



Revisiting the cosmological constraints on the interacting dark energy models

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ABSTRACT

In this work, we consider the cosmological constraints on the interacting dark energy models. We generalize the models considered previously by Guo et al. (2007) [1], Costa and Alcaniz (2010) [2], and try to discuss two general types of models: type I models are characterized by $\rho_X/\rho_m = f(a)$ and $f(a)$ can be any function of scale factor a , whereas type II models are characterized by $\rho_m = \rho_{m0}a^{-3+\epsilon(a)}$ and $\epsilon(a)$ can be any function of a . We obtain the cosmological constraints on the type I and II models with power-law, CPL-like, logarithmic $f(a)$ and $\epsilon(a)$ by using the latest observational data.

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The dark energy has been one of the most active fields in modern cosmology since the discovery of the accelerated expansion of our universe (see e.g. [3] for reviews). Among the conundrums in the dark energy cosmology, the so-called cosmological coincidence problem is the most familiar one. This problem is asking why are we living in an epoch in which the densities of dark energy and matter are comparable? Since their densities scale differently with the expansion of our universe, there should be some fine-tunings. To alleviate the cosmological coincidence problem, it is natural to consider the possible interaction between dark energy and dark matter in the literature (see e.g. [4–13]). In fact, since the nature of both dark energy and dark matter are still unknown, there is no physical argument to exclude the possible interaction between them. On the contrary, some observational evidences of this interaction have been found recently. For example, in a series of papers by Bertolami et al. [14], they shown that the Abell Cluster A586 exhibits evidence of the interaction between dark energy and dark matter, and they argued that this interaction might imply a violation of the equivalence principle. On the other hand, in [15], Abdalla et al. found the signature of interaction between dark energy and dark matter by using optical, X-ray and weak lensing data from 33 relaxed galaxy clusters. Therefore, it is reasonable to consider the interaction between dark energy and dark matter in cosmology.

We consider a flat Friedmann–Robertson–Walker (FRW) universe. In the literature, it is usual to assume that dark energy and dark matter interact through a coupling term Q , according to

$$\dot{\rho}_X + 3H\rho_X(1 + w_X) = Q, \quad (1)$$

$$\dot{\rho}_m + 3H\rho_m(1 + w_m) = -Q, \quad (2)$$

where ρ_m and ρ_X are densities of dark matter and dark energy (we assume that the baryon component can be ignored); w_X is the equation-of-state parameter (EoS) of dark energy and it is assumed to be a constant; a dot denotes the derivative with respect to cosmic time t ; $H \equiv \dot{a}/a$ is the Hubble parameter; $a = (1+z)^{-1}$ is the scale factor (we have set $a_0 = 1$; the subscript “0” indicates the present value of corresponding quantity; z is the redshift). Notice that Eqs. (1) and (2) preserve the total energy conservation equation

$$\dot{\rho}_{tot} + 3H\rho_{tot}(1 + w_{eff}) = 0, \quad (3)$$

where $\rho_{tot} = \rho_m + \rho_X$ is the total energy; w_{eff} is the total (effective) EoS. Since there is no natural guidance from fundamental physics on the coupling term Q , one can only discuss it to a phenomenal level. The most familiar coupling terms extensively considered in the literature are $Q = \alpha\kappa\rho_m\dot{\phi}$, $Q = 3\beta H\rho_{tot}$, and $Q = 3\eta H\rho_m$. The first one arises from, for instance, string theory or scalar-tensor theory (including Brans–Dicke theory) [6–8]. The other two are phenomenally proposed to alleviate the coincidence problem in the other dark energy models [9–11]. In the usual approach, one should firstly write down the coupling term Q , and then obtain the evolutions of ρ_m and ρ_X from Eqs. (1) and (2), respectively. In fact, this is the common way to study the interacting dark energy models in the literature.

However, there is also an alternative way in the literature [1,2,16–18]. One can reverse the logic mentioned above. Due to the interaction Q , the evolutions of ρ_m and ρ_X should deviate from the ones without interaction, i.e., $\rho_m \propto a^{-3}$ and $\rho_X \propto a^{-3(1+w_X)}$, respectively. If the deviated evolutions of ρ_m and/or ρ_X are given,

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one can find the corresponding interaction Q from Eqs. (1) and (2). Naively, the simplest example has been considered by Wang and Meng [16], namely

$$\rho_m = \rho_{m0} a^{-3+\epsilon}, \quad (4)$$

where ϵ is a constant which measures the deviation from the normal $\rho_m \propto a^{-3}$. Substituting into Eq. (1), it is easy to find the corresponding interaction $Q = \epsilon H \rho_m$ [11,16,17,19]. Alternatively, one can consider another type of interacting dark energy model which is characterized by [1,18]

$$\frac{\rho_X}{\rho_m} = \frac{\rho_{X0}}{\rho_{m0}} a^\xi, \quad (5)$$

where ξ is a constant which measures the severity of the coincidence problem. From Eqs. (1), (2) and (5), one can find that the corresponding interaction is given by [1]

$$Q = -H \rho_m \Omega_X (\xi + 3w_X) = -H \rho_X \Omega_m (\xi + 3w_X), \quad (6)$$

where $\Omega_i \equiv 8\pi G \rho_i / (3H^2)$ for $i = m$ and X , which are the fractional energy densities of dark matter and dark energy, respectively. In fact, Guo et al. [1] considered the cosmological constraints on the interacting dark energy model characterized by Eq. (5) with the 71 SNLS Type Ia supernovae (SNIa) dataset, the shift parameter R from the Wilkinson Microwave Anisotropy Probe 3-year (WMAP3) data, and the distance parameter A of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies. On the other hand, the interacting dark energy model characterized by Eq. (4) has been extended in [2]. It is more realistic that ϵ is a function of time. Costa and Alcaniz [2] considered the interacting dark energy model characterized by

$$\rho_m = \rho_{m0} a^{-3+\epsilon(a)}, \quad (7)$$

in which $\epsilon(a)$ was chosen to be

$$\epsilon(a) = \epsilon_0 a^{\epsilon_1}, \quad (8)$$

where ϵ_0 and ϵ_1 are constants. They obtained the constraints on this model by using the 307 Union SNIa dataset, the CMB constraint $\Omega_{m0} h^2 = 0.109 \pm 0.006$ from the Wilkinson Microwave Anisotropy Probe 5-year (WMAP5) data, and the distance ratio from $z_{\text{BAO}} = 0.35$ to $z_{\text{LS}} = 1089$ measured by SDSS, namely $R_{\text{BAO/LS}} = 0.0979 \pm 0.0036$.

In the present work, we generalize the interacting dark energy models considered in [1,2], and we call them type I and II models, respectively. The type I models are characterized by

$$\frac{\rho_X}{\rho_m} = f(a), \quad (9)$$

where $f(a)$ can be any function of a , beyond the special case in Eq. (5). The type II models are characterized by Eq. (7) but $\epsilon(a)$ can be any function of a , beyond the special case in Eq. (8). In the present work, we consider the constraints on these models by using the latest cosmological observations, namely, the 397 Constitution SNIa dataset [20], the shift parameter R from the newly released Wilkinson Microwave Anisotropy Probe 7-year (WMAP7) data [21], and the distance parameter A of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies [22,23]. In the next section, we briefly introduce these observational data. In Section 3 and Section 4, we discuss the type I and II models, and consider their cosmological constraints, respectively. A brief summary is given in Section 5.

2. Observational data

In the present work, we will consider the latest cosmological observations, namely, the 397 Constitution SNIa dataset [20], the shift parameter R from the newly released Wilkinson Microwave Anisotropy Probe 7-year (WMAP7) data [21], and the distance parameter A of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies [22,23].

The data points of the 397 Constitution SNIa compiled in [20] are given in terms of the distance modulus $\mu_{\text{obs}}(z_i)$. On the other hand, the theoretical distance modulus is defined as

$$\mu_{\text{th}}(z_i) \equiv 5 \log_{10} D_L(z_i) + \mu_0, \quad (10)$$

where $\mu_0 \equiv 42.38 - 5 \log_{10} h$ and h is the Hubble constant H_0 in units of 100 km/s/Mpc, whereas

$$D_L(z) = (1+z) \int_0^z \frac{d\tilde{z}}{E(\tilde{z}; \mathbf{p})}, \quad (11)$$

in which $E \equiv H/H_0$, and \mathbf{p} denotes the model parameters. Correspondingly, the χ^2 from the 397 Constitution SNIa is given by

$$\chi_\mu^2(\mathbf{p}) = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma^2(z_i)}, \quad (12)$$

where σ is the corresponding 1σ error. The parameter μ_0 is a nuisance parameter but it is independent of the data points. One can perform a uniform marginalization over μ_0 . However, there is an alternative way. Following [24,25], the minimization with respect to μ_0 can be made by expanding the χ_μ^2 of Eq. (12) with respect to μ_0 as

$$\chi_\mu^2(\mathbf{p}) = \tilde{A} - 2\mu_0 \tilde{B} + \mu_0^2 \tilde{C}, \quad (13)$$

where

$$\tilde{A}(\mathbf{p}) = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, \mathbf{p})]^2}{\sigma_{\mu_{\text{obs}}}^2(z_i)},$$

$$\tilde{B}(\mathbf{p}) = \sum_i \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, \mathbf{p})}{\sigma_{\mu_{\text{obs}}}^2(z_i)}, \quad \tilde{C} = \sum_i \frac{1}{\sigma_{\mu_{\text{obs}}}^2(z_i)}.$$

Eq. (13) has a minimum for $\mu_0 = \tilde{B}/\tilde{C}$ at

$$\tilde{\chi}_\mu^2(\mathbf{p}) = \tilde{A}(\mathbf{p}) - \frac{\tilde{B}(\mathbf{p})^2}{\tilde{C}}. \quad (14)$$

Since $\chi_{\mu, \text{min}}^2 = \tilde{\chi}_{\mu, \text{min}}^2$ obviously, we can instead minimize $\tilde{\chi}_\mu^2$ which is independent of μ_0 .

There are some other observational data, such as the observations of cosmic microwave background (CMB) anisotropy [21] and large-scale structure (LSS) [22]. However, using the full data of CMB and LSS to perform a global fitting consumes a large amount of computation time and power. As an alternative, one can instead use the shift parameter R from the CMB, and the distance parameter A of the measurement of the baryon acoustic oscillation (BAO) peak in the distribution of SDSS luminous red galaxies. In the literature, the shift parameter R and the distance parameter A have been used extensively. It is argued that they are model-independent [26], while R and A contain the main information of the observations of CMB and BAO, respectively.

As is well known, the shift parameter R of the CMB is defined by [26,27]

$$R \equiv \Omega_{m0}^{1/2} \int_0^{z_*} \frac{d\tilde{z}}{E(\tilde{z})}, \quad (15)$$

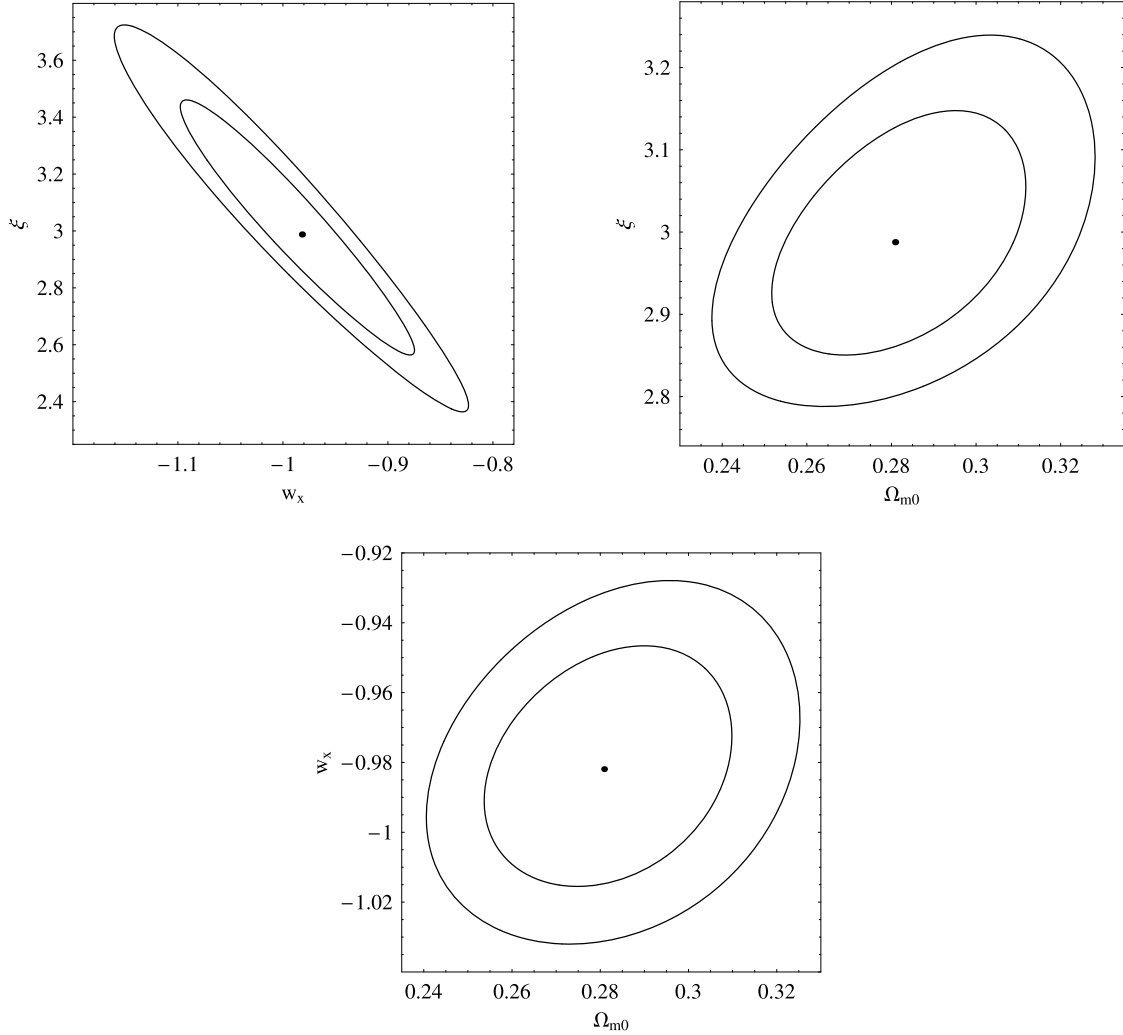


Fig. 1. The 68% and 95% confidence level contours in w_X - ξ plane, Ω_{m0} - ξ plane and Ω_{m0} - w_X plane for the type I model characterized by $f(a) = f_0 a^\xi$. The best-fit parameters are also indicated by the black solid points.

where the redshift of recombination $z_* = 1091.3$ which has been updated in the Wilkinson Microwave Anisotropy Probe 7-year (WMAP7) data [21]. The shift parameter R relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at z_* and the angular scale of the first acoustic peak in CMB power spectrum of temperature fluctuations [26,27]. The value of R has been updated to 1.725 ± 0.018 from the WMAP7 data [21]. On the other hand, the distance parameter A of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies [22] is given by

$$A \equiv \Omega_{m0}^{1/2} E(z_b)^{-1/3} \left[\frac{1}{z_b} \int_0^{z_b} \frac{d\bar{z}}{E(\bar{z})} \right]^{2/3}, \quad (16)$$

where $z_b = 0.35$. In [23], the value of A has been determined to be $0.469 (n_s/0.98)^{-0.35} \pm 0.017$. Here the scalar spectral index n_s is taken to be 0.963, which has been updated from the WMAP7 data [21]. So, the total χ^2 is given by

$$\chi^2 = \tilde{\chi}_\mu^2 + \chi_{CMB}^2 + \chi_{BAO}^2, \quad (17)$$

where $\tilde{\chi}_\mu^2$ is given in Eq. (14), $\chi_{CMB}^2 = (R - R_{obs})^2 / \sigma_R^2$ and $\chi_{BAO}^2 = (A - A_{obs})^2 / \sigma_A^2$. The best-fit model parameters are determined by minimizing the total χ^2 . As in [28,29], the 68% confidence level is

determined by $\Delta\chi^2 \equiv \chi^2 - \chi_{min}^2 \leq 1.0, 2.3$ and 3.53 for $n_p = 1, 2$ and 3 , respectively, where n_p is the number of free model parameters. Similarly, the 95% confidence level is determined by $\Delta\chi^2 \equiv \chi^2 - \chi_{min}^2 \leq 4.0, 6.17$ and 8.02 for $n_p = 1, 2$ and 3 , respectively.

3. Type I models

3.1. Equations

As mentioned in Section 1, the type I models are characterized by Eq. (9), whereas $f(a)$ can be any function of a . From Eq. (9), it is easy to obtain

$$\Omega_X = \frac{f}{1+f}, \quad \Omega_m = \frac{1}{1+f}. \quad (18)$$

Substituting $\rho_X = \rho_m f(a)$ into Eq. (2) and using $\dot{\rho}_m$ from Eq. (1), we can find that the corresponding interaction term is given by

$$Q = -H\rho_m\Omega_X \left(a \frac{f'}{f} + 3w_X \right) = -H\rho_X\Omega_m \left(a \frac{f'}{f} + 3w_X \right), \quad (19)$$

where a prime denotes the derivative with respect to a . Obviously, if $f(a) \propto a^\xi$, Eq. (19) reduces to Eq. (6) which has been obtained in [1]. On the other hand, one can recast Eq. (3) as

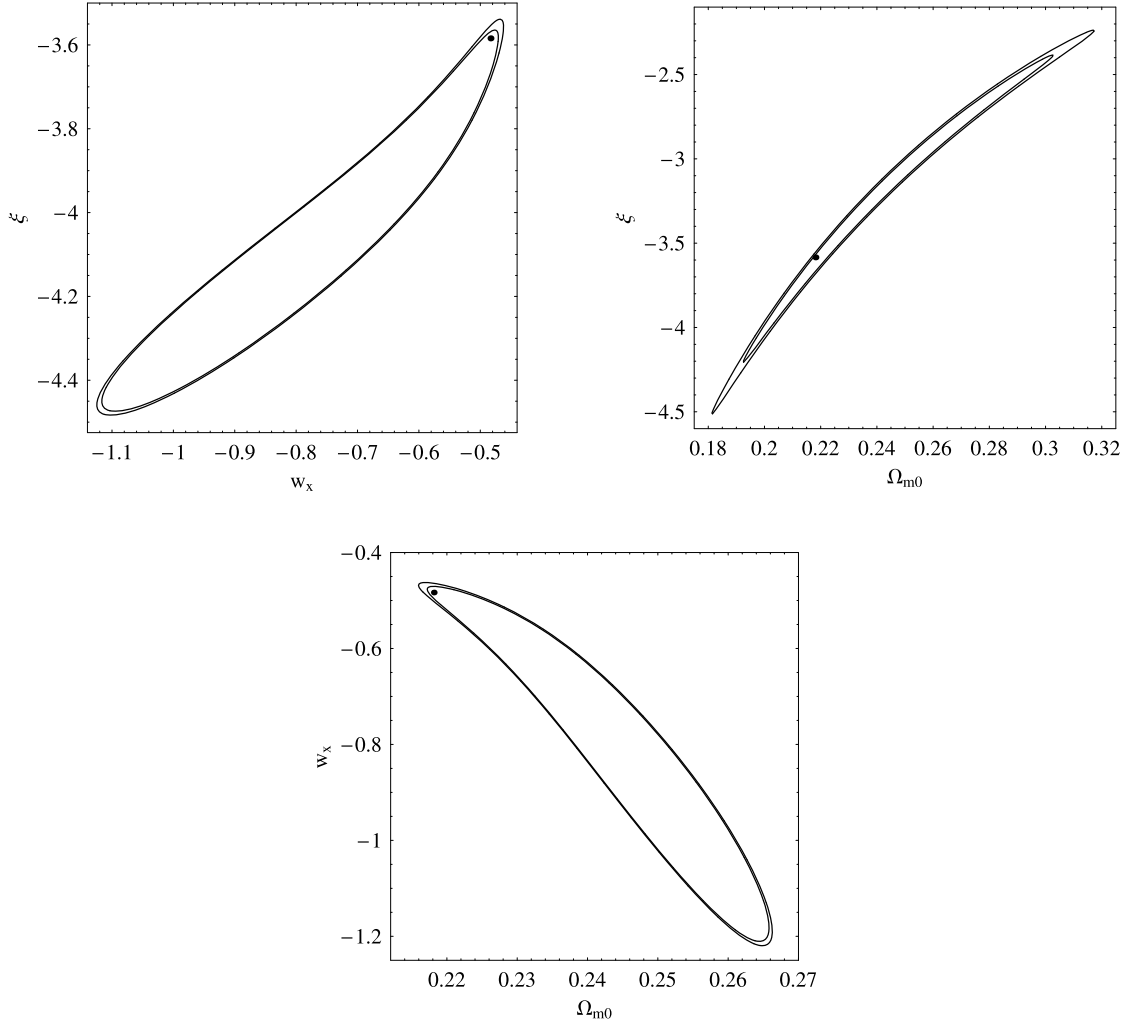


Fig. 2. The same as in Fig. 1, except for the type I model characterized by $f(a) = f_0 + \xi(1 - a)$ with the condition $\xi \geq 1 - \Omega_{m0}^{-1}$. See the text for details.

$$\frac{d \ln \rho_{tot}}{d \ln a} = -3(1 + w_{eff}) = -3(1 + \Omega_X w_X). \quad (20)$$

Using Eq. (18), we can integrate Eq. (20) to obtain

$$\rho_{tot} = a^{-3} \exp\left(-\int \frac{3w_X f}{1+f} d \ln a\right) \cdot const., \quad (21)$$

where *const.* is an integral constant, which can be determined by requiring the condition $\rho_{tot}(a=1) = \rho_{tot,0} = 3H_0^2/(8\pi G)$. Once ρ_{tot} is on hand, we can readily find the corresponding $E \equiv H/H_0$ from Friedmann equation, and then fit it to the observational data. Correspondingly, $\rho_X = \Omega_X \rho_{tot}$ and $\rho_m = \Omega_m \rho_{tot}$ are also available from Eqs. (18) and (21). Finally, it is worth noting that by definition (9), we have

$$f_0 = f(a=1) = \frac{\rho_{X0}}{\rho_{m0}} = \frac{1}{\Omega_{m0}} - 1, \quad (22)$$

which is useful to fix one of parameters in $f(a)$.

3.2. Cosmological constraints on type I models

In this subsection, we consider the cosmological constraints on type I models by using the observational data given in Section 2. At first, we consider the power-law case with

$$f(a) = f_0 a^\xi, \quad (23)$$

where ξ is a constant; f_0 can be determined by definition (9) to be the one given in Eq. (22), and hence it is not an independent parameter. In this case, there are three free model parameters, namely, Ω_{m0} , w_X and ξ . Although the cosmological constraints on the model characterized by Eq. (23) has been considered by Guo et al. [1], as mentioned in Section 1, they have used the earlier observational data. Therefore, it is still worthwhile to consider the cosmological constraints once more in the present work by using the latest observational data mentioned in Section 2. Substituting Eq. (23) into Eq. (21) and requiring $\rho_{tot}(a=1) = \rho_{tot,0}$, we can determine the integral constant, and finally obtain

$$\rho_{tot} = \rho_{tot,0} a^{-3} [\Omega_{m0} + (1 - \Omega_{m0})a^\xi]^{-3w_X/\xi}. \quad (24)$$

Substituting into Friedmann equation, we find that

$$E^2 = \frac{H^2}{H_0^2} = a^{-3} [\Omega_{m0} + (1 - \Omega_{m0})a^\xi]^{-3w_X/\xi} \\ = (1+z)^3 [\Omega_{m0} + (1 - \Omega_{m0})(1+z)^{-\xi}]^{-3w_X/\xi}. \quad (25)$$

By minimizing the corresponding total χ^2 in Eq. (17), we find the best-fit parameters $\Omega_{m0} = 0.281$, $w_X = -0.982$ and $\xi = 2.988$, whereas $\chi_{min}^2 = 465.604$. In Fig. 1, we also present the corresponding 68% and 95% confidence level contours in w_X - ξ plane, Ω_{m0} - ξ plane and Ω_{m0} - w_X plane. It is easy to see that these constraints

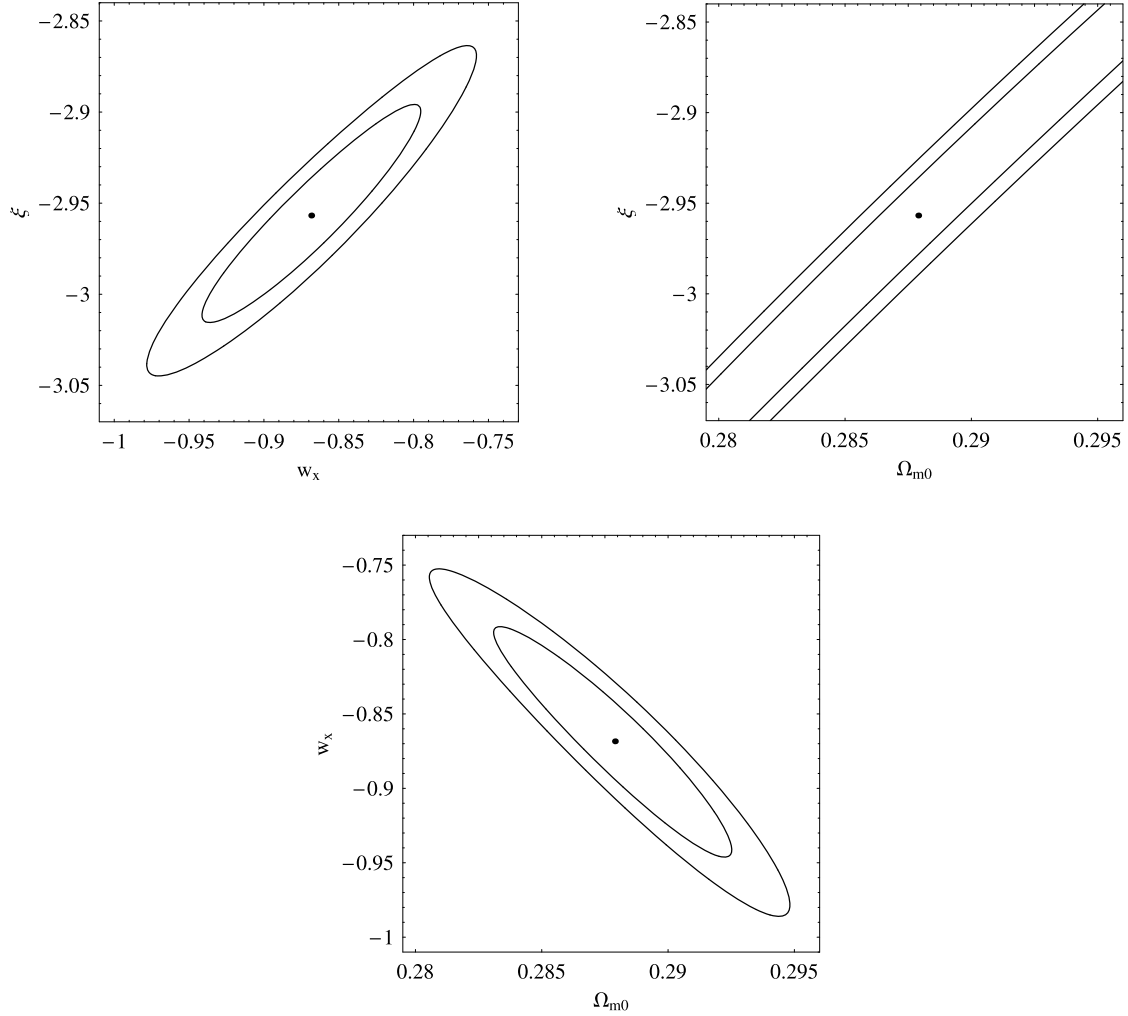


Fig. 3. The same as in Fig. 1, except for the type I model characterized by $f(a) = f_0 + \xi(1 - a)$ without the condition $\xi \geq 1 - \Omega_{m0}^{-1}$. See the text for details.

on the model characterized by $f(a) = f_0 a^\xi$ are much tighter than the ones obtained by Guo et al. [1], thanks to the latest observational data.

Next, we consider a new case with

$$f(a) = f_0 + \xi(1 - a), \tag{26}$$

which can be regarded as a linear expansion of $f(a)$ with respect to a , similar to the familiar Chevallier-Polarski-Linder (CPL) parameterization for EoS $w(a) = w_0 + w_a(1 - a)$ [30]. Again, f_0 can be determined by definition (9) to be the one given in Eq. (22), and hence it is not an independent parameter. Thus, there are three free model parameters, namely, Ω_{m0} , w_X and ξ . Substituting Eq. (26) into Eq. (21) and requiring $\rho_{tot}(a = 1) = \rho_{tot,0}$, we can determine the integral constant, and finally obtain

$$\rho_{tot} = \rho_{tot,0} a^{-3(1+w_X)} \times [(1 + \xi \Omega_{m0}) a^{-1} - \xi \Omega_{m0}]^{-3w_X \Omega_{m0} / (1 + \xi \Omega_{m0})}. \tag{27}$$

Substituting into Friedmann equation, we find that

$$E^2 = \frac{H^2}{H_0^2} = a^{-3(1+w_X)} [(1 + \xi \Omega_{m0}) a^{-1} - \xi \Omega_{m0}]^{-3w_X \Omega_{m0} / (1 + \xi \Omega_{m0})}$$

$$= (1 + z)^{3(1+w_X)} \times [(1 + \xi \Omega_{m0})(1 + z) - \xi \Omega_{m0}]^{-3w_X \Omega_{m0} / (1 + \xi \Omega_{m0})}. \tag{28}$$

Noting Eq. (9) and imposing the condition $\rho_X \geq 0$, we have $\xi \geq 1 - \Omega_{m0}^{-1}$ from Eqs. (26) and (22). Under this condition, by minimizing the corresponding total χ^2 in Eq. (17), we find the best-fit parameters $\Omega_{m0} = 0.218$, $w_X = -0.483$ and $\xi = -3.584$, whereas $\chi_{min}^2 = 563.77$. In Fig. 2, we also present the corresponding 68% and 95% confidence level contours in w_X - ξ plane, Ω_{m0} - ξ plane and Ω_{m0} - w_X plane. It is easy to see that the 68% and 95% confidence level contours are very close. On the other hand, $\chi_{min}^2 = 563.77$ is fairly larger than the degree of freedom $dof \sim 400$.

So, we give up the condition $\xi \geq 1 - \Omega_{m0}^{-1}$ in the case of $f(a) = f_0 + \xi(1 - a)$. This means that ρ_X might be negative in the early universe. In fact, Guo et al. [1] also explicitly include this possibility. Since such a negative energy appears in phantom models [31] and modified gravity models [32], it is reasonable to consider this possibility. Without the condition $\xi \geq 1 - \Omega_{m0}^{-1}$, by minimizing the corresponding total χ^2 in Eq. (17), we find the best-fit parameters $\Omega_{m0} = 0.288$, $w_X = -0.868$ and $\xi = -2.957$, whereas $\chi_{min}^2 = 467.718$. In Fig. 3, we also present the corresponding 68% and 95% confidence level contours in w_X - ξ plane, Ω_{m0} - ξ plane and Ω_{m0} - w_X plane. Obviously, these results are significantly better than the ones with the condition $\xi \geq 1 - \Omega_{m0}^{-1}$, whereas the corresponding $\chi_{min}^2 = 467.718$ is also better.

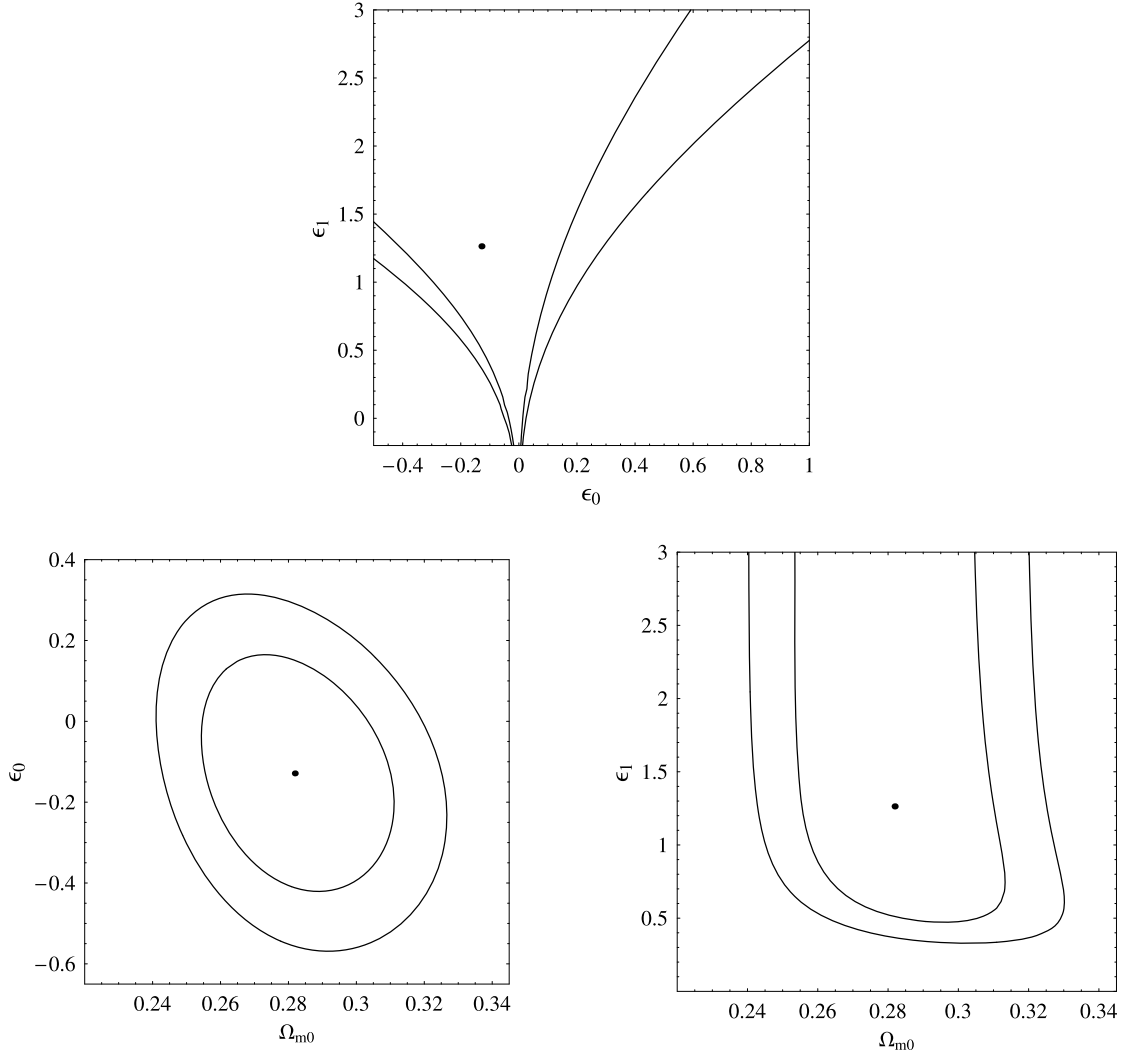


Fig. 4. The 68% and 95% confidence level contours in ϵ_0 - ϵ_1 plane, Ω_{m0} - ϵ_0 plane and Ω_{m0} - ϵ_1 plane for the type II model characterized by $\epsilon(a) = \epsilon_0 a^{\epsilon_1}$. The best-fit parameters are also indicated by the black solid points.

Finally, one might consider the logarithmic case with

$$f(a) = f_0 + \xi \ln a, \quad (29)$$

which can be regarded as a linear expansion of $f(a)$ with respect to the so-called e -folding time $\mathcal{N} = \ln a$ in the literature. This case seems attractive since in Eq. (21) the integration is with respect to $\ln a$. However, in this case, $f(a)$ (and hence ρ_X) diverges when $a \rightarrow 0$ in the early universe. So, we do not consider the logarithmic case in type I model.

4. Type II models

4.1. Equations

In this section, we turn to type II models, which are characterized by Eq. (7) whereas $\epsilon(a)$ can be any function of a . Substituting Eq. (7) into Eq. (1), we can easily find that the corresponding interaction term is given by

$$Q = H \rho_m [\epsilon(a) + a \epsilon'(a) \ln a]. \quad (30)$$

It is worth noting that in type II models, there is no condition like Eq. (22) in type I models to reduce the number of free model parameters. If there are at least two parameters in $\epsilon(a)$, adding Ω_{m0}

and w_X , we should have four free model parameters or even more. In this case, the constraints will be very loose, and the calculations will be very involved. Instead, we follow Costa and Alcaniz [2] to consider the case with a fixed $w_X = -1$, namely, the role of dark energy is played by a decaying Λ [2,16]. Therefore, Eq. (2) becomes

$$\dot{\rho}_\Lambda = -Q = -H \rho_m [\epsilon(a) + a \epsilon'(a) \ln a]. \quad (31)$$

Substituting Eq. (7) into Eq. (31), we have

$$\frac{d\rho_\Lambda}{da} = -\rho_{m0} a^{-4+\epsilon(a)} [\epsilon(a) + a \epsilon'(a) \ln a]. \quad (32)$$

We can integrate Eq. (32) to obtain

$$\rho_\Lambda = \rho_{m0} \int_a^1 \tilde{a}^{-4+\epsilon(\tilde{a})} [\epsilon(\tilde{a}) + \tilde{a} \epsilon'(\tilde{a}) \ln \tilde{a}] d\tilde{a} + \rho_{\Lambda 0}. \quad (33)$$

Substituting Eqs. (33) and (7) into Friedmann equation, we find that

$$E^2 = \frac{H^2}{H_0^2} = \Omega_{m0} \theta(a) + (1 - \Omega_{m0}), \quad (34)$$

where

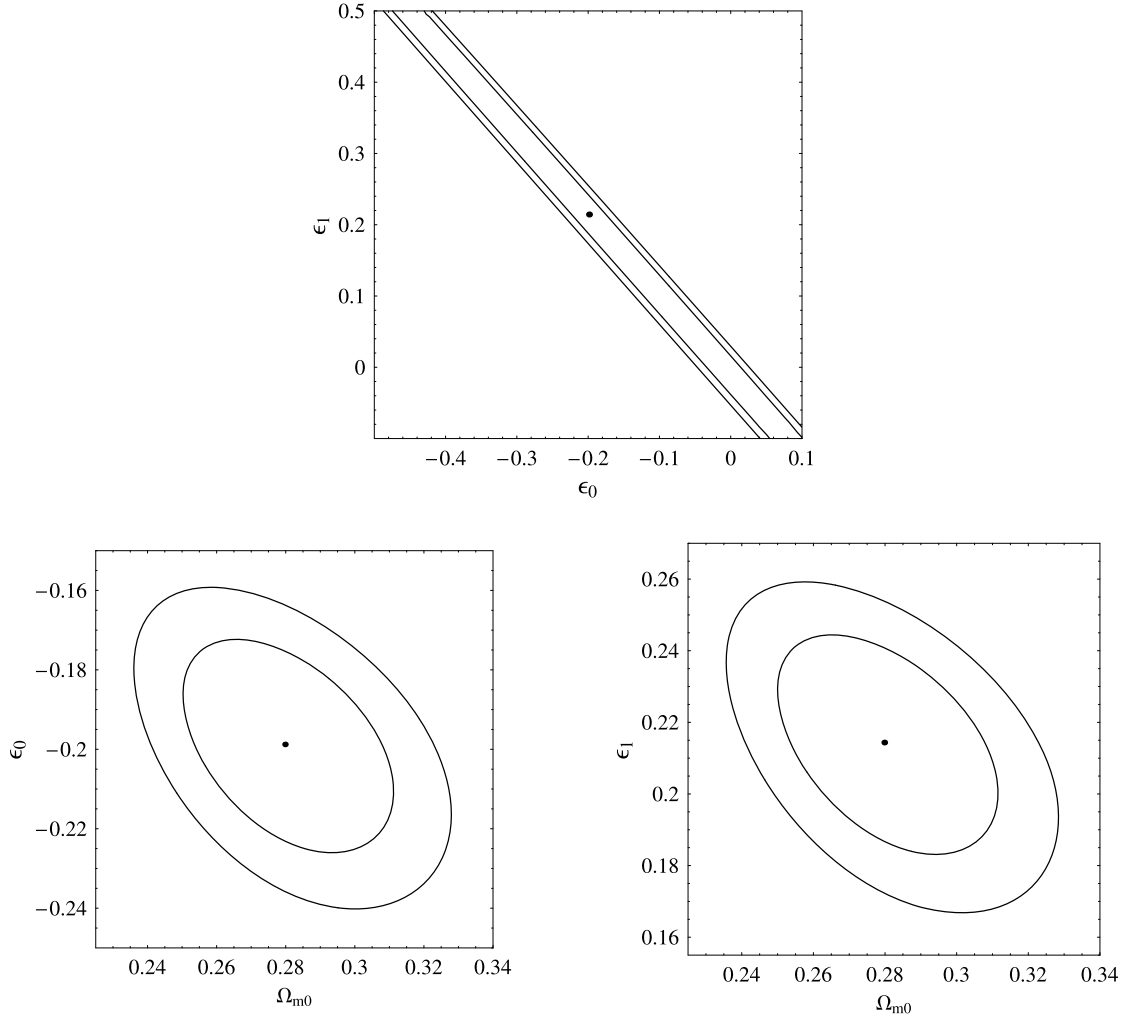


Fig. 5. The same as in Fig. 4, except for the type II model characterized by $\epsilon(a) = \epsilon_0 + \epsilon_1(1 - a)$.

$$\theta(a) \equiv a^{-3+\epsilon(a)} + \int_a^1 \tilde{a}^{-4+\epsilon(\tilde{a})} [\epsilon(\tilde{a}) + \tilde{a}\epsilon'(\tilde{a}) \ln \tilde{a}] d\tilde{a}. \quad (35)$$

4.2. Cosmological constraints on type II models

In this subsection, we consider the cosmological constraints on type II models by using the observational data given in Section 2. At first, we consider the power-law case with

$$\epsilon(a) = \epsilon_0 a^{\epsilon_1}, \quad (36)$$

where ϵ_0 and ϵ_1 are constants. In this case, there are three free model parameters, namely, Ω_{m0} , ϵ_0 and ϵ_1 . Although the cosmological constraints on the model characterized by Eq. (36) has been considered by Costa and Alcaniz [2], as mentioned in Section 1, they have used the earlier observational data. Therefore, it is still worthwhile to consider the cosmological constraints once more in the present work by using the latest observational data mentioned in Section 2. Substituting Eq. (36) into Eqs. (34) and (35), we can then fit this model to the observational data. By minimizing the corresponding total χ^2 in Eq. (17), we find the best-fit parameters $\Omega_{m0} = 0.282$, $\epsilon_0 = -0.129$ and $\epsilon_1 = 1.263$, whereas $\chi_{min}^2 = 465.635$. In Fig. 4, we also present the corresponding 68% and 95% confidence level contours in ϵ_0 - ϵ_1 plane, Ω_{m0} - ϵ_0 plane and Ω_{m0} - ϵ_1 plane. It is easy to see from the ϵ_0 - ϵ_1 plane that if ϵ_0

is close to zero, ϵ_1 cannot be too negative. On the other hand, the constraint on the parameter ϵ_1 is still very loose.

Next, we turn to the CPL-like case with

$$\epsilon(a) = \epsilon_0 + \epsilon_1(1 - a), \quad (37)$$

which can be regarded as a linear expansion of $\epsilon(a)$ with respect to a , similar to the well-known CPL parameterization for EoS $w(a) = w_0 + w_a(1 - a)$ [30]. Substituting Eq. (37) into Eqs. (34) and (35), we can then fit this model to the observational data. By minimizing the corresponding total χ^2 in Eq. (17), we find the best-fit parameters $\Omega_{m0} = 0.280$, $\epsilon_0 = -0.199$ and $\epsilon_1 = 0.214$, whereas $\chi_{min}^2 = 465.604$. In Fig. 5, we also present the corresponding 68% and 95% confidence level contours in ϵ_0 - ϵ_1 plane, Ω_{m0} - ϵ_0 plane and Ω_{m0} - ϵ_1 plane.

Finally, we consider the logarithmic case with

$$\epsilon(a) = \epsilon_0 + \epsilon_1 \ln a, \quad (38)$$

which can be regarded as a linear expansion of $\epsilon(a)$ with respect to the so-called e -folding time $\mathcal{N} = \ln a$ in the literature. Although $\epsilon(a)$ diverges when $a \rightarrow 0$ in the early universe, unlike in the same case of type I model, it does not cause any problem in type II model, since $a^{-3+\epsilon(a)} \rightarrow 0^\infty \rightarrow 0$ which is regular when $a \rightarrow 0$. Substituting Eq. (38) into Eqs. (34) and (35), we can then fit this model to the observational data. By minimizing the corresponding total χ^2 in Eq. (17), we find the best-fit parameters $\Omega_{m0} = 0.278$,

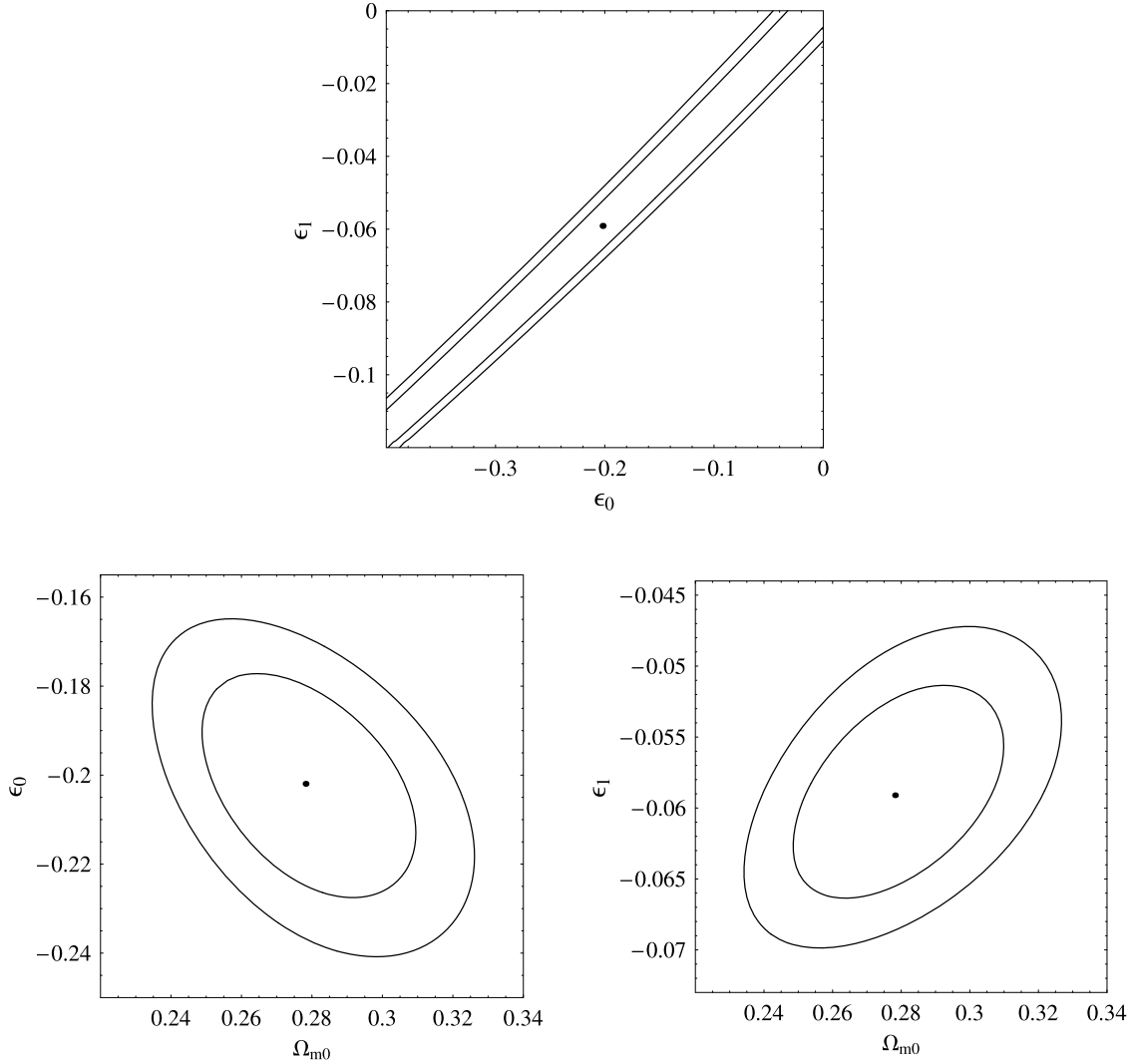


Fig. 6. The same as in Fig. 4, except for the type II model characterized by $\epsilon(a) = \epsilon_0 + \epsilon_1 \ln a$.

$\epsilon_0 = -0.202$ and $\epsilon_1 = -0.059$, whereas $\chi_{min}^2 = 465.516$. In Fig. 6, we also present the corresponding 68% and 95% confidence level contours in ϵ_0 - ϵ_1 plane, Ω_{m0} - ϵ_0 plane and Ω_{m0} - ϵ_1 plane.

5. Summary and discussions

In this work, we considered the cosmological constraints on the interacting dark energy models. We generalized the models considered previously by Guo et al. [1], Costa and Alcaniz [2], and we have discussed two general types of models: type I models are characterized by $\rho_X/\rho_m = f(a)$ and $f(a)$ can be any function of scale factor a , whereas type II models are characterized by $\rho_m = \rho_{m0} a^{-3+\epsilon(a)}$ and $\epsilon(a)$ can be any function of a . We obtained the cosmological constraints on the type I and II models with power-law, CPL-like, logarithmic $f(a)$ and $\epsilon(a)$ by using the latest observational data.

Some remarks are in order. Firstly, here we briefly justify the interaction forms considered in the present work. We take type I models as examples. For the power-law case with $f(a) = f_0 a^\xi$ in Eq. (23), noting that in the case without interaction $\rho_X \propto a^{-3(1+w_X)}$ and $\rho_m \propto a^{-3}$, from definition Eq. (9), it is reasonable to parameterize $f(a) = \rho_X/\rho_m \propto a^\xi$, where ξ measures the severity of the coincidence problem [1,18]. For the CPL case with

$f(a) = f_0 + \xi(1-a)$ in Eq. (26) and the logarithmic case with $f(a) = f_0 + \xi \ln a$ in Eq. (29), noting that the Taylor series expansion of any function $F(x)$ is given by $F(x) = F(x_0) + F_1(x-x_0) + (F_2/2!)(x-x_0)^2 + (F_3/3!)(x-x_0)^3 + \dots$, the CPL and logarithmic cases can be regarded as the Taylor series expansion of f with respect to the scale factor a and the e -folding time $\mathcal{N} = \ln a$ up to first order (linear expansion), similar to the well-known EoS parameterizations $w(a) = w_0 + w_a(1-a)$ and $w(z) = w_0 + w_1 z$.

Secondly, we would like to briefly consider the comparison of these models. For convenience, we also consider the well-known Λ CDM model in addition. Fitting Λ CDM model to the observational data considered in the present work, it is easy to find the corresponding best-fit parameter $\Omega_{m0} = 0.278$, whereas $\chi_{min}^2 = 466.317$. A conventional criterion for model comparison in the literature is χ_{min}^2/dof , in which the degree of freedom $dof = N - k$, whereas N and k are the number of data points and the number of free model parameters, respectively. We present the χ_{min}^2/dof for all the 7 models in Table 1. On the other hand, there are other criterions for model comparison in the literature, such as Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC). The BIC is defined by [33,34]

$$\text{BIC} = -2 \ln \mathcal{L}_{max} + k \ln N, \quad (39)$$

Table 1

Summarizing all the 7 models considered in this work. Here, we label the type I models characterized by $f(a) = f_0 a^\xi$, $f(a) = f_0 + \xi(1-a)$ with the condition $\xi \geq 1 - \Omega_{m0}^{-1}$, and $f(a) = f_0 + \xi(1-a)$ without the condition $\xi \geq 1 - \Omega_{m0}^{-1}$ as IPL, ICPLw and ICPLwo, respectively. Also, we label the type II models characterized by $\epsilon(a) = \epsilon_0 a^{\epsilon_1}$, $\epsilon(a) = \epsilon_0 + \epsilon_1(1-a)$, and $\epsilon(a) = \epsilon_0 + \epsilon_1 \ln a$ as IIPL, IICPL and IILog, respectively.

Model	Λ CDM	IPL	ICPLw	ICPLwo	IIPL	IICPL	IILog
Best fits	$\Omega_{m0} = 0.278$	$\Omega_{m0} = 0.281$ $w_\chi = -0.982$ $\xi = 2.988$	$\Omega_{m0} = 0.218$ $w_\chi = -0.483$ $\xi = -3.584$	$\Omega_{m0} = 0.288$ $w_\chi = -0.868$ $\xi = -2.957$	$\Omega_{m0} = 0.282$ $\epsilon_0 = -0.129$ $\epsilon_1 = 1.263$	$\Omega_{m0} = 0.280$ $\epsilon_0 = -0.199$ $\epsilon_1 = 0.214$	$\Omega_{m0} = 0.278$ $\epsilon_0 = -0.202$ $\epsilon_1 = -0.059$
χ_{min}^2	466.317	465.604	563.77	467.718	465.635	465.604	465.516
k	1	3	3	3	3	3	3
χ_{min}^2/dof	1.172	1.176	1.424	1.181	1.176	1.176	1.176
Δ BIC	0	11.265	109.431	13.379	11.296	11.265	11.177
Δ AIC	0	3.287	101.453	5.401	3.318	3.287	3.199
Rank	1	3–4	7	6	5	3–4	2

where \mathcal{L}_{max} is the maximum likelihood. In the Gaussian cases, $\chi_{min}^2 = -2 \ln \mathcal{L}_{max}$. So, the difference in BIC between two models is given by Δ BIC = $\Delta \chi_{min}^2 + \Delta k \ln N$. The AIC is defined by [34,35]

$$\text{AIC} = -2 \ln \mathcal{L}_{max} + 2k. \quad (40)$$

The difference in AIC between two models is given by Δ AIC = $\Delta \chi_{min}^2 + 2\Delta k$. In Table 1, we also present the Δ BIC and Δ AIC of all the 7 models considered in this work. Notice that Λ CDM has been chosen to be the fiducial model when we calculate Δ BIC and Δ AIC. From Table 1, it is easy to see that the rank of models is coincident in all the 3 criterions (χ_{min}^2/dof , BIC and AIC). The Λ CDM model is the best one, whereas ICPLw model is the worst one. This result is consistent with the one obtained in e.g. [34]. However, it is well known that Λ CDM model is plagued with the cosmological constant problem and the coincidence problem (see e.g. [3]). On the other hand, as mentioned in the beginning of Section 1, there are some observational evidences for the interaction between dark energy and dark matter, and the coincidence problem can be alleviated in the interacting dark energy models. Therefore, it is still worthwhile to study the interacting dark energy models.

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