Estimation of hydraulic jump on corrugated bed using artificial neural networks and genetic programming

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Abstract: Artificial neural networks (ANNs) and genetic programming (GP) have recently been used for the estimation of hydraulic data. In this study, they were used as alternative tools to estimate the characteristics of hydraulic jumps, such as the free surface location and energy dissipation. The dimensionless hydraulic parameters, including jump depth, jump length, and energy dissipation, were determined as functions of the Froude number and the height and length of corrugations. The estimations of the ANN and GP models were found to be in good agreement with the measured data. The results of the ANN model were compared with those of the GP model, showing that the proposed ANN models are much more accurate than the GP models.

Key words: artificial neural networks; genetic programming; corrugated bed; Froude number; hydraulic jump

1 Introduction

The transition of a supercritical open channel flow into a subcritical flow is associated with the formation of a hydraulic jump. Hydraulic jumps have been extensively studied because of their frequent occurrence in nature and their use as energy dissipators in outlet works of hydraulic structures (Hager 1992).

The classical equation for estimating the relationship between the ratio of the downstream flow depth to the upstream flow depth, \( y_2^*/y_1^* \), and the upstream Froude number \( F_{1r} = u_1/\sqrt{g y_1} \) (where \( u_1 \) is the mean velocity at the beginning of the jump and \( g \) is the acceleration due to gravity) originates from the momentum equation. Applying the momentum equation to the control volume bounded by the upstream and downstream cross-sections of the jump, the subcritical sequent depth \( y_2^* \) is obtained from Eq. (1) (Vischer and Hager 1995):

\[
\frac{y_2^*}{y_1^*} = \frac{1}{2} \left( \sqrt{1 + 8 F_{1r}^2} - 1 \right)
\]

A complete description of a hydraulic jump also involves its length \( L_j \), which is the distance between the two cross-sections with the sequent depth \( y_2^* \) and upstream supercritical depth \( y_1^* \). From the practical point of view, the jump length is an important variable to define the downstream limit beyond which no bed protection is necessary. The jump length is hard to
define in actual experiments, mainly because the end cross-section of the hydraulic jump is difficult to locate due to surface waves and residual turbulence (Hager 1992).

Preliminary investigations by Hager (1992), Hughes and Flack (1984), Mohamed Ali (1991), Negm (2002), and Carollo et al. (2007) indicate that if the bed of the channel on which the jump is formed is rough, the sequent depth $y_2$ could be significantly lower than the corresponding subcritical sequent depth $y'_2$. Ead and Rajaratnam (2002) installed corrugated sheets on the bed in such a way that the crests of sinusoidal corrugations were at the same level as the upstream bed on which the supercritical flow was produced by a sluice gate. The roughness height was assumed equal to the corrugation height. Tokyay (2005) experimentally investigated the effects of channel bed corrugations on hydraulic jumps. In his experiment, wave steepness values of 0.20 and 0.26 were used, and the Froude number values range from was 5 to 12. Abbaspour et al. (2009) studied the effects of a sinusoidal corrugated bed on the characteristic parameters of a hydraulic jump within a wide range of wave steepness values. The sequent depth ratio $y_2/y_1$, the jump length, the energy dissipation, and the bed shear stress of the jump were determined as functions of the Froude number, and the results were compared with the data obtained from a smooth channel bed.

Because of the complexity of hydraulic jumps, more practical tools are required to model hydraulic jump processes. Regressions have been most commonly used to estimate jump characteristics. However, regression analysis may have large uncertainties, and the computed jump depth and length can be far from the actual ones. Also, the regression analysis has some limitations caused by predefined equations for modeling.

Recently, artificial neural networks (ANNs) and genetic programming (GP) have been used to model hydraulic jump processes. They have been used to estimate the scouring around piles by Kambekar and Deo (2003) and the scouring below spillways by Azmathullah et al. (2008). Also, a combination of the fuzzy inference system (FIS) with ANNs, ANFIS, has been employed to estimate the wave characteristics by Mahjoobi et al. (2008). GP and ANNs have been successfully applied in maritime engineering (Kalra and Deo 2007; Singh et al. 2007; Gaur and Deo 2008).

The purpose of this study was to investigate the characteristics of hydraulic jumps in a horizontal flume with a corrugated bed using the ANN and GP methods. These soft computing tools can evaluate the relative importance of input parameters, such as the relative roughness, the corrugation wavelength, and the Froude number, on the jump process.

2 Materials and methods

2.1 Experimental setup

The experimental setup consisted of a main flume in a discharge collection channel. The main flume was 0.25 m wide and 0.50 m deep, and had a bed slope of 0.002. A triangular weir was placed at the end of the channel to measure the discharge. A supercritical approach flow
was produced using a sluice gate. A corrugated polyethylene sheet with sinusoidal corrugations of wavelength $s$ and height $t$ was installed perpendicular to the flow direction in the flume so that the corrugation crests were at the level of the upstream bed carrying the supercritical flow. The flow channel section of the experiment is illustrated in Fig. 1 (Abbaspour et al. 2009). A total of 123 experimental groups were conducted. Ranges of the variables in the experiment are shown in Table 1. Hydraulic jumps on the corrugated bed were produced for different Froude numbers, and the hydraulic parameters were measured. The water surface profiles of the jumps on the corrugated bed were measured at the centerline of the flume with a point gauge with an accuracy of 0.1 mm. The supercritical depth $y_1$ and sequent depth $y_2$ of the jumps were continuously measured using ultra sonic sensors, and the data was saved on a computer and processed with the VisiDAQ software. The length of the jump, $L_j$, in the experiment was recorded. The values of the Reynolds number in this experiment were in the range of 61 200 to 175 600.

Using the Buckingham $\pi$ theorem, non-dimensional equations in functional forms can be obtained. The following functional relationships have been worked out in the present study, as shown in Eqs. (2) through (4), where $y_2/y_1$, $L_j/y_1$, and $E_L/E_i$ are, respectively, the sequent depth ratio, length ratio, and energy dissipation in a hydraulic jump:

$$\frac{y_2}{y_1} = f_1\left(F_{Ri}, \frac{t}{y_1}, \frac{s}{y_1}\right)$$

$$\frac{L_j}{y_1} = f_2\left(F_{Ri}, \frac{t}{y_1}, \frac{s}{y_1}\right)$$

$$\frac{E_L}{E_i} = \frac{E_2 - E_1}{E_i} = f_3\left(F_{Ri}, \frac{t}{y_1}, \frac{s}{y_1}\right)$$

where $E_1$ is the difference between the specific energy before and after the jump, and $E_L = E_2 - E_1$.

### 2.2 Artificial neural network (ANN)

An artificial neural network (ANN) is an information processing paradigm that is inspired
by the way biological nervous systems, such as the brain, process information. It is composed of a large number of highly interconnected processing elements (neurons) working in unison to solve specific problems. Neurons are arranged in layers, including an input layer, hidden layers, and an output layer. There is no specific rule that dictates the number of hidden layers. The function is established largely based on the connections between the elements of the network. In the input layer, each neuron is designated for one of the input parameters. The network learns by applying the back-propagation algorithm, which compares the neural network simulated values with the actual values and calculates the estimation errors. The data set in the network is divided into a learning data set, which is used to train the network, and a validation data set, which is used to test the network performance. In the present study, the neural network fitting tool (nftool) of MATLAB 7.5 was used.

After training the network, verification is conducted until the success of the training can be established. In the simulation of hydraulic jumps, characteristic data were investigated with the neural network using the Levenberg-Marquardt algorithm, which is an approximation of Newton’s method. In order to check the sensitivity of the neural networks, a simulation study was carried out with hidden nodes of different numbers, 5, 10, 15, and 20.

The parameters considered in the study are the dimensionless jump depth $y_2/y_1$, dimensionless jump length $L_j/y_1$, and dimensionless energy dissipation $E_{11}/E_{1}$. The dimensionless parameters, $s/y_1$, $t/y_1$, and $Fr_1$, were used as inputs to the ANN model to estimate the hydraulic jump characteristics. One hundred and twenty-three experimental data sets were used for the ANN simulations. They were divided into three parts, 60% for training, 20% for validation, and 20% for testing.

The correlation coefficient ($R$), the root mean square error ($RMSE$), the mean absolute error ($MAE$), and the Nash-Sutcliffe efficiency coefficient ($NSE$) statistics were used to evaluate the model accuracy. $R$ shows the degree to which two variables were linearly related. Different types of information about the predictive capabilities of the model are measured through $RMSE$ and $MAE$. An efficiency of 1 ($NSE = 1$) corresponds to a perfect match of the modeled values to the observed data.

$$
R = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2 \sum_{i=1}^{n} (Y_i - \bar{Y})^2}} \quad (5)
$$

$$
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (X_i - Y_i)^2}{n}} \quad (6)
$$

$$
MAE = \frac{\sum_{i=1}^{n} (X_i - Y_i)}{n} \quad (7)
$$
where \( X_i \) is the observed values, \( \bar{X} \) is the mean of \( X_i \), \( Y_i \) is the estimated values, \( \bar{Y} \) is the mean of \( Y_i \), and \( n \) is the number of data sets.

From the simulation study that was carried out on different numbers of hidden nodes, it was found that good estimation accuracy was achieved with 15 neurons in the hidden layer in four trials. The sigmoid, \( f(x) = \frac{1}{1 + e^{-x}} \), and linear activation functions were used for the hidden and output nodes, respectively.

### 2.3 Genetic programming (GP)

In artificial intelligence, genetic programming (GP) is an evolutionary algorithm-based methodology inspired by biological evolution to find computer programs that perform a user-defined task. GP initializes a population consisting of random members known as chromosomes, and the fitness of each chromosome is evaluated with respect to a target value. The principle of Darwinian natural selection is used to select and reproduce fitter programs. GP creates computer programs that consist of variables and several mathematical function sets as the solution. The function set of a system can be composed of arithmetic operations (+, −, ×, ÷), function calls (such as \( e^x \), \( x \), sqrt, and power), even relational operators (>, <, =) or conditional operators, and a terminal set with variables and constants \( (x_1, x_2, \ldots, x_n) \). An initial population is randomly created with a number of individuals formed by nodes (operators, variables, and constants) and previously defined according to the problem domain. An objective function must be defined to evaluate the fitness of each individual. Selection, crossover, and mutation operators are then applied to the best individuals and a new population is created. The whole process is repeated until the given generation number is reached (Koza 1992).

The fitness of a GP individual may be computed using Eq. (9):

\[
 f = \sum_{j=1}^{n} \left| X_j - Y_j \right| 
\]

where \( X_j \) is the value returned by a chromosome for the fitness case \( j \), and \( Y_j \) is the expected value for the fitness case \( j \).

In the GP model many operators, like sin, cos, and log, and mathematical functions were used, and it was found that the functions of the proposed GP model were complex. Also, the GP model using more operators has larger estimated difference. In this study, for simplicity, only four arithmetic operators (+, −, ×, ÷) were used. The functional and operational parameter settings used in the GP model are shown in Table 2. The performance of the GP model in training and testing sets was validated in terms of the common statistical measures \( R, \text{RMSE}, \text{MAE}, \text{and NSE} \).
Table 2 Parameters of GP Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter description</th>
<th>Setting of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>Function set</td>
<td>+, -, x, +</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Generation number</td>
<td>5000</td>
</tr>
<tr>
<td>$P_3$</td>
<td>Mutation frequency (%)</td>
<td>50</td>
</tr>
<tr>
<td>$P_4$</td>
<td>Number of chromosomes</td>
<td>30</td>
</tr>
<tr>
<td>$P_5$</td>
<td>Number of generations</td>
<td>3</td>
</tr>
<tr>
<td>$P_6$</td>
<td>Linking function</td>
<td>Multiplication</td>
</tr>
</tbody>
</table>

3 Results and discussion

In order to investigate the hydraulic jumps on the corrugated bed, the characteristics, such as water surface profile, jump length, and bed energy dissipation, were evaluated. During model developments in this study, the dimensionless jump depth, jump length, and energy dissipation ($y_j/y_i$, $L_j/y_i$, and $E_j/E_i$) were selected as outputs and three dimensionless parameters $Fr_i$, $t/y_i$, and $s/y_i$ as inputs.

3.1 Hydraulic jump estimation using ANN model

Different ANN structures were tried in terms of hidden layer node numbers. In this study, the number of neurons in the hidden layer was obtained using the trial and error method. From the simulation study, which was carried out using the ANN model, it was found that with 15 neurons in the hidden layer, the estimation accuracy increased to some extent.

For the better understanding of the model performance, plots of $y_j/y_i$, $L_j/y_i$, and $E_j/E_i$ simulations taken from the training, validation, and testing data sets are given in Figs. 2, 3, and 4. In general, an $R$ value greater than 0.9 and a $NSE$ value greater than 0.7 indicate a very satisfactory model performance. The comparison between the estimated values of $y_j/y_i$, $L_j/y_i$, and $E_j/E_i$ with measured data (Figs. 2, 3, and 4) shows that an excellent estimation using the ANN model can be observed.

![Fig. 2 Comparison of measured and estimated $y_j/y_i$ values using ANN model for training, validation, and testing data](image-url)
3.2 Hydraulic jump estimation using GP model

Figs. 5, 6, and 7 show the estimated results of \( y_2/y_1 \), \( L_j/y_1 \), and \( E_L/E_i \) for the training and testing data. An almost perfect agreement of the measured values with the GP estimations is clearly seen. For the GP model, referring to Figs. 5, 6, and 7, the GP model has good ability in estimating \( y_2/y_1 \) and \( E_L/E_i \), as reflected in low values of RMSE and MAE and a high value of \( R \).

The superior performance of the GP model, compared with other methods, is attributed to the powerful artificial intelligence techniques for computer learning inspired by natural evolution to find an appropriate mathematical model to fit a set of points. GP employs a population of functional expressions and also numerical constants, based on how closely they fit to the corresponding data (Koza 1992).

The simplified analytical forms of the proposed GP model for \( y_2/y_1 \), \( L_j/y_1 \), and \( E_L/E_i \) may be, respectively, expressed as shown in Eqs. (10) through (12).
Fig. 5 Comparison of measured and estimated $y_2/y_1$ values using GP model for training and testing data

$$y = 0.969x + 0.183$$
$$R = 0.984$$

(a) Training data

Fig. 6 Comparison of measured and estimated $L_1/y_1$ values using GP model for training and testing data

$$y = 0.920x + 1.714$$
$$R = 0.936$$

(a) Training data

Fig. 7 Comparison of measured and estimated $E_{1}/E_1$ values using GP model for training and testing data

$$y = 0.979x + 0.013$$
$$R = 0.984$$

(a) Training data

$$\frac{y_2}{y_1} = \frac{(C_x^4t/y_1 + C_1)[s/y_1(C_2C_4 - Fr_t + 1) - C_4]}{(C_2C_1C_4^4)s/y_1}$$

(10)
\[ L_{1y} = \left( C_3 + \frac{F_{1y}}{t/y_1 - C_3} + t/y_1 \right) \left[ \frac{(t/y_1)^2}{(s/y_1)^2} + (C_6 + C_5) + (t/y_1)^2 \right] \]  
\[ E_{1y} = \left[ \frac{(C_2 s/y_1 + F_{1y}^2) C_4}{C_2 F_{1y} (s/y_1 + C_4)} \right] \left[ \frac{C_4 (s/y_1) F_{1y} + t/y_1}{C_4 (s/y_1)(t/y_1)} \right] \left[ \frac{t/y_1}{C_5 (F_{1y} + 1) + t/y_1} \right] \]  

where \( C_1, C_2, C_3, C_4, C_5, \) and \( C_6 \) are constant coefficients that are determined by the GP model (Table 3).

**Table 3 Constant coefficients in GP model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
<th>( C_5 )</th>
<th>( C_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_1/y_1 )</td>
<td>7.174</td>
<td>1.000</td>
<td>-5.440</td>
<td>-8.916</td>
<td>5.137</td>
<td>2.118</td>
</tr>
<tr>
<td>( L_{1y}/y_1 )</td>
<td>-2.818</td>
<td>6.821</td>
<td>-1.481</td>
<td>5.772</td>
<td>2.848</td>
<td>8.680</td>
</tr>
<tr>
<td>( E_{1y}/E_1 )</td>
<td>7.677</td>
<td>7.857</td>
<td>8.543</td>
<td>8.440</td>
<td>1.101</td>
<td>-9.671</td>
</tr>
</tbody>
</table>

### 3.3 Comparison of ANN model with GP model

The ANN and GP models are compared in Figs. 2 through 7. It can be seen from the fit line equations (the equations are assumed to be \( y = ax + b \)) in the scatter plots of the GP model that the coefficients \( a \) and \( b \) for the ANN model, with a higher \( R \) value, are, respectively, closer to 1 and 0 than the GP model. This can be clearly observed from its fit line equation coefficients.

Table 4 compares the ANN and GP models, with all statistical measures, \( R, \) \( RMSE, \) \( NSE, \) and \( MAE, \) of the training and testing data. According to Table 4, the ANN model has lower absolute error as compared with the GP model, showing that the proposed ANN models are much more accurate than the GP models for water engineering.

**Table 4 RMSE, MAE, R, and NSE statistics of training and testing data of ANN and GP models**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model</th>
<th>( RMSE )</th>
<th>( MAE )</th>
<th>( R )</th>
<th>( NSE )</th>
<th>( RMSE )</th>
<th>( MAE )</th>
<th>( R )</th>
<th>( NSE )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_1/y_1 )</td>
<td>ANN</td>
<td>0.026</td>
<td>0.013</td>
<td>0.999</td>
<td>0.99</td>
<td>0.035</td>
<td>0.02</td>
<td>0.995</td>
<td>0.99</td>
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<tr>
<td></td>
<td>GP</td>
<td>0.210</td>
<td>0.170</td>
<td>0.984</td>
<td>0.98</td>
<td>1.130</td>
<td>0.83</td>
<td>0.919</td>
<td>0.91</td>
</tr>
<tr>
<td>( L_{1y}/y_1 )</td>
<td>ANN</td>
<td>0.036</td>
<td>0.020</td>
<td>0.999</td>
<td>0.99</td>
<td>0.027</td>
<td>0.03</td>
<td>0.998</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>GP</td>
<td>2.549</td>
<td>2.061</td>
<td>0.936</td>
<td>0.98</td>
<td>3.773</td>
<td>3.09</td>
<td>0.920</td>
<td>0.96</td>
</tr>
<tr>
<td>( E_{1y}/E_1 )</td>
<td>ANN</td>
<td>1×10^{-4}</td>
<td>8×10^{-4}</td>
<td>0.998</td>
<td>0.98</td>
<td>2×10^{-5}</td>
<td>1×10^{-4}</td>
<td>0.999</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>GP</td>
<td>0.016</td>
<td>0.013</td>
<td>0.984</td>
<td>0.98</td>
<td>0.039</td>
<td>0.03</td>
<td>0.949</td>
<td>0.98</td>
</tr>
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</table>

### 4 Conclusions

In general, the performance of the ANN and GP models are superior to the statistical regression schemes. The ANN and GP models were developed to determine the hydraulic jump on a corrugated bed. The input parameters used for the ANN and GP simulations are the dimensionless wavelength (\( s/y_1 \)), the dimensionless height (\( t/y_1 \)), and the upstream Froude number (\( F_{1y} \)). These models can be successfully used in the computation of hydraulic jump.
characteristics. The optimum ANN model was obtained after different structures were tried in terms of hidden layer node numbers. The estimations of the ANN model were compared with those of the GP model. According to Table 4, the model performance can be evaluated as satisfactory if $NSE > 0.7$ and $R > 0.9$ with low values of $RMSE$ and $MAE$. The proposed ANN models are much more accurate than the GP models. The GP models are much more practical than the ANN models because they provide nonlinear mathematical equations. This study used limited field data from available literature and further work using more data from various areas may be required to provide additional support for these conclusions.

References


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