# Thinned Concentric Circular Antenna Array Synthesis using Particle Swarm Optimization 

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#### Abstract

This paper presents an optimal thinning of a large multiple concentric circular ring arrays of uniformly excited isotropic antennas based on Particle Swarm Optimization method. Circular Antenna Array (CAA) has gained immense popularity in the field of communications nowadays. It has proved to be a better alternative over other types of antenna array configuration due to its all-azimuth scan capability, and the beam pattern which can be kept invariant. In this paper, a 9 ringed Concentric Circular Antenna Array (CCAA) with central element feeding is considered. Extensive simulation results justify the optimization efficacy of the proposed approach for antenna array synthesis. The simulation results show that the number of effective antenna elements can be brought down from 279 to 139 with simultaneous reduction in Side Lobe Level by 20.37 dB relative to the main beam with a fixed half power beamwidth using PSO. Real coded Genetic Algorithm (RGA) as well is also adopted to compare the results of PSO algorithm.


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## 1. Introduction

Concentric Circular Antenna Array (CCAA) [1-11] has received considerable interest for its symmetric and compactness in structure. The CCAA is an array that consists of many concentric rings of different radii and a number of elements on its circumference. Since a CCAA does not have edge elements, directional patterns synthesized with a concentric circular array can be electronically rotated in the plane of the array without a significant change in the beam shape. Uniform CCA (UCCA) is the CCA where all the elements in the array are uniformly excited and the inter-element spacing in individual ring is kept as almost half of the wavelength. For larger number of rings with uniform excitations, the side lobe in the

UCCA drops to about 17.4 dB .
A lot of research has gone into optimizing antenna structures such that the radiation pattern has sidelobe level at the lowest possible magnitude. This very fact has driven researchers to optimize the CCAA design. Although, uniformly excited and equally spaced antenna arrays have high directivity but at the same time, they suffer from high side lobe level. Reduction in side-lobe level can be brought about in either of the following ways: either by keeping excitation amplitudes uniform but changing the position of antenna elements or by using equally spaced array with radially tapered amplitude distribution. These processes are referred to as thinning. Thinning not only reduces side lobe level but also brings down the manufacturing cost by decreasing the number of antenna elements [12-13].

Classical optimization methods have several disadvantages such as: i) highly sensitive to starting points when the number of solution variables and hence the size of the solution space increase, ii) frequent convergence to local optimum solution or divergence or revisiting the same suboptimal solution etc. But there are various global optimization tools for thinning such as Genetic Algorithms (GA) [8], Particle Swarm Optimization (PSO) [14-18] etc. which does not suffer from above disadvantages may be adopted. The PSO algorithm has proven to be a better alternative to other evolutionary algorithms such as Genetic Algorithms (GA), Ant Colony Optimization (ACO) etc. for optimal design of antenna array.

In this work, for the optimization of complex, highly non-linear, discontinuous, and non-differentiable array factors for thinning a large multiple concentric circular ring arrays of isotropic antennas, Particle Swarm Optimization (PSO) is adopted. Thus, optimal CCAA design achieved by PSO would have the near-global optimized set of current excitation.

The rest of the paper is organized as follows: In section 2, the general design equations for the CCAA are discussed. Then, in section 3, a brief introduction for the RGA and PSO are presented. Simulation results are shown and discussed in section 4 . Finally section 5 concludes the paper.

## 2. Design Equation

Fig. 1 shows the general configuration of CCAA with $M$ concentric circular rings, where the $m^{\text {th }}(\mathrm{m}=$ $1,2, \ldots, \mathrm{M}$ ) ring has a radius $\mathrm{r}_{\mathrm{m}}$ and the corresponding number of elements is $\mathrm{N}_{\mathrm{m}}$. If all the elements (in all the rings) are assumed to be isotopic sources, the radiation pattern of this array can be written in terms of its array factor only.

Referring to Fig. 1, the far field radiation pattern of a thinned CCAA in $x-y$ plane may be written as [15-18]:
$A F(\theta, I)=\sum_{m=1}^{M} \sum_{i=1}^{N_{m}} I_{m i} \exp \left[j\left(k r_{m} \sin \theta \cos \left(\phi-\phi_{m i}\right)+\alpha_{m i}\right)\right]$
where
$I_{m i}=$ Excitation of $i^{\text {th }}$ element of $m^{\text {th }}$ ring $= \begin{cases}1 & \text { on } \\ 0 & \text { off }\end{cases}$
$k=2 \pi / \lambda ; \lambda$ being the signal wave-length. If the elevation angle, $\phi=$ constant, then (1) may be written as a periodic function of $\theta$ with a period of $2 \pi$ radian i.e. the radiation pattern will be a broadside array pattern. The azimuth angle of the $i^{\text {th }}$ element of the $m^{\text {th }}$ ring is $\phi_{m i}$. The elements in each ring are assumed to be uniformly distributed. $\phi_{m i}$ and $\alpha_{m i}$ are also obtained from [13] as:

$$
\begin{align*}
& \phi_{m i}=2 \pi\left((i-1) / N_{m}\right)  \tag{3}\\
& \alpha_{m i}=-K r_{m} \sin \theta_{0} \cos \left(\phi-\phi_{m i}\right)
\end{align*}
$$

$\theta_{0}$ is the value of $\theta$ where peak of the main lobe is obtained. After defining the array factor, the next
step in the design process is to formulate the objective function which is to be minimized. The objective function "Cost Function" ( $C F$ ) may be written as (5):

$$
\begin{equation*}
C F=W_{F 1} \times \frac{\left|A F\left(\theta_{m s l 1}, I_{m i}\right)+A F\left(\theta_{m s l 2}, I_{m i}\right)\right|}{\left|A F\left(\theta_{0}, I_{m i}\right)\right|}+W_{F 2} \times\left(F N B W_{\text {computed }}-F N B W\left(I_{m i}=1\right)\right) \tag{5}
\end{equation*}
$$



Fig. 1 Concentric circular antenna array (CCAA).
$F N B W$ is an abbreviated form of first null beamwidth, or, in simple terms, angular width between the first nulls on either side of the main beam. $C F$ is computed only if $F N B W_{\text {computed }}<F N B W\left(I_{m i}=1\right)$ and corresponding solution of current excitation weights is retained in the active population, otherwise the solution is not retained. $W_{F 1}$ (unitless) and $W_{F 2}\left(\right.$ radian $\left.^{-1}\right)$ are the weighting factors. $\theta_{0}$ is the angle where the highest maximum of central lobe is attained in $\theta \in[-\pi, \pi] . \theta_{m s l}$ is the angle where the maximum sidelobe $\left(A F\left(\theta_{m s_{1}}, I_{m i}\right)\right)$ is attained in the lower band and $\theta_{m s_{2}}$ is the angle where the maximum sidelobe $\left(A F\left(\theta_{m s l}, I_{m i}\right)\right)$ is attained in the upper band. $W_{F 1}$ and $W_{F 2}$ are so chosen that optimization of SLL remains dominant than optimization of $F N B W_{\text {computed }}$ and $C F$ never becomes negative. In (4) the two beam widths, $F N B W_{\text {computed }}$ and $F N B W\left(I_{m i}=1\right)$ refer to the computed first null beamwidths in radian for the non-uniform excitation case and for uniform excitation case, respectively. Minimization of $C F$ indicates the maximum reduction of SLL both in lower and upper sidebands and lesser $F N B W_{\text {computed }}$ as compared to $F N B W\left(I_{m i}=1\right)$. The evolutionary optimization techniques employed for optimizing the current excitation weights result in the minimization of $C F$ and hence reductions in both SLL and $F N B W$.
In this case, $I_{m i}$ is 1 if the $m_{i}^{\text {th }}$ element is turned "on" and 0 if it is "off." All the elements have same excitation phase of zero degree.

An array taper efficiency can be calculated from:

$$
\begin{equation*}
\eta_{a r}=\frac{\text { number of elements in the array turned on }}{\text { total number of elements in the array }} \tag{6}
\end{equation*}
$$

## 3. Evolutionary Techniques Employed

RGA and PSO as implemented for the optimization of current excitation weights of the CCAA are given in [19]. So, the steps of RGA and PSO are not described due to limitation in space.

## 4. Simulation Results and Discussions

This section shows and describes simulation results for the CCAA synthesis obtained by RGA, PSO techniques. Each CCAA maintains a fixed optimal inter-element spacing between the elements in each ring. The limits of the radius of a particular ring of CCAA are decided by the product of number of elements in the ring and the inequality constraint for the inter-element spacing, $d,(d \in[\lambda / 2, \lambda])$. For all the cases, $\theta_{0}=0^{0}$ is considered so that the peak of the main lobe starts from the origin.

Since GA and PSO techniques are sometimes quite sensitive to certain parameters, the simulation parameters should be carefully chosen. The best chosen parameters values are shown in Table I.

Fig. 2 is a diagram of a 279 element concentric ring array. Nine rings $\left(\mathrm{N}_{1}, \mathrm{~N}_{2}, \ldots \ldots ., \mathrm{N}_{9}\right)$ are considered for synthesis having ( $6,12,18,25,31,37,43,50,56$ ) elements with central element feeding. For this case $r_{m}=m \times \frac{\lambda}{2}$ and inter-element spacing in each ring is $d_{m} \cong \frac{\lambda}{2}$. The number of equally spaced elements in ring $m$ is given by (7).

$$
\begin{equation*}
N_{m}=\frac{2 \pi r_{m}}{d_{m}}=2 \pi m \tag{7}
\end{equation*}
$$

Since the number of elements must be an integer, the value in (7) must be rounded up or down. To keep $d_{m} \geq \frac{\lambda}{2}$, the digits to the right of the decimal point are dropped. Table II lists the ring spacing and number of elements in each ring for a uniform concentric ring array with nine rings as shown in Fig. 2.

The RGA, PSO individually generate a set of optimal uniform current excitation for the CCAA set. $I_{m i}$ is 1 if the $m_{i}$ th element is turned "on" and 0 if it is "off". The optimal results are shown in Tables III- V.

### 4.1. Analysis of Radiation Patterns of CCAA

Figs. 3-4 show the array pattern found by RGA, PSO for 9 rings CCAA set with central element feeding. From Figs. 3-4 it is observed that a substantial reduction in SLL is achieved with optimal current excitation weights after thinning, as compared to the case of uniform current excitation weights and of fully populated array (considering fixed inter-element spacing, $\mathrm{d} \approx \lambda / 2$ ).

As seen from Table V the SLL reduces to more than 20 dB for the given set, with respect to -17.4 dB for uniform excitation and $d=\lambda / 2$ for all algorithms. Further, the above improvement is achieved for an array of $43.4 \%$, and $50.18 \%$ elements turned off for RGA, and PSO respectively.

The above results reveal that the thinned 9 rings CCAA set with central element feeding results in the reductions in SLL as compared to that of the fully populated same array.

### 4.2. Convergence Profiles

The minimum $C F$ values against number of iteration cycles are recorded to get the convergence profile of each set algorithm. Figs. 5-6 portray the convergence profiles of RGA, PSO respectively. All computations were done in MATLAB 7.5 on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

TABLE I. AlGORITHMS PARAMETERS

| Parameters | RGA | PSO |
| :--- | :---: | :---: |
| Population size | 120 | 120 |
| Iteration cycles | 300 | 80 |
| Crossover rate | 1 |  |
| Crossover | Two Point |  |
| Mutation rate, Mutation | 0.01, Gaussian Mutation |  |
| Selection, Selection <br> probability <br> C1 \& C2 | Roulette, $1 / 3$ |  |
| $v_{i}^{\min }, v_{i}^{\max }$ |  | 1.5 |
| $w_{\min }, w_{\max }$ |  | 1,10 |



Fig. 2. Concentric ring array with nine rings spaced $\square \lambda \square / 2$ apart and $\mathrm{d}_{\mathrm{m}} \approx \lambda / 2$.

TABLE II. RING RADIUS AND NUMBER OF ELEMENTS PER RING FOR A 9-RING CONCENTRIC CIRCULAR ANTENNA ARRAY

| $\mathbf{M}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $r_{m}(\lambda)$ | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 |
| $N_{m}$ | 6 | 12 | 18 | 25 | 31 | 37 | 43 | 50 | 56 |

TABLE III. Excitation Amplitude Distribution $\left(I_{m n}\right)$ Using RGA

| Concentric Ring No. | No. of elements in the corresponding rings | Element Excitations corresponding to the Concentric Circle |
| :---: | :---: | :---: |
| central element | 1 | 1 |
| 1 | 6 | $\begin{array}{llllll}1 & 0 & 1 & 1 & 1 & 1\end{array}$ |
| 2 | 12 | $\begin{array}{llllllllllllll}1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1\end{array}$ |
| 3 | 18 |  |
| 4 | 25 | $\begin{array}{llllllllllllllllllllll} \hline 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & & & & & & & & & & & & & & & & & & & \end{array}$ |
| 5 | 31 | $\begin{array}{llllllllllllllllllllll} \hline 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & & & & & & & & & & & & & \end{array}$ |
| 6 | 37 | $\begin{array}{ccccccccccccccccccccccc} \hline 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & & & & & & & & & \end{array}$ |
| 7 | 43 | $\begin{array}{cccccccccccccccccccccccccc} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & & & \end{array}$ |
| 8 | 50 | 1 1 1 1 1 0 1 0 1 1 1 1 0 0 1 0 0 1 1 1 1 1  <br> 0 0 1 0 0 0 0 0 1 0 1 0 0 0 0 1 1 1 1 1 0 1 0 <br> 0 0 0 0 1                   |
| 9 | 56 | 1 0 0 1 1 1 0 0 1 0 1 1 1 0 1 1 0 1 1 0 1 0 <br> 0 0 1 1 1 1 0 0 0 0 0 1 1 1 0 0 0 1 1 1 0 1 <br> 0 0 0 0 0 0 0 1 0 1 0 1           |



Fig. 3. Array pattern found by RGA for 9 rings CCAA set with central element feeding.


Fig. 4. Array pattern for 9 rings CCAA set with central element feeding for PSO.

TABLE IV. Excitation Amplitude Distribution ( $I_{m n}$ ) Using PSO


TABLE V. Thinned and Fully Populated Array Results for RGA and PSO

| Parameters | thinned array (RGA) | thinned array (PSO) | Fully populated <br> array |
| :--- | :--- | :--- | :--- |
| Side Lobe Level (SLL) | -20.78 dB | -20.37 dB | -17.4 dB |
| Half-power beamwidth <br> (HPBW, in degree) | 6.37 | 6.406 | 6.2 |
| Number of elements turned off | $\mathbf{1 2 1}$ | $\mathbf{1 4 0}$ | $\mathbf{0}$ |
| Number of elements turned on | 158 | 139 | 279 |



Fig. 5. Convergence profile for RGA in case thinned CCAA


Fig. 6. Convergence profile for PSO in case thinned CCAA

## 5. Conclusions

This paper illustrates Real coded Genetic Algorithm (RGA), and Particle Swarm Optimization (PSO) for thinning of large Concentric Circular Antenna Array of isotropic elements. RGA has proved to be inferior and sub-optimal. The PSO algorithm can efficiently handle the thinning of large Concentric Circular Antenna Array of isotropic elements with a reduction to $50.18 \%$ of the total elements as used in case of a fully populated array with a simultaneous reduction in side lobe level to more than 20 dB by PSO. The half power beamwidth of the synthesized array pattern with fixed inter-element spacing is remain same as that of a fully populated array of same shape and size, but with reduced side level for all algorithms.

## References

[1] C. Stearns and A. Stewart, "An investigation of concentric ring antennas with low sidelobes," IEEE Trans. Antennas Propag., vol. 13(6), pp. 856-863, Nov. 1965.
[2] R. Das, "Concentric ring array," IEEE Trans. Antennas Propag., vol. 14(3), pp. 398-400, May 1966.
[3] N. Goto and D. K. Cheng, "On the synthesis of concentric-ring arrays," IEEE Proc., vol. 58(5), pp. 839-840, May 1970.
[4] L. Biller and G. Friedman, "Optimization of radiation patterns for an array of concentric ring sources," IEEE Trans. Audio Electroacoust., vol. 21(1), pp. 57-61, Feb. 1973.
[5] M D. A. Huebner, "Design and optimization of small concentric ring arrays," In Proc. IEEE AP-S Symp., 1978, pp. 455-458.
[6] M G. Holtrup, A. Margulnaud, and J. Citerns, "Synthesis of electronically steerable antenna arrays with element on concentric rings with reduced sidelobes," In Proc. IEEE AP-S Symp., 2001, pp. 800-803.
[7] R. L. Haupt, "Optimized element spacing for low sidelobe concentric ring arrays," IEEE Trans. Antennas Propag., vol. 56(1), pp. 266-268, Jan. 2008.
[8] R. L. Haupt, and D. H. Werner, Genetic Algorithms in Electromagnetics, IEEE Press Wiley-Interscience, 2007.
[9] M. Dessouky, H. Sharshar, and Y. Albagory, "Efficient sidelobe reduction technique for small-sized concentric circular arrays," Progress In Electromagnetics Research, vol. PIER 65, pp. 187-200, 2006.
[10] M. A. Panduro, A. L. Mendez, R. Dominguez and G. Romero, "Design of non-uniform circular antenna arrays for side lobe reduction using the method of genetic algorithms," Int. J. Electron. Commun. (AEÜ) vol. 60 pp. $713-717,2006$.
[11] K. -K. Yan and Y. Lu, "Sidelobe Reduction in Array-Pattern Synthesis Using Genetic Algorithm," IEEE Trans. Antennas Propag., vol. 45(7), pp. 1117-1122, July 1997.
[12] R. L. Haupt, "Thinned concentric ring arrays," Antennas and Propagation Society International Symposium, 2008. AP-S 2008. IEEE , pp.1-4, 5-11 July 2008.
[13] R. L. Haupt, "Thinned interleaved linear arrays," Wireless Communications and Applied Computational Electromagnetics, 2005. IEEE/ACES International Conference on, pp. 241-244, 3-7 April 2005
[14] R.C. Eberhart and Y.Shi, "Particle swarm optimization: developments, applications and resources, evolutionary computation," Proceedings of the 2001 Congress on Evolutionary Computation, vol. $1 \mathrm{pp} .81-86,2001$.
[15] D. Mandal, S. P. Ghoshal, and A. K. Bhattacharjee, "Design of Concentric Circular Antenna Array With Central Element Feeding Using Particle Swarm Optimization With Constriction Factor and Inertia Weight Approach and Evolutionary Programing Technique," Journal of Infrared Milli Terahz Waves, vol. 31 (6), pp. 667-680, 2010.
[16] D. Mandal, S. P. Ghoshal, and A. K. Bhattacharjee, "Application of Evolutionary Optimization Techniques for Finding the Optimal set of Concentric Circular Antenna Array," Expert Systems with Applications, (Elsevier), vol. 38, pp. 2942-2950, 2010.
[17] D. Mandal, S. P. Ghoshal, and A. K. Bhattacharjee, "Radiation Pattern Optimization for Concentric Circular Antenna Array With Central Element Feeding Using Craziness Based Particle Swarm Optimization," International Journal of $R F$ and Microwave Computer-Aided Engineering, vol. 20, Issue. 5, pp. 577-586, Sept. 2010.
[18] D. Mandal, S. P. Ghoshal, and A. K. Bhattacharjee, "Optimized Radii and Excitations with Concentric Circular Antenna Array for Maximum Sidelobe Level Reduction Using Wavelet Mutation Based Particle Swarm Optimization Techniques," Telecommunications System, (Springer), DOI 10.1007/s11235-011-9482-8, 2011.
[19] D. Mandal, S. P. Ghoshal, and A. K. Bhattacharjee, "Swarm Intelligence Based Optimal Design of Concentric Circular Antenna Array," Journal of Electrical Engineering, vol. 10, no. 3, pp. 30-39, 2010.

