{J^+_{zz} - \eta J^+_{xx} - 2\sqrt{\eta}J^+_{zx}}{(1+\eta)}, \quad (9)$$

with the matrix elements of the plus component of the current evaluated between LF polarization states denoted as $I^+_{m'm}$.

The angular condition satisfied by the matrix elements of the plus component of the current, in the Breit-frame with $q^+ = 0$, is given by (see e.g. [36]):

$$\Delta(q^2) = (1+2\eta)I^+_{11} + I^+_{1-1} - \sqrt{8\eta}I^+_{10} - I^+_{00}$$
$$= (J^+_{yy} - J^+_{zz})(1+\eta) = 0. \quad (10)$$

After Eq. (10), the angular condition in the IF spin basis takes a remarkable simple form: $J^+_{yy} = J^+_{zz}$ [32].

We remind that, different prescriptions to extract the form factors using the valence wave function choose three matrix elements among the four independent ones, or any other three linearly independent combinations of them.

Evaluation of zero-mode contributions to the current. The contribution of the LF Z-diagram or the nonvalence contribution to the matrix elements of the current for the vertex model (1) is computed below. Technically, we are able to separate out the pair terms using the pole dislocation method, i.e., by using the limit of $q^+ = \delta^+ \to 0_+$ (see e.g. [15–17,33]). The LF projection of the impulse approximation formula of the current (6) is done by integration over k^- in the momentum loop.

The zero-modes originated from pair-term contributions to the matrix elements of the current in the limit $\delta^+ \to 0_+$ arise from powers of k^- coming along with the Dirac trace in (6). The potential sources are the instantaneous terms of the fermion propagators and the derivative coupling of the fermions to the vector particle, as in our model (1) of the vertex. The angular condition is violated if we do not perform the limit carefully. We have to take into account all possible non-vanishing contributions coming from terms of the trace carrying powers of k^-.

The different terms contributing to the Z-diagram in the matrix elements of J^+ in the IF cartesian spin basis from the vertex (1), consisting of γ^μ and derivative couplings, are analyzed separately. They have distinct singular behavior near the end points, which can allow zero-modes in the limit $\delta^+ \to 0_+$. We compute: (i) the direct term with γ^μ vertices (gg) from the initial and final states of the vector particle; (ii) the cross term with γ^μ and derivative coupling (dg); (iii) the direct term with two derivative couplings (dd). This strategy simplifies the separation of zero-mode contributions present in the different terms of the matrix elements of the electromagnetic current and electromagnetic form factors of the composite vector state.

(i) *Direct term with γ^μ couplings.* We start by computing the trace with γ^μ's from the vertex (1) of both the initial and final states:

$$Tr[gg]_{ji} = Tr\big[\not\epsilon'_j(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\not\epsilon_i(\not k + m)\big], \quad (11)$$

where the k^- powers are separated out according to [17] to pin down the zero-modes. The instantaneous term of the Dirac propagator (see e.g. [17]) brings the k^- momentum dependence in the trace:

$$Tr[gg]^Z_{ji} = \frac{1}{2}k^- Tr\big[\not\epsilon'_j(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\not\epsilon_i\gamma^+\big], \quad (12)$$

where the terminology “bad” [35], denoted by Z, indicates that Z-diagrams or pair terms can potentially survive the limit $\delta^+ \to 0_+$ and become a zero-mode contribution to the current. The four independent matrix elements corresponding to the initial (ϵ_i) and final (ϵ'_j) polarization states, respectively given by (7) and (8), are:

$$Tr[gg]^Z_{xx} = -\eta\, Tr[gg]^Z_{zz}, \qquad Tr[gg]^Z_{zx} = -\sqrt{\eta}\, Tr[gg]^Z_{zz},$$
$$Tr[gg]^Z_{zz} = R_{gg}, \quad (13)$$

where $R_{gg} = \frac{k^-}{2}Tr[(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\gamma^-]$ and $Tr[gg]^Z_{yy} = 4k^-(p^+ - k^+)^2$.

The projection over the LF hyperplane demands the integration over k^- in (6). In detail, the Z-diagram contribution is:

$$J^{+Z}_{ji}[gg] = \lim_{\delta^+\to 0_+} \sum_{r=4,5;\, s=4,6} \int [d^4k]^Z \frac{Tr[gg]^Z_{ji}}{\{1\}\{2\}\{3\}\{r\}^2\{s\}^2}, \quad (14)$$

where $[d^4k]^Z = [d^4k]\theta(p'^+ - k^+)\theta(k^+ - p^+)$, and the denominators are given by: $\{1\} = [k^2 - m^2 + \imath\epsilon]$, $\{2\} = [(k-p)^2 - m^2 + \imath\epsilon]$, $\{3\} = [(k-p')^2 - m^2 + \imath\epsilon]$, $\{4\} = [(k-p)^2 - m^2_R + \imath\epsilon]$, $\{5\} = [(k-p')^2 - m^2_R + \imath\epsilon]$, $\{6\} = [k^2 - m^2_R + \imath\epsilon]$. The integration over k^- is spoiled by an end-point singularity which is taken care by using *the pole dislocation method* [15], i.e., making $p'^+ = p^+ + \delta^+$ in the denominator $\{3\}$. It has been shown in [16] that it is enough to dislocate one of the poles to pick end-point singularities.

After the k^- integration of (14), the Z-diagram contributions, with support in the interval $p^+ < k^+ < p'^+$ and in the limit $\delta^+ \to 0_+$, vanish for terms carrying the dependence $(k^-)^{m+1}(p^+ - k^+)^n$

if $m < n$ [17]. Therefore, $J^{+Z}_{yy} = 0$, and the matrix element J^{+}_{yy} does not have a pair-term contribution, as has been already verified explicitly in [2,17].

Due to the trace relations (13) the zero-mode contributions to the matrix elements of the current from the direct term with γ^μ couplings are obtained from $J^{+Z}_{zz}[gg]$ as:

$$J^{+Z}_{xx}[gg] = -\eta J^{+Z}_{zz}[gg], \qquad J^{+Z}_{zx}[gg] = -\sqrt{\eta} J^{+Z}_{zz}[gg]. \tag{15}$$

These relations are the main result of this work, which are kept valid when considering the other parts of the vector particle vertex. The success of the prescription [27] in canceling the zero-modes in the vector particle form factors is due to the validity of (15), extending previous findings to more general vertex forms.

The matrix element $J^{+Z}_{zz}[gg]$ does not vanish in the limit of $\delta^+ \to 0_+$, after the k^- integration in (14) is performed. In view of (15) all matrix elements of the plus component of the current, excepting the yy one, have contribution from a zero-mode. The evaluation is performed by dislocating the position of the zero from the denominator $\{3\}$ using $p'^+ = p^+ + \delta^+$. The other denominators, namely $\{i\}$ for $i \neq 3$ are maintained unaltered by keeping $p'^+ = p^+$ in their expressions. Then, the Cauchy integration in k^- of (14) with k^+ in the interval $0 < k^+ - p^+ < \delta^+$, can be done by closing the contour in the upper-half of the k^- complex plane. The arc contribution vanishes in this case. Only the residue from the pole associated with the dislocated denominator $\{3\} = 0$, i.e.,

$$k^{-Z} = p'^- - \frac{(\vec{p}'_\perp - \vec{k}_\perp)^2 + m^2 - \imath\epsilon}{\delta^+ - (k^+ - p^+)}, \tag{16}$$

has to be evaluated, and then the non-vanishing zero-mode contribution to the matrix element of the current $J^{+Z}_{zz}[gg]$ is obtained. Note that for $\delta^+ \to 0_+$, the position of the pole k^{-Z} diverges towards infinity as $\sim 1/\delta^+$. Therefore, terms containing powers of k^- in the numerator of the integrand are potential sources of end-point singularities, which can produce a non-vanishing result of the integration in k^+, even when the interval of integration shrinks to zero, as happens in the present case.

The above limiting behavior can be explained by taking into account that at the pole, one has that the trace contributes with $k^- \sim 1/\delta^+$, and the denominators behave as $\{1\} \sim 1/\delta^+$ and $\{i\} \sim (\delta^+)^0$ for $i = 2, 4, 5$. Furthermore, one has the phase-space factor $(p'^+ - k^+) \sim \delta^+$ from the decomposition $\{3\} = (p'^+ - k^+)(p'^- - k^- - (p' - k)^-_{on} + i\epsilon)$ where $(p' - k)^-_{on} = ((p' - k)^2_\perp + m^2)/(p'^+ - k^+)$. Putting all together, one finds that the residue is $\mathcal{O}[1/\delta^+]$ for $s = 4$ in (14). Then, after the integration in k^+, one has that $J^{+Z}_{zz}[gg]$ is finite and nonzero when $\delta^+ \to 0_+$.

We add that, the LF projection of the impulse approximation formula (6) has the valence interval $0 < k^+ < p^+$ from the residue at the poles $k^2 = m^2$ and/or $k^2 = m^2_R$ in the k^- integration. These poles come from the zero of the denominators $\{1\}$ and $\{4\}$ defined below Eq. (14).

(ii) *Cross term with γ^μ and derivative couplings.* The trace with γ^μ and derivative coupling of the constituents to the vector particle from the vertices (1) of the initial and final states, which enters in the impulse approximation formula for the microscopic current, reads:

$$Tr[dg]_{ji} = \epsilon'_j \cdot (2k - p') Tr\big[(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\not\epsilon_i(\not k + m)\big], \tag{17}$$

where we separate out the powers in k^- to access the zero-modes. We remind that, the Z-diagram contribution from the k^- integration in the interval $p^+ < k^+ < p'^+$ in the limit of $\delta^+ \to 0_+$ vanishes for terms of the form $(k^-)^{m+1}(p^+ - k^+)^n$ for $m < n$. In evaluating (17), we keep only terms in the trace with $m \geqslant n$, which needs to be considered for the zero-mode calculation. Under this restriction, we get:

$$Tr[dg]^Z_{ji} = \epsilon'^+_j \epsilon^+_i R_{dg} - 4mk^- k^+ \epsilon'^+_j \vec{\epsilon}_{i\perp} \cdot \vec{q}_\perp, \tag{18}$$

where $R_{dg} = 4mk^-(k^-(k^+ - p^+) + (\vec{k}_\perp - \vec{p}'_\perp) \cdot (\vec{k}_\perp - \vec{p}_\perp) + q_\perp \cdot k_\perp + m^2)$. In the trace (18), we use $p^+ = p'^+$, as this identification is irrelevant for the dislocation of the pole in the denominator of the integrand in the impulse approximation formula. Immediately, one observes that the part of the trace, (18), which could carry a zero-mode contribution for the yy polarizations vanishes. This happens because the y-polarization four-vector has plus component identical to zero (cf. Eqs. (7) and (8)).

However, differently from the previous case, the analogous relations to (13) do not hold for the trace (18). Thus, the analysis of the k^- integration should be done carefully taking into account these traces to verify the presence of zero-modes. In fact, the terms proportional to k^- and $(k^-)^2(k^+ - p^+)$ should be checked against a zero-mode contribution. Note that at the pole k^{-Z}, the product $(k^-)^2(k^+ - p^+)$ diverges like k^- when $\delta^+ \to 0_+$. Then for our analysis, it is enough to discuss the case when k^- appears in the numerator of the integrand. As we show below, no end-point singularity appears in this case, and the contribution to the Z-diagram vanishes when $\delta^+ \to 0_+$.

The contribution of the interval $0 < k^+ - p^+ < \delta^+$ to the light-front projection of the current having γ^μ and derivative couplings has two possible combinations. We choose one of them to perform our analysis without loosing generality:

$$J^{+Z}_{ji}[dg] = \lim_{\delta^+ \to 0_+} \int \big[d^4k\big]^Z \frac{Tr[dg]^Z_{ji}}{\{1\}\{2\}\{3\}\{6\}^2} \left[\frac{1}{\{4\}^2} + \frac{1}{\{5\}^2}\right] \times \frac{m_v}{2(p' \cdot k + mm_v - i\epsilon)}, \tag{19}$$

where the zero of $\{3\}$ is dislocated by using $p'^+ = p^+ + \delta^+$, while the other denominators remain the same. The Cauchy integration in k^- with k^+ in the interval $0 < k^+ - p^+ < \delta^+$ can be performed by closing the contour in the upper-half of the k^- complex plane. There, two poles are present: one from the dislocated denominator $\{3\} = 0$, Eq. (16), and another one

$$k^- = \frac{1}{p^+}\big(2\vec{p}'_\perp \cdot \vec{k}_\perp - k^+ p^- - 2mm_v + \imath\epsilon\big). \tag{20}$$

Next, we perform the analysis of the diverging behavior of the several terms entering in the computation of the residue, as we have done before. Then, we obtain that the residue from the zero of $\{3\}$ is $\mathcal{O}[(\delta^+)^2]$, taking into account the traces from (18). The residue from the pole (20) gives a contribution $\mathcal{O}[(\delta^+)^0]$. Thus, after integration in k^+, one has that:

$$J^{+Z}_{ji}[dg] \sim \mathcal{O}\big[\delta^+\big]. \tag{21}$$

An analogous analysis of the other possibility for the couplings γ^μ and derivative of the constituents to the vector particle in the kinematical region $0 < k^+ - p^+ < \delta^+$, shows that it vanishes for $\delta^+ \to 0_+$.

(iii) *Direct term with derivative couplings.* The trace for the case of derivative vertex couplings from (1), corresponding to the initial and final states in the impulse approximation formula for the microscopic current, is given by:

$$Tr[dd]_{ji} = \left[A_{dd}\frac{k^-}{2} + B_{dd}\right]\epsilon'_j \cdot (2k - p')\epsilon_i \cdot (2k - p), \tag{22}$$

where

$$A_{dd} = Tr\big[(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\gamma^+\big] = 8\big(p^+ - k^+\big)^2,$$
$$B_{dd} = Tr\bigg[\big(\not k - \not p' + m\big)\gamma^+(\not k - \not p + m)\bigg(\frac{\gamma^-}{2}k^+ - \vec{\gamma}_\perp.\vec{k}_\perp + m\bigg)\bigg]. \tag{23}$$

For our analysis and with the aim of isolating terms carrying the k^- dependence, we have separated out the light-front instantaneous terms of the Dirac propagators. However, differently from the case (i), the analogous relations to (13), for the trace with *xx*, *zx* and *zz* polarizations do not hold for (22).

In this case among the four possible combinations of derivative couplings in the impulse approximation originated from the vertex, Eq. (1), we choose to analyze, without loosing generality, the following term:

$$J^{+Z}_{ji}[dd] = \lim_{\delta^+\to 0_+} \int \big[d^4k\big]^Z \times \frac{\frac{m_v^2}{4}Tr[dd]_{ji}}{\{1\}\{2\}\{3\}\{5\}^2\{6\}^2(p\cdot k + mm_v - \imath\epsilon)(p'\cdot k + mm_v - \imath\epsilon)}, \tag{24}$$

where the zero of {3} is dislocated by using $p'^+ = p^+ + \delta^+$, while the other denominators are kept unchanged. The Cauchy integration in k^- is performed by closing the contour in the upper-half of the k^- complex plane. There, three poles are present: one from the dislocated denominator $\{3\} = 0$, Eq. (16), and two others, one of them is (20) and a new one:

$$k^- = \frac{1}{p^+}\big(2\vec{p}_\perp \cdot \vec{k}_\perp - k^+p^- - 2mm_v + \imath\epsilon\big). \tag{25}$$

The residue computed at the poles (20) and (25) are finite and trivially do not carry an end-point singularity. Note that the matrix element of the current with *yy* polarizations does not carry a zero-mode because the k^- dependence in the trace appears multiplied by $(p^+ - k^+)^2$, which is enough to kill the divergence coming from the pole k^{-Z}.

In the following analysis we discuss only the residue of the integration of (24) in k^- at the pole k^{-Z} (16). Because, this residue can potentially give a zero-mode contribution to the matrix elements of the current picking up end-point singularities. The scalar product in (22) computed for the cartesian polarization four-vectors *x* and *z* given by Eqs. (7) and (8), provides terms up to $(k^-)^2$, and then Eq. (22) carries terms in k^- up to the third power. However, it is sufficient to analyze terms proportional to $(k^-)^2$ from (22), because terms with $(k^-)^3$ come multiplied by $(k^+ - p^+)^2$ in this case. The product $(k^{-Z})^3(k^+ - p^+)^2$ behaves as k^{-Z} when approaching the end point of the k^+ integral in (24). Considering the denominators and $(k^-)^2$ in the numerator of (24), the residue at k^{-Z} is $\mathcal{O}[(\delta^+)^2]$. Thus, after integrating in k^+, one has that:

$$J^{+Z}_{ij}[dd] \sim \mathcal{O}\big[\delta^+\big]. \tag{26}$$

Evoking the same analysis as we did above, the other terms coming from derivative couplings in the kinematical region $0 < k^+ - p^+ < \delta^+$ vanish when δ^+ goes to zero.

Cancellation of zero-modes in the electromagnetic form factors. The contributions from *Z*-diagrams survive the limit $q^+ \to 0_+$ in the matrix elements of J^+, only for the direct term with γ^μ couplings. The relations (15) are valid for the full matrix elements of the current, because, as we have shown, the zero-modes vanish for the other possible combinations from the vertices of the initial and final states of the vector particle. Then, for the full $J^{+Z}_{zz} \sim \mathcal{O}[(\delta^+)^0]$, the relations

$$J^{+Z}_{xx} + \eta J^{+Z}_{zz} = 0, \qquad J^{+Z}_{zx} + \sqrt{\eta}J^{+Z}_{zz} = 0 \quad \text{and} \quad J^{+Z}_{yy} = 0 \tag{27}$$

are satisfied. From the angular condition, $J^+_{zz} = J^+_{yy}$, and considering that the *yy* matrix element does not carry a zero-mode, one can express the non-vanishing zero-mode matrix element in terms of the matrix elements computed in the valence region as $J^{+Z}_{zz} = J^{+V}_{yy} - J^{+V}_{zz}$ (the superscript *V* indicates the valence terms). Then, we have that:

$$J^+_{xx} = J^{+V}_{xx} - \eta\big(J^{+V}_{yy} - J^{+V}_{zz}\big),$$
$$J^+_{zx} = J^{+V}_{zx} - \sqrt{\eta}\big(J^{+V}_{yy} - J^{+V}_{zz}\big), \qquad J^+_{zz} = J^{+V}_{yy}, \tag{28}$$

which isolates the zero-modes in the matrix elements and can be computed considering only the valence region.

In particular, introducing the relations (27) in (9), one obtains the corresponding to matrix elements in the LF spin basis as:

$$I^{+Z}_{11} = 0, \qquad I^{+Z}_{10} = 0,$$
$$I^{+Z}_{1-1} = 0 \quad \text{and} \quad I^{+Z}_{00} = (1+\eta)J^{+Z}_{zz}, \tag{29}$$

with $\lim_{\delta^+\to 0_+} J^{+Z}_{zz} \neq 0$, which gives a zero-mode contribution only in this case.

Within the prescription of Grach and Kondratyuk (GK) [27] the matrix element I^+_{00} is eliminated from the form factors. It trivially excludes the zero-modes due to the validity of (29). To appreciate this finding in the IF spin basis, we write below the form factors,

$$G^{GK}_0 = \frac{1}{3}\big(J^+_{xx} + \eta J^+_{zz} + (2-\eta)J^+_{yy}\big),$$
$$G^{GK}_1 = J^+_{yy} - \frac{1}{\sqrt{\eta}}\big(J^+_{zx} + \sqrt{\eta}J^+_{zz}\big),$$
$$G^{GK}_2 = \frac{\sqrt{2}}{3}\big(J^+_{xx} + \eta J^+_{zz} - (1+\eta)J^+_{yy}\big), \tag{30}$$

by transforming the matrix elements from the LF spin basis [27] to the IF spin basis [32]. One immediately recognizes from (30) and the relations (27) for the contributions of the zero-mode the matrix elements in the IF spin basis that

$$G^{GK,Z}_0 = G^{GK,Z}_1 = G^{GK,Z}_2 = 0, \tag{31}$$

with form factors (30) giving by considering only the valence region. Note that, if the relations (28) are taken into account, the computation of the form factors are independent of the prescription chosen.

The above findings in Eq. (31) generalize the conclusion of [3] obtained with a smeared photon vertex model to the symmetrical vector meson vertex model, where only the valence region contributes to the form factors in the GK prescription. Also found in a numerical calculation of the ρ-meson form factors by comparing the LF calculation considering only the valence region with the covariant results of the model [2].

Summary. We analyzed the rotational symmetry properties of the matrix elements of the plus component of the electromagnetic current in the Breit-frame with $q^+ = 0$ for a symmetric and analytic model of a spin-1 composite particle vertex, considering the projection onto the light-front. If only the valence region is computed in the impulse approximation formula, rotations are not properly accounted by the matrix elements, and the angular condition is violated. This is why different prescriptions for extracting form factors from the microscopic matrix elements do not provide a unique answer, which led to alternative proposals to calculate form factors like, e.g., the Lev–Pace–Salmè frame ($\vec{q}_\perp = 0$).

The naive computation of the microscopic matrix elements of J^+ for $q^+ = 0$, relying only on the valence region, leads to the violation of the angular condition even in analytical models [2] and [3] of composite spin-1 particles. Here, we have used a more general form of the spin-1 vertex to compute the matrix elements of J^+ using the instant-form polarization basis, in the limit of $q^+ \to 0_+$. We showed how to single out the contribution of zero-modes making use of the limit $q^+ \to 0_+$. We prove that the prescription suggested by Grach and Kondratyuk to extract the form factors from the microscopic current, which excludes in the light-front helicity basis the $0 \to 0$ matrix element of J^+ among the four independent ones, eliminates unwanted zero-modes, keeping contribution only from the valence region. Our derivation generalizes to symmetric models the above conclusion found for a simplified non-symmetrical form of the ρ-meson vertex with point-like quarks and also for a model of a smeared quark–photon vertex. Our methods are suitable for applications, e.g., to study vector meson elastic form factors, and also can be easily extended to study transition form factors involving spin-1 composite particles.

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References

[1] F. Cardarelli, I.L. Grach, I.M. Narodetskii, G. Salme, S. Simula, Phys. Lett. B 349 (1995) 393.
[2] J.P.B.C. de Melo, T. Frederico, Phys. Rev. C 55 (1997) 2043.
[3] B.L.G. Bakker, H.-M. Choi, C.-R. Ji, Phys. Rev. D 65 (2002) 116001.
[4] W. Jaus, Phys. Rev. D 67 (2003) 094010.
[5] H.-M. Choi, C.R. Ji, Phys. Rev. D 70 (2004) 053015.
[6] C. Adamuscin, G.I. Gakh, E. Tomasi-Gustafsson, Phys. Rev. C 75 (2007) 065202.
[7] H.R. Grigoryan, Phys. Lett. B 662 (2008) 158.
[8] T.M. Aliev, A. Ozpineci, M. Savci, Phys. Lett. B 678 (2009) 470.
[9] D. Garcia Gudino, G. Toledo Sanchez, Phys. Rev. D 81 (2010) 073006.
[10] H.Z. Sang, X.H. Wu, arXiv:1004.4392 [hep-ph].
[11] H.L.L. Roberts, A. Bashir, L.X. Gutierrez-Guerrero, C.D. Roberts, D.J. Wilson, Phys. Rev. C 83 (2011) 065206.
[12] H.-M. Choi, C.-R. Ji, Phys. Lett. B 696 (2011) 518.
[13] M.V. Terentév, Sov. J. Nucl. Phys. 24 (1976) 106;
L.A. Kondratyuk, M.V. Terentév, Sov. J. Nucl. Phys. 31 (1980) 561.
[14] S.J. Brodsky, H.-C. Pauli, S.S. Pinsky, Phys. Rep. 301 (1998) 299.
[15] J.P.B.C. de Melo, J.H.O. Sales, T. Frederico, P.U. Sauer, Nucl. Phys. A 631 (1998) 574c.
[16] H.W.L. Naus, J.P.B.C. de Melo, T. Frederico, Few-Body Syst. 24 (1998) 99.
[17] J.P.B.C. de Melo, T. Frederico, H.W.L. Naus, P.U. Sauer, Nucl. Phys. A 660 (1999) 219.
[18] J.H.O. Sales, T. Frederico, B.V. Carlson, P.U. Sauer, Phys. Rev. C 61 (2000) 044003;
J.H.O. Sales, T. Frederico, B.V. Carlson, P.U. Sauer, Phys. Rev. C 63 (2001) 064003.
[19] J.A.O. Marinho, T. Frederico, P.U. Sauer, Phys. Rev. D 76 (2007) 096001;
J.A.O. Marinho, T. Frederico, E. Pace, G. Salmè, P.U. Sauer, Phys. Rev. D 77 (2008) 116010;
E. Pace, J.A.O. Marinho, G. Salmè, T. Frederico, Few-Body Syst. 50 (2011) 431.
[20] T. Frederico, G. Salmè, Few-Body Syst. 49 (2011) 163.
[21] T. Frederico, Few-Body Syst. 50 (2011) 9.
[22] D.S. Hwang, V.A. Karmanov, Nucl. Phys. B 696 (2004) 413.
[23] J. Carbonell, B. Desplanques, V.A. Karmanov, J.F. Mathiot, Phys. Rep. 300 (1998) 215.
[24] F.M. Lev, E. Pace, G. Salmè, Nucl. Phys. A 641 (1998) 229;
F.M. Lev, E. Pace, G. Salmè, Few-Body Syst. Suppl. 10 (1999) 135;
F.M. Lev, E. Pace, G. Salmè, Phys. Rev. Lett. 83 (1999) 5250;
F.M. Lev, E. Pace, G. Salmè, Nucl. Phys. A 663 (2000) 365;
F.M. Lev, E. Pace, G. Salmè, Phys. Rev. C 62 (2000) 064004.
[25] B.L.G. Bakker, H.-M. Choi, C.-R. Ji, Phys. Rev. D 63 (2001) 074014.
[26] J.P.B.C. de Melo, T. Frederico, E. Pace, G. Salmé, Nucl. Phys. A 707 (2002) 399;
J.P.B.C. de Melo, T. Frederico, E. Pace, G. Salmé, Braz. J. Phys. 33 (2003) 301.
[27] I.L. Grach, L.A. Kondratyuk, Sov. J. Nucl. Phys. 38 (1984) 198.
[28] I.L. Grach, L.A. Kondratyuk, M. Strikman, Phys. Rev. Lett. 62 (1989) 387.
[29] V.A. Karmanov, Nucl. Phys. A 608 (1996) 316.
[30] P.L. Chung, W.N. Polyzou, F. Coester, B.D. Keister, Phys. Rev. C 37 (1988) 2000.
[31] S.J. Brodsky, J.R. Hiller, Phys. Rev. D 46 (1992) 2141.
[32] L.L. Frankfurt, T. Frederico, M. Strikman, Phys. Rev. C 48 (1993) 2182.
[33] J.P.B.C. de Melo, H.W.L. Naus, T. Frederico, Phys. Rev. C 59 (1999) 2278.
[34] V.A. Karmanov, J.F. Mathiot, A.V. Smirnov, Phys. Rev. D 75 (2007) 045012.
[35] L.L. Frankfurt, M. Strikman, Nucl. Phys. B 148 (1979) 107;
L.L. Frankfurt, M. Strikman, Phys. Rep. 76 (1981) 215.
[36] B.D. Keister, W.N. Polizou, Adv. Nucl. Phys. 20 (1991) 225.