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A dynamic axially symmetric goal and its extended solution for a fixed rigid circular multi-layer plate

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Abstract

The subject matter of the article is a fixed circular multi-layer plate and its reaction to the axially symmetrical force (direct stress) affecting its front face. The force itself is an arbitrary time and radial coordinate function. The authors apply the method of finite integral transformation (based on the theory of electroelasticity in a three-dimentional model) to develop a new closed solution. The improvement of this new solution lies in the fact that boundary conditions for cylinder and front faces of a multi-layer plate here are completely fulfilled when compared with the data obtained in previous research. The achieved ratio design allows to further analyze this multi-layer plate fluctuations frequency content as well as to analyze changing characteristics of its stress-strain behaviour and their dependence on different layers stress-strain properties.

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Keywords: Multi-layer plate; Boundary problem; Theory of elasticity; Dynamic load; Integral transformation.

1. Introduction

Nowadays materials with different physical mechanical properties are used in various multilayer structures. It allows to create elastic systems of comparatively small weight which are at the same time characterized by high bending stiffness. Besides, these systems offer vibration resistance and are able to absorb impact effect energy. That is why they are widely used in aircraft and civil engineering [1-3]. To study multilayer systems stress-strain behaviour, researchers usually use applicable theories for thin elements in connection with some kinematical

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hypothesis [4-11]. This approach, though, does not allow to deeply go into nonstationary process. It does not ley us describe the construction work in real conditions, either.

To solve this problem we suggest using the theory of elasticity and its design ratio. Let us specially stress that one has to encounter considerable mathematical difficulties when analyzing a three-dimentional model of a simple one-layer plate. To illustrate it we should mention some calculation methodologies connected with mixed boundary conditions on end faces [12, 13] and those involving partial solutions [13, 14]. In the previous paper the researches developed a dynamical problem closed solution for a fixed rigid circular multi-layer plate but several conditions for a cylindrical surface were not fulfilled [15].

This paper introduces a new calculation model which gives an extended solution to the problem [15] and meets all boundary conditions for a multi-layer plate.

2. Problem specification

Let a fixed rigid circular multi-layer plate in the cylindrical coordinate system occupies (r_*, θ, z_*) region Ω : $\{0 \le r_* \le b,\ 0 \le \theta \le 2\pi,\ 0 \le z_* \le h^*\}$. For definiteness, we adopt that this thin structure consists of three layers which are in hard contact. Their inner and outer layers thickness is h_1^* and h_2^* ($h^* = 2h_1^* + h_2^*$). Bending oscillations in this case are generated by the axially symmetric dynamic load $q^*(r_*, t_*)$.

Differential equations of motion, as well as boundary conditions in a cylindrical coordinates system and dimensionless form, are as follows [17]:

$$\nabla_1^2 U + \frac{C_{55}^{(s)}}{C_{11}^{(s)}} \frac{\partial^2 U}{\partial z^2} + \frac{\left(C_{13}^{(s)} + C_{55}^{(s)}\right)}{C_{11}^{(s)}} \frac{\partial^2 W}{\partial r \partial z} - \Phi(\rho) \frac{\partial^2 U}{\partial t^2} = 0 , \quad (1.1)$$

$$\frac{C_{55}^{(s)}}{C_{11}^{(s)}} \nabla_2^2 W + \frac{C_{33}^{(s)}}{C_{11}^{(s)}} \frac{\partial^2 W}{\partial z^2} + \frac{\left(C_{13}^{(s)} + C_{55}^{(s)}\right)}{C_{11}^{(s)}} \frac{\partial}{\partial z} \nabla U - \Phi(\rho) \frac{\partial^2 W}{\partial t^2} = 0 ,$$

$$r = 0,1 \quad W(0,z,t) < \infty ,\ U(0,z,t) < \infty ,\ U(1,z,t) = 0 ,\ W(1,z,t) = 0 ; \quad (1.2)$$

$$z = 0 \quad \sigma_{zz}^{(1)} = \frac{C_{13}^{(1)}}{C_{11}^{(1)}} \nabla U^{(1)} + \frac{C_{33}^{(1)}}{C_{11}^{(1)}} \frac{\partial W^{(1)}}{\partial z} = q(r,t) ,\ \sigma_{rz}^{(1)} = \frac{C_{55}^{(1)}}{C_{11}^{(1)}} \left(\frac{\partial W^{(1)}}{\partial r} + \frac{\partial U^{(1)}}{\partial z} \right) = 0 ; \quad (1.3)$$

$$z = h \quad \sigma_{zz}^{(1)} = \frac{C_{13}^{(1)}}{C_{11}^{(1)}} \nabla U^{(1)} + \frac{C_{33}^{(1)}}{C_{11}^{(1)}} \frac{\partial W^{(1)}}{\partial z} = 0 ,\ \sigma_{rz}^{(1)} = \frac{C_{55}^{(1)}}{C_{11}^{(1)}} \left(\frac{\partial W^{(1)}}{\partial r} + \frac{\partial U^{(1)}}{\partial z} \right) = 0 ; \quad (1.4)$$

$$z = h_1, h_1 + h_2 \quad \left(\frac{C_{13}^{(1)}}{C_{11}^{(1)}} \nabla U^{(1)} + \frac{C_{33}^{(1)}}{C_{11}^{(1)}} \frac{\partial W^{(1)}}{\partial z} \right) - \left(\frac{C_{13}^{(2)}}{C_{11}^{(1)}} \nabla U^{(2)} + \frac{C_{33}^{(2)}}{C_{11}^{(1)}} \frac{\partial W^{(2)}}{\partial z} \right) = 0 , \quad (1.5)$$

$$\frac{C_{55}^{(1)}}{C_{11}^{(1)}} \left(\frac{\partial W^{(1)}}{\partial r} + \frac{\partial U^{(1)}}{\partial z} \right) - \frac{C_{55}^{(2)}}{C_{11}^{(1)}} \left(\frac{\partial W^{(2)}}{\partial r} + \frac{\partial U^{(2)}}{\partial z} \right) = 0 ,\ U^{(1)} = U^{(2)} ,\ W^{(1)} = W^{(2)} ;$$

$$t=0 \;\; U(r,z,0)=U_0(r,z), \;\; \dot{U}(r,z,0)=\dot{U}_0(r,z), \tag{1.6}$$

$$W(r,z,0)=W_0(r,z), \;\; \dot{W}(r,z,0)=\dot{W}_0(r,z);$$

where

$$\{U,W,r,z,h,h_1,h_2\}=\{U^*,W^*,r_*,z_*,h^*,h_1^*,h_2^*\}/b\,; \;\; t=t_* b^{-1}\sqrt{C_{11}^{(2)}/\rho^{(2)}}\,;$$

$$\{U,W,\Phi(\rho)\}=\left\{U^{(1)},W^{(1)},\frac{C_{11}^{(2)}\rho^{(1)}}{C_{11}^{(1)}\rho^{(2)}}\right\}\left[1-H(z-h_1)+H(z-h_1-h_2)\right]++\left\{U^{(2)},W^{(2)},1\right\}\times$$

$$\times\left[H(z-h_1)-H(z-h_1-h_2)\right],\;\; q(r,t)=q^*(r,t)/C_{11}\,;\; U^*(r_*,z_*,t_*),\; W^*(r_*,z_*,t_*),\; \sigma_{zz}(r_*,z_*,t_*),$$

$\sigma_{rz}(r_*,z_*,t_*)-$ are components of movement vector and mechanical stress tensor; $\rho^{(s)},C_{mk}^{(s)}$ – is spatial density and elastic modules of the outer $(s=1)$ and inner $(s=2)$ construction layers; $U_0,\dot{U}_0,\;W_0,\dot{W}_0$ – known at the starting point of movement displacement rate; $H(\ldots)-$ Heaviside step function [17];

$$\nabla_1^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2},\;\; \nabla_2^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r},\;\; \nabla=\frac{\partial}{\partial r}+\frac{1}{r}\,.$$

"Point" is differentiation according to t .

Formulae 1.1-1.6 represent a mathematical statement for the theory of elasticity boundary problem.

3. General solution

The general solution is achived with the method of finite integral transformation. Firstly, formulae 1.1 – 1.6 are reduced to a standard form making it possibly to carry put a procedure of separation of variables. For that purpose, Formula 1.2 is changed for that of the vertical component action of support resistance on the cylindrical surface of the plate $N(z,t)$. Taking into consideration shearing stress formula 1.3, we show them as follows:

$$\left(\frac{\partial W}{\partial r}+\frac{\partial U}{\partial z}\right)_{|r=1}=N(z,t)\,. \tag{2.1}$$

As a result we have a new elasticity problem when the front face of a multilayer plate is affected by a definite mechanical stress $q(r,t)$ and the cylindrical surface of the construction is affected by an unknown shearing stress $N(z,t)$.

To solve the problems 1.1 – 1.5 and 2.1 we introduce new functions $u(r,z,t)$, $w(r,z,t)$, which are corelated with $U(r,z,t)$ $W(r,z,t)$:

$$U(r,z,t)=\left(r-r^3\right)q(r,t)+u(r,z,t),\; W(r,z,t)=rN(z,t)+w(r,z,t), \tag{2.2}$$

They also let us find a solution for the arbitrary function $q(r,t)$, as well as make similar boundary conditions according to coordinate r .

Here $N-$ we have an indeterminate function which can be calculated if the cylindrical surface of the plate is stationary (that is boundary condition 1.2) and the plate is balanced because of axially symmetrical loading:

$$\int_0^h N(z,t)dz = -\int_0^1 q(r,r)rdr \;, \tag{2.3}$$

Consequently, after placing (2.2) in (1.1) – (1.6), (2.1) we get a new boundary value problem concerning $u(r,z,t)$, $w(r,z,t)$ functions.

Besides, the conditions 1.2 with account of 2.1 are characterized by $r=1$:

$$u(1,z,t)=0 \;,\; \frac{\partial w}{\partial r}_{|r=1} = 0 \tag{2.4}$$

Then, for the boundary problem u,w we use Hankel transformation [18] with the finite bound of r argument and general final transformation (instruments and controls) [13] of z radial coordinate. The authors introduce this solution in papers {15] and [19].

To finally find $U(r,z,t)$, $W(r,z,t)$ functions, we use transformations with account of Formula 2.2:

$$U(r,z,t) = (r-r^3)q(r,t) + 2\sum_{n=0}^{\infty} J_0(j_n)^{-2}\left[S_4 + \sum_{i=1}^{\infty} G_{in}K_{1in}\|K_{in}\|^{-2}\right] J_1(j_n r) \;, \tag{2.5}$$

$$W(r,z,t) = rN(z,t) + 2\sum_{n=0}^{\infty} J_0(j_n)^{-2}\left[Y_H + \sum_{i=1}^{\infty} G_{in}K_{2in}\|K_{in}\|^{-2}\right] J_0(j_n r).$$

where $G_{in}, K_{1in}, K_{2in}, \|K_{in}\| -$ is load transformant, components of vector-function transformation core [12]; $Y_H -$ standardizing function along the axial coordinate; $j_n -$ characteristic value [17].

$N(z,t)$ function can be found on condition when the cylindrical surface is stationary with account of $r=1$.

4. Numerical results and conclusions

As a model plate we take $b = 14\times10^{-3}$ m plate. Its layers elastic properties correspond to nonpolarized piezoceramic material ZTS-19 [20].

As a result we obtained basic frequency values of the plate fluctuations (with plate of various thickness) ω_{11}, and some experimental data. Investigation results which were obtained while using the algorithm introduced in this paper prove the results of the experiment. On the other hand, the classic applicable theory and Timoshenko's theory yield undercharged values.

We also analyzed a multi-layer plate stress-strain behaviour (its inner layer elasticity being 1.5 times higher than those of its outer layer, $h_2^* = 2h_1^* = 0.25\times10^{-3}$ m) under evenly distributed load:

$$q(r,t) = q_0 \sin\theta t \;,$$

where $q_0, \theta -$ is amplitude and external force frequency in nondimensional form.

On the basis of our research and calculations the following conclusions can be formulated:

1) Calculation model specification introduced in this paper (that is shearing stress involvement $N(z,t)$) leads to displacement reduction. For example, when we calculate $W(0,0,t)$ with account of $N(z,t)$ the displacement is 18% less ($\theta = 0.5\lambda_{11},\ \lambda_{11} = 0.21$) when compared with the solution given in Paper [15].
2) Frequency increase θ leads to gradient increase $N(z,t)$ in cross-sectional height. Besides, when $N(z,t)$ is calculated the twoness law of shearing stress in angular points with account of $z = 0, h$ is violated. Paper [12] yields similar results.
3) The hypothesis of straight normals for multi-layer thin plates can be applied when their elasticity height along characteristics vary considerably.

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