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Axial amplitudes for Δ excitation in chiral quark models

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Abstract

We study the axial amplitudes for the $N-\Delta$ transition in models with quarks and chiral mesons. A set of constraints on the pion field is imposed which enforces PCAC and the off-diagonal Goldberger–Treiman relation. The quark contribution to the amplitudes in general strongly underestimates the C_5^A amplitude as well as the $\pi N\Delta$ strong coupling constant. We show that the results are considerably improved in models that, in addition to the pion cloud, incorporate a fluctuating σ -field inside the baryon.

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1. Introduction

The structure of the weak axial $N-\Delta$ transition currents is ideally probed in neutrino or charged-lepton scattering experiments on deuterium or hydrogen. The experimental efforts so far have been focused on the determination of the dipole axial mass parameter [1], without an attempt to break down the transition current into form-factors [2]. Although a number of phenomenological predictions for the dominant coupling $C_5^A(0)$ exist (see Table 1 of [3] for an exhaustive list), the dependence of the form-factors on momentum transfer is very poorly known. Data on the non-leading form-factors $C_3^A(Q^2)$ and $C_4^A(Q^2)$ are especially scarce [4]. New information on the weak axial form-factors is expected from parity-violating electron scattering experiments planned at Jefferson Laboratory [5].

Theoretical investigation of axial transition amplitudes in different versions of the quark model is of particular interest since it may reveal the importance of non-quark degrees of freedom in baryons, in particular the chiral

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mesons. Yet, except for the calculation in the non-relativistic quark model [6], there exist almost no model predictions for the axial transition amplitudes. This can be traced back to the difficulty of incorporating consistently the pion field which is necessary to describe the proper low- Q^2 behaviour of the amplitudes.

The lack of experimental and theoretical knowledge in the weak sector is in contrast to the case of electro-excitation of the Δ resonance, which has been extensively studied theoretically in the constituent quark models [7] as well as chiral models [8], and experimentally [9]. In [10] we have pointed out the important role played by the pion cloud in the determination of electro-production amplitudes, in particular to the E2/M1 and C2/M1 ratios. This has later been confirmed in other chiral models [11] and dynamical approaches [12].

The aim of this Letter is to study some general properties of the axial amplitudes in chiral quark models and present theoretical predictions in two typical representatives of such models, the linear σ -model with quarks and the cloudy bag model. We derive a set of constraints on the pion field which enforce the proper behaviour of the amplitudes in the vicinity of the pion pole. We also address the long-standing problem of a too low $\pi N\Delta$ coupling constant which rather systematically appears in all quark models. Comparing the results in the two models we are able to draw some general conclusions regarding the contribution of chiral mesons to the weak amplitudes as well as to the strong $\pi N\Delta$ form factor.

2. The axial transition amplitude and the off-diagonal Goldberger–Treiman relation

The axial $N\text{--}\Delta$ transition amplitude is usually parameterized in terms of the Adler form-factors $C_i^A(Q^2)$ as¹

$$\langle \Delta^+(p') | A_{\alpha(a=0)} | N^+(p) \rangle = \bar{u}_{\Delta\alpha} \frac{C_4^A(Q^2)}{M_N^2} p'_{\mu} q^{\mu} u_N - \bar{u}_{\Delta\mu} \frac{C_4^A(Q^2)}{M_N^2} p'_{\alpha} q^{\mu} u_N + \bar{u}_{\Delta\alpha} C_5^A(Q^2) u_N + \bar{u}_{\Delta\mu} \frac{C_6^A(Q^2)}{M_N^2} q^{\mu} q_{\alpha} u_N, \quad (1)$$

where $p'_{\mu} = (M_{\Delta}; 0, 0, 0)$, $u_{\Delta\alpha}$ is the corresponding Rarita–Schwinger spinor, p is the four-momentum of the nucleon and $q^{\mu} = (k_0; 0, 0, k)$ is the four-momentum of the incident weak boson. Then $k_0^2 - k^2 = q^2 \equiv -Q^2$ and $k_0 = (M_{\Delta}^2 - M_N^2 - Q^2)/2M_{\Delta}$. For simplicity, we take the third isospin component ($a = 0$) of the axial current. We have omitted from (1) the $C_3^A(Q^2)$ term [2], which is consistent with the prediction of quark models in which quarks occupy only the $l = 0$ state.

It is convenient to work with helicity amplitudes²

$$\tilde{S}^A = -\langle \Delta^+(p'), s_{\Delta} = \frac{1}{2} | A_0^0(0) | p(p), s_N = \frac{1}{2} \rangle, \quad (2)$$

$$\tilde{A}_{\frac{3}{2}}^A = -\langle \Delta^+(p'), s_{\Delta} = \frac{3}{2} | \boldsymbol{\epsilon}_+ \cdot \mathbf{A}^0(0) | p(p), s_N = \frac{1}{2} \rangle, \quad (3)$$

$$\tilde{A}_{\frac{1}{2}}^A = -\langle \Delta^+(p'), s_{\Delta} = \frac{1}{2} | \boldsymbol{\epsilon}_+ \cdot \mathbf{A}^0(0) | p(p), s_N = -\frac{1}{2} \rangle, \quad (4)$$

$$\tilde{L}^A = -\langle \Delta^+(p'), s_{\Delta} = \frac{1}{2} | \boldsymbol{\epsilon}_0 \cdot \mathbf{A}^0(0) | p(p), s_N = \frac{1}{2} \rangle, \quad (5)$$

where s denotes the third spin component, and $\boldsymbol{\epsilon}$ are the usual polarisation vectors. The helicity amplitudes are related to the C_i^A form-factors by

$$C_6^A = \frac{M_N^2}{k^2} \left[-\tilde{A}_{\frac{3}{2}}^A + \sqrt{\frac{3}{2}} \tilde{L}^A \right], \quad (6)$$

¹ Definition of the transition current with respect to the Δ^{++} brings an additional isospin factor $\sqrt{3}$ to RHS of (1).

² The helicity amplitudes are normally defined as the matrix elements of the interaction Hamiltonian and contain an additional factor $\sqrt{4\pi\alpha_W/2K_0}$, e.g., $S^A = \sqrt{4\pi\alpha_W/2K_0} \tilde{S}^A$, where $K_0 = k_0(Q^2 = 0)$ and α_W is the weak fine-structure constant.

$$C_5^A = \sqrt{\frac{3}{2}} \left(\frac{k_0}{k} \tilde{S}^A - \frac{k_0^2}{k^2} \tilde{L}^A \right) + \frac{k_0^2 - k^2}{k^2} \tilde{A}_{\frac{3}{2}}^A, \quad (7)$$

$$C_4^A = \frac{M_N^2}{kM_\Delta} \left[-\sqrt{\frac{3}{2}} \tilde{S}^A + \frac{k_0 k}{M_N^2} C_6^A \right]. \quad (8)$$

In the approximation with $C_3^A = 0$ we have only one independent transverse amplitude, since in this case $\tilde{A}_{\frac{3}{2}}^A = \sqrt{3} \tilde{A}_{\frac{1}{2}}^A$.

From (1) it follows that the divergence of the transition axial current vanishes in the chiral limit provided $C_6^A(Q^2) = M_N^2 C_5^A(Q^2)/Q^2$. The pole behaviour of the C_6^A amplitude suggests that it is related to the term in the axial current responsible for the pion decay, $A_{a(\text{pole})}^\alpha(x) = f_\pi \partial^\alpha \pi_a(x)$, where $f_\pi = 93$ MeV is the pion decay constant. Taking a finite mass for the pion the divergence does not vanish but is replaced by PCAC:

$$\langle \Delta^+(p') | \partial^\alpha A_{\alpha a} | N^+(p) \rangle = -m_\pi^2 f_\pi \langle \Delta^+(p') | \pi_a(0) | N^+(p) \rangle, \quad (9)$$

where the transition matrix element of the pion field is related to the strong form factor $G_{\pi N \Delta}(Q^2)$ by

$$\langle \Delta^+(p') | \pi_0(0) | N^+(p) \rangle = i \frac{G_{\pi N \Delta}(Q^2)}{2M_N} \frac{\bar{u}_{\Delta\mu} q^\mu u_N}{Q^2 + m_\pi^2} \sqrt{\frac{2}{3}}. \quad (10)$$

Assuming that $A_{a(\text{pole})}^\alpha(x)$ dominates the C_6^A amplitude for $Q^2 \rightarrow -m_\pi^2$, we obtain the *off-diagonal Goldberger–Treiman relation* [2,13,14]:

$$C_5^A(Q^2) = f_\pi \frac{G_{\pi N \Delta}(Q^2)}{2M_N} \sqrt{\frac{2}{3}}, \quad Q^2 \rightarrow -m_\pi^2. \quad (11)$$

For a smooth interpolating pion field we expect that (11) holds also for moderate Q^2 in the physically accessible region.

3. Helicity amplitudes in chiral quark models

For a variety of models involving quarks interacting with chiral fields σ and $\vec{\pi}$ the Hamiltonian can be written in the form

$$H = H_q^0 + H_\sigma + \int d\mathbf{r} \left\{ \frac{1}{2} [\vec{P}_\pi^2 + (\nabla^2 + m_\pi^2) \vec{\pi}^2] + U(\sigma, \vec{\pi}) + \sum_a j_a \pi_a \right\}, \quad (12)$$

where j_a is the quark source, \vec{P}_π is the pion conjugate momentum, H_q^0 and H_σ are the free-quark and the σ -meson terms, and $U(\sigma, \vec{\pi})$ is the meson self-interaction term. In the cloudy bag model the σ -field and the self-interaction term are absent, while in the linear σ -model all terms are present and the self-interaction term is the Mexican-hat potential (see (22) below).

Let $|N\rangle$ and $|\Delta\rangle$ be the exact solution of the Hamiltonian for the ground state and for the Δ , respectively, with $H|N\rangle = E_N|N\rangle$ and $H|\Delta\rangle = E_\Delta|\Delta\rangle$. Then $\langle N|[H, \vec{P}_\pi]|N\rangle = \langle \Delta|[H, \vec{P}_\pi]|\Delta\rangle = 0$ and $\langle \Delta|[H, \vec{P}_\pi]|N\rangle = i(E_\Delta - E_N)^2 \langle \Delta|\vec{\pi}|N\rangle$. Evaluating the commutators using (12) for $a = 0$, we obtain

$$(-\Delta + m_\pi^2) \langle N|\pi_0(\mathbf{r})|N\rangle = -\langle N|J_0(\mathbf{r})|N\rangle, \quad (13)$$

$$(-\Delta + m_\pi^2) \langle \Delta|\pi_0(\mathbf{r})|\Delta\rangle = -\langle \Delta|J_0(\mathbf{r})|\Delta\rangle, \quad (14)$$

$$(-\Delta + m_\pi^2 - (E_\Delta - E_N)^2) \langle \Delta|\pi_0(\mathbf{r})|N\rangle = -\langle \Delta|J_0(\mathbf{r})|N\rangle. \quad (15)$$

The sources on the RHS of (13)–(15) consist of the quark term and the term originating from the meson self-interaction (if present):

$$J_0(\mathbf{r}) = j_0(\mathbf{r}) + \frac{\partial U(\sigma, \vec{\pi})}{\partial \pi_0(\mathbf{r})}. \quad (16)$$

These relations hold for the exact solutions of (12). In an approximate computational scheme they can be used as constraints.

We now show an important property of the axial transition amplitudes between states which satisfy these virial relations. We split the axial current into the non-pole and the pole part, $\vec{A}^\alpha = \vec{A}_{(\text{non-pole})}^\alpha + \vec{A}_{(\text{pole})}^\alpha$, where

$$\vec{A}_{(\text{non-pole})}^\alpha = \bar{\psi} \gamma^\alpha \gamma_5 \frac{1}{2} \vec{\tau} \psi + (\sigma - f_\pi) \partial^\alpha \vec{\pi} - \vec{\pi} \partial^\alpha \sigma, \quad (17)$$

$$\vec{A}_{(\text{pole})}^\alpha = f_\pi \partial^\alpha \vec{\pi}. \quad (18)$$

Since the pole part involves only the pion field we can use (15) to evaluate its contribution to the amplitudes. Note that (15) is equivalent to (10) since in our model we can write the strong N– Δ transition form-factor as

$$\frac{G_{\pi N \Delta}(Q^2)}{2M_N} = \frac{1}{ik} \langle \Delta || J_0(0) || N \rangle. \quad (19)$$

We find $\tilde{A}_{\frac{3}{2}(\text{pole})}^A = 0$ and

$$\tilde{S}_{(\text{pole})}^A = \frac{k_0}{k} \tilde{L}_{(\text{pole})}^A = \frac{2}{3} \frac{G_{\pi N \Delta}(Q^2)}{2M_N} \frac{f_\pi k k_0}{Q^2 + m_\pi^2}. \quad (20)$$

The pole term (18) contributes only to C_6^A ,

$$C_{6(\text{pole})}^A = f_\pi \frac{G_{\pi N \Delta}(Q^2)}{2M_N} \frac{M_N^2}{Q^2 + m_\pi^2} \sqrt{\frac{2}{3}}, \quad (21)$$

while $C_{4(\text{pole})}^A = C_{5(\text{pole})}^A = 0$. We conclude that in models in which the pion contribution to the axial current has the simple form $f_\pi \partial^\alpha \pi_a$ and the pion field satisfies the virial relation (15) *there is no pion contribution to the C_4^A and C_5^A amplitudes* while C_6^A is almost entirely dominated by the pion pole. In such models only the quarks contribute to the C_4^A and C_5^A amplitudes. In this respect, the calculation of C_5^A in a constituent quark model calculation (e.g., [6]), is still legitimate.

4. Constrained calculation in the linear σ -model

The linear σ -model assumes the following form of j_i and \mathcal{U} [15]:

$$j_i = ig \sum_{i=1}^3 \bar{q}_i \gamma_5 \tau_i q_i, \quad \mathcal{U} = \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - f_\pi^2)^2. \quad (22)$$

Here q_i is the quark bispinor for the valence orbit (assumed to be different for the nucleon and the Δ), and $\lambda = (m_\sigma^2 - m_\pi^2)/2f_\pi^2$. The free parameters of the model are the coupling strength g related to the ‘‘constituent’’ mass of the quark gf_π , and the mass of the σ -meson m_σ . The model has been successfully applied to the description of the nucleon and Δ properties. So far the physical states have been constructed from the mean-field solution using either cranking [16] or the Peierls–Yoccoz projection [17]. In the latter method the mean-field solution for the pion field is interpreted as a coherent state.

The mean-field solution fulfills the diagonal virial relations (13), (14) but not the off-diagonal relation (15). To satisfy this relation it is necessary to include a channel representing the Δ decay, i.e., a term that asymptotically represents the nucleon and a free pion. We have therefore taken a more general ansatz for the Δ :

$$|\Delta\rangle = \mathcal{N}_\Delta \left\{ P^{\frac{3}{2}} \Phi_\Delta |\Delta_q\rangle + \int dk \eta(k) [a_{m\pi}^\dagger(k)|N\rangle]^{\frac{3}{2}} \right\}, \quad (23)$$

where the first term represents the bare Δ state surrounded by a cloud of pions and σ -mesons, $P^{\frac{3}{2}}$ is the projection operator on the subspace with isospin and angular momentum $\frac{3}{2}$, $|N\rangle$ is the nucleon ground state, and $[]^{\frac{3}{2}}$ denotes a pion–nucleon state with isospin- $\frac{3}{2}$ and spin- $\frac{3}{2}$. Requiring that the energy of this state is stationary, the denominator of $\eta(k)$ takes the form $\omega_k - (E_\Delta - E_N)$ which is also the form implied by (15). For the nucleon we assume:

$$|N\rangle = \mathcal{N}_N P^{\frac{1}{2}} [\Phi_N |N_q\rangle + \Phi_{N\Delta} |\Delta_q\rangle]. \quad (24)$$

Here Φ_N and $\Phi_{N\Delta}$ stand for hedgehog coherent states describing the pion cloud around the bare nucleon and bare Δ , respectively. To match the third constraint, (15), the denominator of the pion state in the second term of (24) should behave as $\omega_k + \omega_0$ with $\omega_0 = (E_\Delta - E_N)$. In the above ansatz, only one profile for the σ -field is assumed.³

The properties of the ground state are dominated by the first term in (24), and imposing the off-diagonal constraint influences only slightly the results. For the Δ , the inclusion of the decaying channel modifies the long-range behaviour of the pion field, and yields the correct low- Q^2 behaviour of the transition amplitudes as explained in the previous section. The calculated Δ -N splitting is typically only (50–70)% of the experimental value. In order to make a sensible comparison of the transition amplitudes with the experimental ones, it is necessary to have the correct kinematical relations in the model. This can be achieved by including an additional phenomenological term in the Hamiltonian mimicking either the chromo-magnetic or the instanton-induced interaction between quarks and adjusting its strength such as to bring the Δ -N splitting to the experimental value.

5. Calculation of the amplitudes

We calculated the amplitudes in two models: in the linear σ -model and in the cloudy bag model. In the cloudy bag model we assume the usual perturbative form for the pion profiles [18] using the experimental masses for the nucleon and Δ , which fulfills the virial constraints (13)–(15). Since the pion contribution to the axial current in the cloudy bag model has the form of the pole term in (18), only the quarks contribute to the C_5^A and C_4^A amplitudes.

The amplitudes (2)–(5) are defined between states with good 4-momenta p' and p , respectively, while in the model calculations localised states are used. We can use such states in our calculation of amplitudes by interpreting them as wave packets of states with good linear momentum. Extending the method explained in [14] we find for a chosen component of the axial current evaluated between localised states, $\langle \Delta | A(\mathbf{r}) | N \rangle$:

$$\int d\mathbf{p} \varphi_\Delta^*(\mathbf{p} + \mathbf{k}) \varphi_N(\mathbf{p}) \langle \Delta(\mathbf{p} + \mathbf{k}) | A(0) | N(\mathbf{p}) \rangle = \int d\mathbf{r} e^{i\mathbf{k}\mathbf{r}} \langle \Delta | A(\mathbf{r}) | N \rangle. \quad (25)$$

Here the matrix element of (1) is taken on the LHS and $\varphi_N(\mathbf{p})$ and $\varphi_\Delta(\mathbf{p})$ are (normalised) functions describing the center-of-mass motion of the localised solution for the nucleon and the Δ , respectively. We assume that the spread of the wave packet is of the order of the inverse baryon mass (M^{-1}) and use for simplicity the same spread for the nucleon and the delta. The Adler form-factors (6)–(8) are then modified in such a way that C_6^A and C_5^A are multiplied by the factor $2M_\Delta/(M_\Delta + M_N)$, while

$$C_4^A = \frac{M_N^2}{kM_\Delta} \left[-\sqrt{\frac{3}{2}} \tilde{S}^A + \frac{k_0 k}{M_N^2} \frac{M_\Delta + M_N}{2M_\Delta} C_6^A \right] - \frac{M_N^2}{2M_\Delta^2} C_5^A. \quad (26)$$

³ Since the σ -field is scalar its analog of (15) is identically zero.

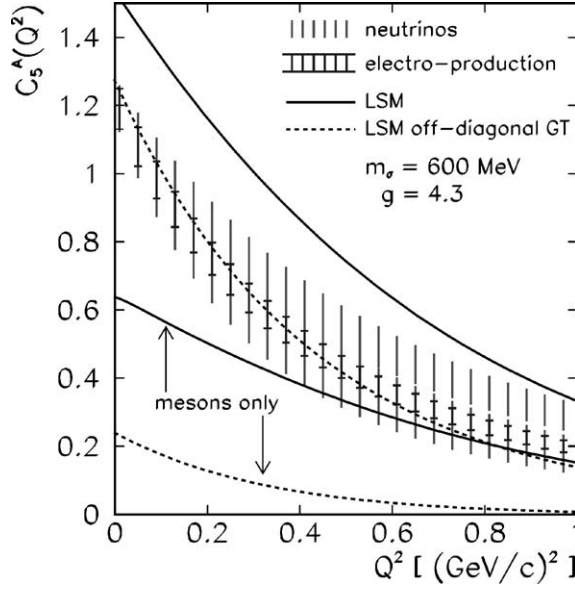


Fig. 1. The amplitude $C_5^A(Q^2)$ in the linear σ -model. The experimental value of 1.22 ± 0.06 at $Q^2 = 0$ [21] is based on data from ANL and BNL [22,23]. The error ranges are given by the spread in the axial-mass parameter M_A as determined from neutrino scattering experiments (broader range) and from electro-production of pions [1] ($M_A = (1.077 \pm 0.039)$ GeV, narrower range). Full curves: wave-packet result; dashed curves: calculation from $G_{\pi N\Delta}$ (11).

We have neglected terms of the order k^2/M^2 . Similarly, the strong $G_{\pi N\Delta}$ form factor (19) acquires the same correction factor. The essential property that the pole contribution cancels out in C_4^A and C_5^A still persists as well as the relation (21) for C_6^A .

Fig. 1 shows the C_5^A amplitude in the linear σ -model with $g = 4.3$ and $m_\sigma = 600$ MeV compared to the experimental weak axial form-factors given in the convention of Adler [19,20], with a phenomenological dipole parameterisation $C_i^A(Q^2) = C_i^A(0)/(1 + Q^2/M_A^2)^2$. The $C_5^A(0)$ is 25% higher than the experimental estimate, while the M_A from a dipole fit to our calculated values matches the experimental M_A to within a few percent.

We note that for the nucleon we obtain $g_A = 1.41$ which is roughly the same amount higher than the experimental value of 1.27. On the other hand, if we determine $C_5^A(Q^2)$ from the calculated strong $\pi N\Delta$ form-factor using the Goldberger–Treiman relation (11) we obtain a better agreement, yet with a steeper fall-off corresponding to $M_A \approx 0.80$ GeV. The discrepancy between the two calculated values (17% at $Q^2 = -m_\pi^2$ where (11) holds) is a measure for the quality of our approximate computational approach. It can be attributed to a too large meson contribution originating from the last two terms in (17). Since in this model only the meson fields bind the quarks it is reasonable that their strength is overestimated in the variational calculation. The effect of the meson self-interaction (the second term in (16)) is relatively less pronounced in the strong coupling constant (only $\sim 20\%$) than in $C_5^A(Q^2)$. Both $G_{\pi N\Delta}(0)$ and $G_{\pi NN}(0)$ are over-estimated in the model by $\sim 10\%$. Still, the ratio $G_{\pi N\Delta}(0)/G_{\pi NN}(0) = 2.01$ is considerably higher than either the familiar SU(6) prediction $\sqrt{72/25}$ or the mass-corrected value of 1.65 [14], and compares reasonably well with the experimental value of 2.2. This improvement is mostly a consequence of the renormalisation of the strong vertices due to pions.

The value of C_5^A grows with g and m_σ in contrast to $G_{\pi N\Delta}$ which remains almost constant over a large range of model parameters. In our calculation we cannot use much lower values for g since the solution becomes numerically unstable.

In the cloudy bag model the picture is reversed. Here only the first term in (17) contributes to the amplitudes; as a result the C_5^A amplitude is less than $2/3$ of the experimental value (see Fig. 2). The behaviour of C_5^A

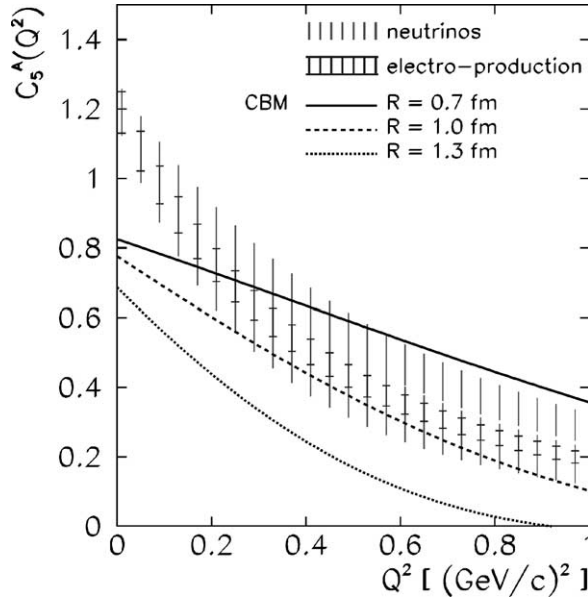


Fig. 2. The amplitude $C_5^A(Q^2)$ in the cloudy bag model for three values of the bag radius. Experimental uncertainties are as in caption to Fig. 1.

is similar as in the pure MIT bag model (to within 10%), with fitted $M_A \sim 1.2 \text{ GeV fm}/R$. The off-diagonal Goldberger–Treiman relation is satisfied in the cloudy bag model, but C_5^A from $G_{\pi N\Delta}$ has a steeper fall-off with fitted $M_A \sim 0.8 \text{ GeV fm}/R$. The ratio $C_5^A(0)/g_A$ is close to the model-independent prediction of [24].

The large discrepancy can be to some extent attributed to the fact that the cloudy bag model predicts a too low value for $G_{\pi NN}$, and consequently $G_{\pi N\Delta}$. Taking a smaller value of f_π in order to increase the strong coupling constants does not improve the results since f_π on the RHS of (11) compensates for the change in $G_{\pi N\Delta}$. We have found that the pions increase the $G_{\pi N\Delta}/G_{\pi NN}$ ratio by $\sim 15\%$ through vertex renormalisation. The effect is further enhanced by the mass-correction factor $2M_\Delta/(M_\Delta + M_N)$, yet suppressed in the kinematical extrapolation of $G_{\pi N\Delta}(Q^2)$ to the SU(6) limit. This suppression is weaker at small bag radii R : the ratio drops from 2.05 at $R = 0.7 \text{ fm}$ to 1.60 (below the SU(6) value) at $R = 1.3 \text{ fm}$.

The determination of the C_4^A is less reliable because the meson contribution to the scalar amplitude is very sensitive to small variations of the profiles. However, the experimental value is very uncertain as well. Neglecting the non-pole contribution to S^A and C_6^A (the pole contribution cancels out) we see from (26) that the value of C_4^A is dominated by the term $-(M_N^2/2M_\Delta^2)C_5^A$, in agreement with the popular parameterisation of the amplitudes. In our models, the non-pole contribution to C_6^A is not negligible and tends to increase C_4^A at small Q^2 , as seen in Fig. 3.

The C_6^A amplitude is governed by the pion pole for small values of Q^2 and hence by the value of $G_{\pi N\Delta}$ which is well reproduced in the linear σ -model, and underestimated by $\sim 35\%$ in the cloudy bag model. Fig. 4 shows that the non-pole contribution becomes relatively more important at larger values of Q^2 .

6. Conclusions

To the best of our knowledge the present work is the first attempt to calculate the axial N– Δ transition amplitudes in a quark model which consistently includes the chiral mesons already at the Lagrangian level. We have derived a set of constraints which ensures the proper treatment of the pion pole dominating the transition at low Q^2 . Though

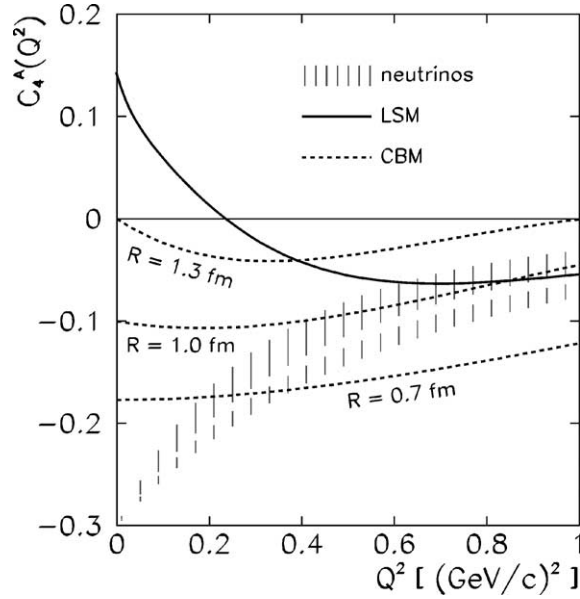


Fig. 3. The amplitude $C_4^A(Q^2)$ in the linear σ -model, with model parameters and experimental uncertainties due to the spread in M_A as in Fig. 1. Experimentally, $C_4^A(0) = -0.3 \pm 0.5$ [4]. For orientation, the value for $C_4^A(0)$ is used without error-bars.

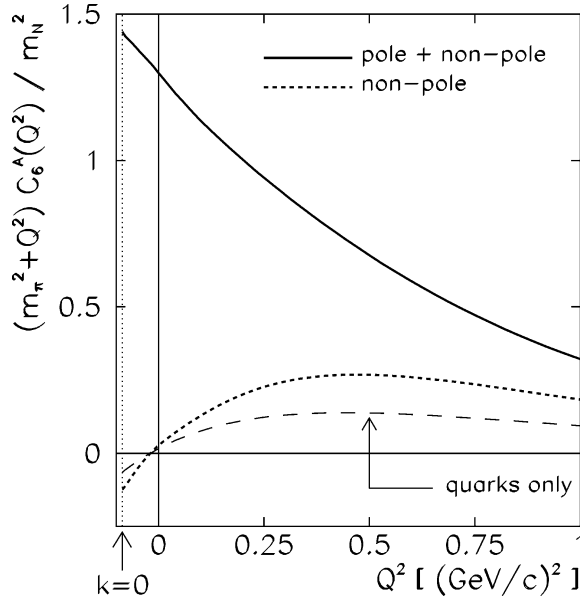


Fig. 4. The non-pole part and the total amplitude $C_6^A(Q^2)$ in the linear σ -model. Model parameters are as in Fig. 1.

there is a rather strong discrepancy between calculated amplitudes in the two models considered here, we are nonetheless able to draw some general conclusions about the role of the chiral mesons.

The quark contribution alone strongly underestimates the C_5^A amplitude. Models in which only a linear coupling of pions to quarks is added do not improve the situation since in such a case the pion term in the axial current does

not contribute to the amplitude. On the other hand, the inclusion of meson self-interaction which allows for a substantial deviation of the σ -field from its vacuum value inside the baryon considerably increases C_5^A . The linear σ -model seemingly overestimates this contribution as it could have been anticipated from the overestimate of g_A obtained in this model.

Regarding the ratio $G_{\pi N\Delta}/G_{\pi NN}$ we find that it is the pion cloud which enhances its value compared to the SU(6) value of $\sqrt{72/25}$; in the linear σ -model as well as in the cloudy bag model for smaller bag radii the ratio is greater than 2 and not far from the experimentally determined value of 2.2.

The Q^2 -behaviour of the axial amplitudes is well reproduced in the linear σ -model. We stress that the behaviour of $G_{\pi N\Delta}(Q^2)$ is considerably softer, with a cut-off parameter (corresponding to the axial mass M_A) of ~ 0.8 GeV. A similar trend is also seen in the cloudy bag model for bag radii above ~ 1 fm. The popular assumption in which the same value for the strong and axial cut-offs is taken is, therefore, not supported by the two models.

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