Isomeric $0^-$ halo-states in $^{12}\text{Be}$ and $^{11}\text{Li}$

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Received 24 August 2007; received in revised form 28 November 2007; accepted 2 December 2007
Available online 3 January 2008

Editor: J.-P. Blaizot

Abstract

We predict the existence of an isomeric $0^-$-state in $^{12}\text{Be}$ at an excitation energy of about 2.5 MeV, and a $0^-$-resonance in $^{11}\text{Li}$ with both energy and width of about 1 MeV corresponding to two-neutron emission. The structure of these halo-like states are like the $1^-$-states which means essentially a core surrounded by two neutrons in single-particle $s$- and $p$-states. The life-time of the $^{12}\text{Be}$ state is determined by $M_1$- or $M_2$-emission, $\tau(M_1) \approx 10^{-11}$ s or $\tau(M_2) \approx 10^{-8}$ s estimated for photon energies of 0.1 MeV and 0.6 MeV, respectively.

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PACS: 21.45.+v; 31.15.Ja; 25.70.Ef

1. Motivation

Although light nuclei exhibit amazingly individual characters [1], identical structures can show up in these otherwise very different quantum systems. A prominent example are the isobaric analog states which in principle can be traced through a sequence of neighboring nuclei [2]. Another type of similarity appears for cluster states when the corresponding threshold is approached [3]. This is seen for halo nuclei of two and three-body character with the Borromean two-neutron nuclei $^6\text{He}$ and $^{11}\text{Li}$ as well-known examples [4].

The individual characters of light nuclei are reflected in relatively large changes of structure by addition of one or a few nucleons. Substantial efforts have been devoted to investigate the changing shell structure as the driplines are approached [5,6]. In particular the $N = 8$ neutron shell is very stable at the $\beta$-stability line for $^{16}\text{O}$ while it has disappeared for $^{12}\text{Be}$ and $^{11}\text{Li}$.

This was recognized very early by Talmi and Unna [7] who traced the $1p_{1/2}$ and $2s_{1/2}$ neutron levels from $^{13}\text{C}$ to $^{11}\text{Be}$. These levels, belonging to different harmonic oscillator shells, approach each other and eventually cross when the neutron excess increases. Close to the neutron dripline they both appear just above the Fermi energy. This phenomenon, known as parity inversion, explains the fact that a nucleus like $^{13}\text{C}$ has a $1/2^-$ ground state, while the ground state of the one-neutron halo nucleus $^{11}\text{Be}$ is $1/2^+$ [8]. For 9 neutrons different ground state spins, $5/2^+$ and $1/2^+$, are also found for $^{17}\text{O}$ and $^{15}\text{C}$. For the particle unstable nucleus $^{10}\text{Li}$ different theoretical works predicted the existence of a similar low-lying $s$-wave intruder state [9–11], in such a way that the ground state in $^{10}\text{Li}$ should correspond to a state with negative parity (the ground state of $^{9}\text{Li}$ has spin and parity $3/2^-$). The available experimental data concerning the ground state properties of $^{10}\text{Li}$ are however controversial, although most of them point towards the existence of such a low-lying virtual $s$-state [12–15]. This result has also been recently supported in [16].

Having in mind the structure of the nuclei in the $N = 7$ isotonic chain, one could wonder about the properties of the spectrum for the nuclei in the $N = 8$ chain. Independently of the existence of an intruder $s$-state, it is clear that some of the excited states of these nuclei should arise from the excitation of the neutrons inside the $sp$-shell. In particular, when one of the neutrons is in the $s_{1/2}$-shell and the other one in the $p_{1/2}$-shell, this structure can obviously lead to either a $1^-$ or a $0^-$ excited state. This doublet can actually be found for instance in $^{14}\text{C}$...
with excitation energies of 6.09 MeV and 6.90 MeV, respectively [17], and in $^{16}$O with excitation energies of 9.59 MeV and 10.97 MeV, respectively [18]. A linear extrapolation in mass from $^{16}$O over $^{14}$C to $^{12}$Be and $^{11}$Li leads to predictions of (2.8 MeV, 2.6 MeV) and (0.80 MeV, 0.84 MeV) for $^{12}$Be and $^{11}$Li, respectively, for $0^+$ and $2^+$. For $^{16}$N and $^{16}$F, for which some of the states should correspond to one neutron and one proton in the $s$-p levels, the spectra also present the same $0^+$-, $1^+$-states at (0.12 MeV, 0.40 MeV) and (ground state, 0.19 MeV). The $0^+$ is close but below the $1^+$-state, as repeated in $^{14}$N but with larger spacing.

For all these reasons, it is surprising that for $^{12}$Be and $^{11}$Li, also belonging to the $N = 8$ chain, information is available only about the $1^+$ excited states. For $^{12}$Be a bound $1^+$-state has been found with excitation energy of 2.68 MeV [5], while for $^{11}$Li experimental evidences about the existence of an unbound $1^+$ excited state has also been given [19]. No predictions have been formulated about the occurrence of $0^+$-states. Only in [20] an unbound $0^+$-state was predicted for $^{12}$Be. However in that work the energies of the negative parity states are systematically overestimated, i.e. the $1^-$-state is also clearly unbound, contrary to the experimental knowledge. Also in shell model calculations the simultaneous inclusion of different parities is a general source of uncertainty due to the requirement of a larger Hilbert space. This problem is still present in recent no-core shell model results obtained for $^{11}$Be [21].

In general high-lying $0^+$- and $1^+$-states of short half-life are abundant throughout the chart of nuclei. On the other hand, such low-lying states are rare especially if they are of simple single-particle structure and with a long lifetime classifying them as isoscalar states. The purpose of this Letter is to present theoretical evidence for the existence of so far unknown (isoscalar) $0^+$-states in both $^{12}$Be and $^{11}$Li. We shall estimate some of the properties of these states and especially predict energies and transition strengths. It turns out that the most probable energy for $^{12}$Be($0^+$) is below the particle emission threshold with magnetic dipole or quadrupole transitions as the only possible decay channels. Its lifetime should therefore be comparable to the recently discovered isoscalar $0^+$-state [22].

2. Theoretical formulation

We use a three-body model to describe both nuclei, with a $^{10}$Be or $^9$Li-core surrounded by two neutrons. In [23] the role played by core excitations in $^{12}$Be is investigated. The lowest excited state in $^{10}$Be is a $2^+$-state, which cannot couple the two neutrons in the $s_{1/2}$- and $p_{1/2}$-states to total angular momentum zero. Therefore, the $^{10}$Be excitations should not contribute significantly in this case and we can then assume an inert core. The three-body wave functions are obtained by solving the Faddeev equations with the hyperspherical adiabatic expansion method [24]. Unbound resonant states are computed by using the complex scaling method [25].

For the $^{10}$Be-neutron interaction we have constructed a simple $\ell$-dependent Gaussian potential containing central and spin–orbit terms. The range of the Gaussians has been chosen equal to 3.5 fm. For $s$-waves a strength of $-8.40$ MeV places then the $1/2^+$-state in $^{11}$Be at $-0.504$ MeV, that matches the experimental value [8]. The use of this shallow $s$-wave potential is an efficient way of taking into account the Pauli principle, since the lowest $s_{1/2}$-shell is fully occupied in the $^{10}$Be-core. This procedure is phase-equivalent to use a deeper potential, binding the neutrons in the Pauli forbidden $s$-state, and afterwards removing it from the active space available for the three-body system [26]. For $p$-waves the strengths of the central and spin–orbit Gaussians are 40.0 MeV and 63.52 MeV, which produces a bound $1/2^+$-state in $^{11}$Be at $-0.184$ MeV, in agreement with the experiment [8]. The $p_{3/2}$-wave is at the same time pushed up to high energies, where it remains unoccupied as it should since this state already is occupied by core–neutrons and therefore Pauli forbidden. This is achieved by using an inverted $p$-wave spin–orbit force.

For the $^{12}$Be calculation we have also included a $d$-wave potential, giving rise to $5/2^+$ and $3/2^+$-resonances at 1.28 MeV and 2.90 MeV, respectively, above threshold, which again match the experimental values [8]. This is obtained by using Gaussian $d_{5/2}$ and $d_{3/2}$-potentials with strengths equal to $-43.8$ MeV and $-199$ MeV, respectively. For the $d_{3/2}$ Gaussian potential a range of 1.7 fm (instead of 3.5 fm) has been used in order to produce a narrower resonance in better agreement with the experiment. The neutron–neutron interaction is from [27].

For the $^{11}$Li calculations we use the simple $^9$Li-neutron interaction quoted as potential IV in Table I of [27]. A more sophisticated potential, like the one used in [26], where the Pauli principle is accounted for by use of phase equivalent potentials could be used, but as shown in [28] the results are indistinguishable. Furthermore potential IV in Table I of [27] can be used assuming zero spin for the $^9$Li-core, which permits us to perform a calculation fully analogous to the one for $^{12}$Be, and at the same time we can investigate the $0^+$-state directly without the entanglement of the coupling to the core-spin of $3/2$. This is as realistic as the full complication of including the $^9$Li core-spin of $3/2$ [27], where the coupling of the $3/2^-$ core-state to a two-neutron $0^-$-state leads to a $3/2^+$-state. For $d$-waves we use the same potential as for $p$-waves. The neutron–neutron interaction is again from [27].

3. Results

When the interactions above are used, the $0^+$ ground states of $^{12}$Be and $^{11}$Li are obtained with two-neutron separation energies equal to $-3.67$ MeV and $-0.30$ MeV, respectively, both of them matching the experimental values [8,29]. For $^{11}$Li the agreement with the experiment has been obtained after using a Gaussian three-body force with a range of 2.5 fm and a strength of $-4.0$ MeV. All components with $\ell = \ell_y = 0, 1, 2$ are included. The weights of the $s$-, $p$-, and $d$-waves in the core–neutron channel turn out to be 74%, 15%, and 11% for $^{12}$Be, and 64%, 35%, and 1% for $^{11}$Li, respectively.

For $^{12}$Be particle-bound $0^+$ and $2^+$ excited states are found with two-neutron separation energies of $-0.59$ MeV and $-0.62$ MeV. These two states are experimentally known to be present in the $^{12}$Be spectrum with binding energies...
Table 1
Components included in the calculations for the $1^-$-states. The left and right parts correspond to the first Jacobi set ($x$ between the two neutrons) and the second and third Jacobi sets ($x$ from core to neutron), respectively. The last row gives the maximum value of the hypermomentum used for each component. The components written in bold letters are used in the calculation of the $0^-$-states.

<table>
<thead>
<tr>
<th>$\ell_x$</th>
<th>$1$</th>
<th>$0$</th>
<th>$2$</th>
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<th>$1$</th>
<th>$1$</th>
<th>$1$</th>
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<th>$1$</th>
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</thead>
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<td>$41$</td>
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<td>$41$</td>
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</tr>
</tbody>
</table>

$-1.43 \pm 0.02$ MeV [22] and $-1.56 \pm 0.01$ MeV [30], respectively. Therefore an effective three-body force is here needed to recover the experimental values.

In Table 1 we give the components included in the calculations for the $1^-$-states. For $^{12}$Be a bound $1^-$-state is found with a binding energy of $-0.97$ MeV, which agrees with the experimental value of $-0.99 \pm 0.03$ MeV [5]. For $^{11}$Li a $1^-$-resonance is found with energy and width $(E_R, \Gamma_R) = (0.64, 0.32)$ MeV, which also agrees with the experimental excitation energy of about $1$ MeV given in [31,32] (the ground state in $^{11}$Li is bound with a two neutron separation energy of $0.3$ MeV). These values have been obtained without inclusion of effective three-body forces.

The components written in bold letters in Table 1 are used in the computation of the $0^-$-states. For $^{12}$Be a bound $0^-$-state is found with a two-neutron separation energy of $-0.96$ MeV, which corresponds to an excitation energy roughly $10$ keV higher than for the $1^-$-state. For $^{11}$Li a $0^-$-resonance has also been found, with energy and width $(E_R, \Gamma_R) = (0.92, 0.82)$ MeV, which corresponds to an excitation energy roughly $300$ keV higher than for the $1^-$-resonance. In Table 2 we summarize the results corresponding to the $0^-$- and $1^-$-states for both nuclei. The labels $%s$- and $%p$-wave refer to the weights in the three-body wave function of the components corresponding to the core and one of the neutrons in a relative $s$- or $p$-wave. Although $d$-waves are included in the calculation, their contributions to the $0^-$- and $1^-$-states are negligible. To test the robustness of the results for the $0^-$-states we have repeated the calculations for $^{12}$Be using the Argonne $v_8$ neutron–neutron potential. The $0^-$- and $1^-$-states appear then more bound by almost $200$ keV and $150$ keV, respectively.

The low-lying spectra are sketched in Fig. 1 for both $^{12}$Be and $^{11}$Li. The results without adjustment by the three-body interaction are compared to the established experimental energies. For $^{12}$Be the excited $0^+$- and $2^+$-states both appear above the experimental results while the $1^-$-state agrees with the measured value. The $0^-$-state is built of the same levels as the $1^-$-state and it therefore is a strong indication that also the predicted $0^-$-state is particle-bound. In any case it would be very unusual to miss these three-body states by $1$ MeV as required to make it unbound. Even in that unlikely event there should be a resonance structure as predicted in $^{11}$Li. However, for $^{12}$Be the spectrum should be much cleaner than for $^{11}$Li where the low-lying continuum already is crowded by the three different $1^-$-excitations on top of the $3/2^-$ ground state.

### Table 2
Energy $E$ of the bound $0^-$- and $1^-$-states in $^{12}$Be and energy and width $(E_R, \Gamma_R)$ of the $0^-$- and $1^-$-states in $^{11}$Li. The last two rows give the percentage of the three-body wave function corresponding to neutron and core in a relative $s$- or $p$-wave.

<table>
<thead>
<tr>
<th>$1^-$</th>
<th>$0^-$</th>
<th>$1^-$</th>
<th>$0^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ or $(E_R, \Gamma_R)$</td>
<td>$-0.97$</td>
<td>$-0.96$</td>
<td>$(0.64,0.32)$</td>
</tr>
<tr>
<td>$%s$-wave</td>
<td>$51%$</td>
<td>$54%$</td>
<td>$43%$</td>
</tr>
<tr>
<td>$%p$-wave</td>
<td>$49%$</td>
<td>$46%$</td>
<td>$57%$</td>
</tr>
</tbody>
</table>

Fig. 1. The computed (without effective three-body forces) and experimental low-lying spectra of $^{12}$Be and $^{11}$Li obtained in two-neutron cluster models with neutron–core $s$-, $p$- and $d$-waves.

### 4. Wave functions and transition strengths
The relatively small binding energy allows halo formation. The dimensionless quantity $(\rho_\theta^2)/\rho_0^2$ ($\theta$ is the hyperradius) used as criterion in [4] is $2.4$ and $2.5$ for the $0^-$-states in $^{12}$Be and $^{11}$Li, i.e. both numbers are larger than $2$, indicating halo configuration. For the $^{11}$Li-resonance the number quoted is $(\rho_\theta^2)/\rho_0^2 = 2.4$ and $2.5$, where $(\rho_\theta^2)$ is complex and $\rho_0$ is real as defined in [4]). The density distributions can be seen in the upper ($^{12}$Be) and lower ($^{11}$Li) parts of Fig. 2. For $^{11}$Li the complex scaled three-body resonance wave function is shown ($\theta = 0.3$ rads). Both the $0^-$-states in $^{12}$Be and $^{11}$Li have the largest probabilities when the two neutrons are well separated by about $5$ fm and $8$ fm, respectively, and the cores are close to the two-neutron center-of-mass by about $2$ fm.

For these $0^-$-states, the average root-mean-square distances between pairs of particles are $\langle r_{nn}^2 \rangle^{1/2} = 9.2$ fm and...
where the constants $g_s$ and $g_\ell$ depend on the constituent particles $i$. We can identify three different sources of uncertainties in the lifetime estimates, i.e. (i) the effective values of the $g$-factor in these expressions, (ii) the precise values of the excitation energies or rather the emitted photon energy, and (iii) contributions from degrees of freedom beyond the three-body model. We believe that uncertainties due to (iii) are relatively small and consider in the following only effects of three-body structures. When the excitation energies are fine-tuned to reproduce experimental information by use of three-body potentials we essentially maintain the three-body structures and thereby the matrix elements.

The $0^-$-state has a very similar composition to the $1^-$-state which reproduces the experimental energy without any three-body force. This indicates that the computed $0^-$-energy also is close to the correct value. The lifetime estimates are then reliable determined by the corresponding matrix elements and the relative energies. Thus the $0^-$-state can decay to both the $1^-$- and $2^+$-states by magnetic dipole and quadrupole emission, respectively.

The core has angular momentum zero and therefore a vanishing effective spin $g^{(c)}_\ell$-factor and $g^{(c)}_c = 4$. We also use the free value of $g^{(n)}_s = -3.826$ and we calculate an effective charge corresponding to $g^{(n)}_\ell = 0.28$ which reproduces the $^{11}$Be transition strength $B(E1, 1/2^- \rightarrow 1/2^+) = 0.115 \pm 0.010 \, e^2 \, fm^2$ [33]. The transition operators are then defined and we can compute $B(M1, 0^+ \rightarrow 1^-)$ and $B(M2, 0^+ \rightarrow 2^+)$. However, the effective values of these $g$-factors are rather uncertain and spin polarization could reduce $g_s$ by a factor of 2, change $g^{(n)}_c$ by perhaps 10%, and vary $g^{(c)}_\ell$ from the assumed effective value, see also the discussion of the empirical evidence in [2].

In practice the present magnetic dipole transition is determined by the motion of the neutrons because the difference in the two states $0^-$ and $1^-$ is essentially a spin flip of one of the neutrons. The contribution from the core is very small, first because it has spin zero, and second, because due to its large mass compared to the mass of the neutrons, the orbital angular momentum of the core from the three-body center of mass is also small. The dependence on the neutron $g$-factors are seen in Table 3. The dependence on the orbital part is a lot smaller than the spin part arising from $g^{(n)}_s$. In total with a rather wide interval of parameter variation the changes are within a factor of 3 and certainly within an order of magnitude in this three-body model.

The lifetimes of the corresponding two possible decay modes are given by trivial factors and specific power law dependences on energy [2]. With the values from Table 3 obtained for $g^{(n)}_s = 0.28$, $g^{(n)}_c = -3.82$ we find

$$
\tau(M1) \approx \left( \frac{0.1 \, MeV}{\hbar \omega_1} \right)^3 \, 1.06 \times 10^{-11} \, s, \\
\tau(M2) \approx \left( \frac{0.6 \, MeV}{\hbar \omega_2} \right)^5 \, 1.12 \times 10^{-8} \, s,
$$

Fig. 2. The probability distributions of the $0^-$-states for $^{12}$Be (upper part) and $^{11}$Li (lower part) as functions of the distances between the two neutrons and their center-of-mass and the cores. The wave functions are computed in two-neutron cluster models with neutron–core $s$-, $p$- and $d$-waves.

$(r_{nn}^2)^{1/2} = 5.9$ fm for the neutron–neutron and neutron–core distances in $^{12}$Be, and $(r_{nn}^2)^{1/2} = 8.7$ fm and $(r_{nc}^2)^{1/2} = 5.1$ fm for the same distances in $^{11}$Li.

The possibility of detecting the $0^-$-state in $^{12}$Be might depend strongly on its life-time which in turn depends on the allowed decay modes. Since the relative energies are uncertain we have to compute decays to both $1^-$- and $2^+$-states which means magnetic dipole or quadrupole transitions. The operators are

$$
\mathcal{M}_\mu(M1) = \frac{e \hbar}{2 M \mathcal{C}} \sqrt{\frac{3}{4 \pi}} \sum_i \left( g_s^{(i)} \bar{s}_i + g_\ell^{(i)} \bar{\ell}_i \right)_{\mu},
$$

$$
\mathcal{M}_\mu(M2) = \frac{e \hbar}{2 M \mathcal{C}} \sqrt{\frac{3}{4 \pi}} \sum_i \sum_{v,q} \left( \frac{1}{v} \frac{1}{q} \frac{2}{-\mu} \right) Y_{1,v}(\Omega_i)
\times \left( g_s^{(i)} \bar{s}_i + \frac{2 g_\ell^{(i)}}{3} \bar{\ell}_i \right)_q,
$$
where the emitted photon energies are $\hbar \omega_1$ and $\hbar \omega_2$. The uncertainties due to the $g$-factors are easily found by combining Table 3 and Eq. (3). Since $\hbar \omega_2$ is expected to deviate much less than $\hbar \omega_1$ from their correct values we deduce that $\tau(M1)$ decides the lifetime of the $0^-$ state, unless the energies of the $0^-$ and $1^-$-states coincides to a remarkable accuracy of less than 10 keV. Thus the lifetime is estimated to be longer than about $10^{-11}$ s and shorter than about $10^{-8}$ s which qualifies the name of an isomeric state. Due to the uncertainties arising from the three-body forces the $0^-$-state could be below the $1^-$-level, and there is even a small probability for being unbound. In case of being below the $1^-$, then only the $M2$ transition to the $2^+$-state would be allowed and the lifetime would be confined to be around $10^{-8}$ s. If the $0^-$-state is above the two-neutron threshold, the state should resemble the corresponding resonance in $^{11}$Li. In that case the width should be comparable to the resonance energy, e.g. an energy below say 0.2 MeV above threshold would be related to a width smaller than that value.

5. Summary and conclusions

We predict a particle-bound isomeric $0^-$-state in $^{12}$Be with a dominant configuration of two neutrons around the $^{10}$Be-core. A similar low-lying but particle-unbound $0^-$-resonance of rather large two-neutron emission width is predicted in $^{11}$Li. The decay of $0^-$ in $^{12}$Be has to be by magnetic dipole or quadrupole photon emission. The lifetime $\tau$ is estimated with large uncertainty to be in the interval $10^{-11}$–$10^{-8}$ s. This $0^-$-state of $^{12}$Be can be reached from the ground state by a single particle excitation from an $s_{1/2}$- to a $p_{1/2}$-state. It has a structure essentially only deviating from the known $1^-$-state by neutron spin couplings to zero instead of 1. This $^{12}$Be-state can be expected to be populated in reactions by a cross section about the statistical factor of three smaller than that known for the similar $1^-\text{-state}$. Due to the long lifetime of the isomeric $0^-\text{-state}$ of $^{11}$Li its production rate from the $^{11}$Li ground state can be expected to be much weaker than for the $1^-\text{-state}$.

Acknowledgements

We highly appreciate discussions with Dr. H.O.U. Fynbo in the initial phase of the present investigation. This work was partly supported by funds provided by DGI of MEC (Spain) under contract No. FIS2005-00640. One of us (C.R.R.) acknowledges support by a predoctoral I3P fellowship from CSIC and the European Social Fund.

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