Rationing and Pricing Strategies for Congestion Mitigation: Behavioral Theory, Econometric Model, and Application in Beijing

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Abstract

Some travel demand management policies such as road pricing have been widely studied in literature. Rationing policies, including vehicle ownership quota and vehicle usage restrictions, have been implemented in several megaregions to address congestion and other negative transportation externalities, but not well explored in literature. Other strategies such as Vehicle Mileage Fee have not been well accepted by policy makers, but attract growing research interest. As policy makers face an increasing number of policy tools, a theoretical framework is needed to analyze these policies and provide a direct comparison of their welfare implications such as efficiency and equity. However, such a comprehensive framework does not exist in literature. To bridge this gap, this study develops an analytical framework for analyzing and comparing travel demand management policies, which consists of a mathematical model of joint household vehicle ownership and usage decisions and welfare analysis methods based on compensating variation and consumer surplus. Under the assumptions of homogenous users and single time period, this study finds that vehicle usage rationing performs better when relatively small percentages of users (i.e., low rationing ratio) are rationed off the roads and when induced demand elasticity resulting from congestion mitigation is low. When the amount of induced demand exceeds a certain level, it is shown analytically that vehicle usage restrictions will always cause welfare losses. When the policy goal is to reduce vehicle travel by a fixed portion, road pricing provides a larger welfare gain. The performance of different policies is influenced by network congestion and congestibility. This paper further generalizes the model to consider heterogenous users and demonstrates how it can be applied for policy analysis on a real network after careful calibration.

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1. Introduction

Policy makers and researchers have proposed various policy tools to mitigate ever-growing congestion by allocating scarce road space more efficiently. Road pricing, the probably most well-known travel demand management policy, has been extensively studied in literature (e.g. Mohring & Harwitz, 1962; Arnott & Small, 1994; Verhoef, 2002; Zhang & Ge, 2004; Brownstone & Small, 2005) among others). The first-best pricing, when achievable, is an efficiency way to mitigate negative externality generated by traffic. However, road pricing has been controversial among the public because of its 1) similarity to another tax; 2) hefty transaction cost; 3) concerns on welfare distributional effects (Giuliano, 1994; Harrington et al., 2001; Yang et al., 2004; Zhang et al., 2008); 4) and privacy concerns due to certain fare collection technology. In contrast, rationing as a policy tool does not involve any fee transaction and has been regarded as more equitable in resource allocation by some researchers in other industries (e.g. Evans, 1983). In general, rationing, or quantity control, does not achieve the first-best situation. But they could become very useful when we are dealing with basic life necessity (e.g. water in Renwick & Archibald (1998)), or when the price elasticity is too small (Guesnerie & Roberts, 1984). However, its role as a policy tool to address congestion problem has barely been studied in literature. Several studies focused on early experience in Singapore (Koh & Lee, 1994; Toh & Phang, 1997; Smith & Chin, 1997; Koh, 2003). A few existing studies in other regions includes (Daganzo, 1995; Eskeland & Feyzioglu, 1997; Davis, 2008; Nakamura & Kockelman, 2002). The authors pointed out several limitations in current studies and recommended further research in this field.

While the effects of rationing in transportation have not been well understood, it has been implemented in several metropolitan areas. For example, Singapore started the Vehicle Quota System(VQS) in 1990 (Barter, 2005), which only releases a limited number of vehicle purchase permits each month through auctions. A similar quota system was adopted in Shanghai, China in 2001. A more drastic vehicle ownership rationing scheme, where residents can only acquire the vehicle purchase permits through monthly lottery, was recently implemented in Beijing, China (Lim, Accessed on Apr 8). Rationing policies can be applied not only to vehicle ownership, but also to vehicle usage. For example, several Latin American countries introduced vehicle usage restriction measures due to emissions and air quality concerns in metropolitan area. Mexico City administration imposed a regulation banning each car from driving on a specific day of the week according to their license plate number in 1989 (dubbed as “Day without a Car”). Similarly regulation has been adopted by Sao Paulo, Brazil, Bogotá, Columbia, Quito, Ecuador, and Santiago, Chile(Davis, 2008). As congestion and air pollution problems deteriorate, vehicle usage restriction has been extended to some smaller cities, such as Medellín and Cali, with less than two million residents each. Outside Latin America, vehicle usage restriction is also seen in Beijing, China, Manila, Philippines, Lagos, Nigeria (Thomson, 1998), Athens, Greece (Kambezidis et al., 1995), as well as Guangzhou, China during the 2010 Asian Games (Hao et al., 2010).

As new technologies such as hybrid and electric vehicles emerge, some researchers propose policies such as Vehicle Mileage Fee that could serve both as solutions to decreasing gas tax (ironically due to the increased fuel efficiency) and travel demand management tools. As policy makers face an increasing number of policy tools, a theory is needed to analyze different pricing and rationing policies under an integrated framework and provide a direct comparison of their welfare implications such as efficiency and equity. However, such a theoretical framework does not exist in literature. To bridge this theoretical gap, this study develops an analytical framework for analyzing transportation pricing and rationing policies, which consists of a mathematical model of joint household vehicle ownership and usage decisions and welfare analysis methods based on compensating variation and consumer surplus. This integrated theoretical model also supports a direct comparison between rationing policies and pricing policy, and to illustrate their difference under various conditions.

2. Theoretical Framework

For a traveler who is facing various travel demand management policies, the decision of owning a vehicle and the decision of using a vehicle are interrelated. Previous studies usually treat road pricing and vehicle ownership decisions separately, which prevents policy makers from analyzing pricing and rationing...
policies under an integrated framework. To correctly capture these behavioral dynamics in reaction to price changes and rationing policies, the proposed framework must be able to jointly model vehicle ownership and usage decisions. This study follows the indirect utility approach initiated by Dubin & McFadden (1984) in their study about residential electric appliance holdings and consumption because of its solid foundation in consumer behavior theory. Hausman (1985) developed similar framework in his work on wages and labour force participation. This approach was first introduced to the field of transportation by Mannering & Winston (1985) (an early version was presented at the 1982 Winter Meeting of the Econometric Society) in their seminal work on household vehicle ownership and utilization. They also extended this approach from static to dynamic models and addressed several econometric issues in model estimation. Train (1985) provided another early application on car ownership study. Following these early works, many researchers, including Winston & Mannering (1984); Hensher et al. (1992); De Jong (1990); Goldberg (1998), and West (2004) have further extended and applied this indirect utility approach in various context (De Jong et al. 2004) provided a review).

In this study, we consider a household who seeks to maximize its utility under a budget constraint. We consider two goods: vehicle usage \( A \) and all other goods \( X \). The household faces a discrete choice of owning vehicles and a continuous choice of vehicle usage conditional on the ownership choice. This joint decision gives the consumption of vehicle usage and determines vehicle ownership. The consumption of vehicle usage and all other goods yield positive marginal utility. Many functional forms have been used as conditional indirect utility in the literature (Mannering & Winston, 1985; Winston & Mannering, 1984; West, 2004). We choose the double log specification in this study because the utility level of choosing not to own a vehicle is endogenous with this form. This function form has previous been used by (De Borger & Mayeres, 2007) in investigating optimal taxation of optimal car ownership, car use, and public transport.

From De Jong (1990), we assume the demand for driving \( A_i \) units of distance (e.g. measured in annual Vehicle Mile Traveled (VMT)) by a household \( i \) with annual income \( Y_i \) is determined by:

\[
\ln A_i = \alpha_i \ln(Y_i - C) - \beta_i p + \eta_i + e_i
\]  

(1)

where: \( p \) is the operating cost per mile for the vehicle;
\( C \) represents the annualized capital cost of owning a car;
\( \eta_i \) summarizes observed heterogeneity such as household socio-economic and demographic characteristics;
\( e_i \) summarizes heterogeneity unobservable to researchers;
and \( \alpha_i \) and \( \beta_i \) are parameters.

Following Burtless & Hausman (1978), the corresponding indirect utility function is:

\[
V(p, Y_i - C) = \frac{1}{\beta_i} \exp(\eta_i + e_i - \beta_i p) + \frac{1}{1 - \alpha_i} (Y_i - C)^{1-\alpha_i}
\]  

(2)

We can easily verify this correspondence by applying Roy’s identity to 2, which gives the demand function 1. Equation 2 gives the maximum utility on the downward-sloping part of the budget line (where people choose to own a vehicle). For each point on that line, we have to compare its utility with the utility of owning no vehicle (which is always an option) to decide the optimal decision. To derive the utility of owning no vehicle, De Jong (1990) observed that the optimal decision would be to not drive at all when operating cost per mile \( p \) goes to infinity. Therefore, we have:

\[
\lim_{p \to \infty} V(p, Y_i) = U(0, Y_i)
\]  

(3)

which yields:

\[
U(0, Y_i) = \frac{1}{1 - \alpha_i} Y_i^{1-\alpha_i}
\]  

(4)

Here \( U(0, Y_i) \) represents the utility of owning no vehicle. Because people are not driving in this situation, the first argument, the annual VMT \( A_i \), is 0.
To avoid unnecessary complexity and to obtain important insights and theoretical findings, we will first focus on the homogeneous user group, while heterogeneity among travelers will be considered in a numerical example. Therefore, the terms summarizing the heterogeneity ($\eta$ and $e$) and the subscript for individual household will first be dropped.

Under the assumption of homogenous users, all people will choose to own a car if $V(p, Y - C) > U(0, Y)$. The demand of driving is conditional on the decision of owning a car, following equation 1. As a road becomes more congested and driving is more costly, some people may give up driving. Because we consider homogenous drivers, drivers must be indifferent between owning a car and driving $A_{\text{min}}$ miles with it, and owning no car, which is $V(p, Y - C) = U(0, Y)$. Following the above reasoning, the travel demand curve can be developed.

To get the network equilibrium and evaluate the welfare implications of different policies, a network supply function must also be specified. To maintain the tractability of the analysis, we consider a stylized network with only one link and one origin-destination pair. We adopt a flexible network performance function that allows us to consider networks with different congestability (travel time elasticities):

$$p = \phi T_0 (1 + \xi \left( \frac{q}{F} \right)^{\varphi})$$

where $\phi$ is the value of time, which converts travel time into generalized monetary travel cost (in practice, this problem could be more complicated since not all time periods are equally valuable and we do have a physical constraint of 24 hours in a day; researchers such as Jara-Diaz (2008) treated time budget separately from money budget, but this study ignores such difference for simplification); $q$ is the aggregated travel demand; and $F$ represents road capacity. $T_0$ captures the free flow time. $\xi$ and $\varphi$ are parameters. In the most common BPR function, $\xi = 0.15$, $\varphi = 4$, while for the less congestable Vickery’s model (Vickrey, 1969), $\varphi = 1$.

With both demand and supply function defined, the network equilibrium could be solved. And we use $(p^*, q^*)$ to denote the equilibrium point before any policy is implemented.


While theoretical studies on road pricing have been abundant in the literature, vehicle usage and ownership rationing policies have not been well analyzed. Therefore, before a direct comparison between these two policies, a theoretical framework must be established for the rationing policies. In this study, we will analyze the two distinct rationing policies: vehicle ownership rationing and vehicle usage rationing.

3.1. Vehicle Ownership Restriction

For vehicle ownership rationing, we consider a rationing policy that directly limits the total number of vehicles in the system (although most of such policies target the newly added vehicles through license quota, they could be interpreted as a control of total vehicles in the future). Under this regulation, only a part of the population ($\theta$) who are willing to own a vehicle can actually own one, regardless of their willingness to pay. While the $1 - \theta$ portion of potential drivers will be rationed out from the market. Although households may choose to own more than one vehicle, we keep the choice between 0 and 1 in this study to avoid further complication. However, this simplification does not prevent us from considering households with multiple cars in future study. Related work on vehicle choice problem includes those by De Jong (1997) and Rouwendal & Pommer (2004), among others.

The number of households who are willing to own a vehicle before the policy implementation is $P^*H$, which can be derived given the initial network equilibrium condition. Here $H$ is the total number of households. After the vehicle ownership restriction is implemented, the total travel demand becomes

$$q_o = \theta PHA(p)$$

Subscript $o$ represents the case of ownership rationing. To be more rigorous, a coefficient need to be introduced to convert A, the total car miles per year, to trip frequency. For theoretical analysis on a
A stylized network with only one OD and one link, this is less important since the link length is fixed and the capacity can be adjusted to yearly capacity. As we will see in following sections, this problem becomes very important when extending the theoretical framework to network analysis. More rigorous treatment will be introduced in network analysis.

By substituting 6 into equation 2 and then combining with supply function 5, the new equilibrium point, \((q_o^{**}, p_o^{**})\), can be obtained by solving the following equation set:

\[
\begin{align*}
q_o &= \theta \text{PHA}(p) = \theta \text{PH}(Y-C)^\alpha \exp(-\beta p), \\
p &= \phi T_0(1 + \xi(q_o)^2).
\end{align*}
\]

The welfare change for the \(\theta\) portion of households who no longer own a vehicle after rationing policy implementation could be captured by calculating their compensating variation \(CV_{o,1-\theta}\) through the following equation:

\[
U(0, Y - CV_{o,1-\theta}) = V(p^*, Y - C)
\]

Equation 8 ensures that by compensating each household \(CV_{o,1-\theta}\), their utility stays the same as the level before the rationing policy is implemented. Similarly, for the \(\theta\) portion of households who still keep their vehicles, compensating variation \(CV_{o,\theta}\) is calculated by following the same logic:

\[
V(p_o^{**}, Y - CV_{o,\theta} - C) = \frac{1}{\beta} \exp(-\beta p_o^{**}) + \frac{1}{1 - \alpha} (Y - C - CV_{o,\theta})^{1-\alpha} = V(p^*, Y - C)
\]

By combining the welfare impacts for these two group of households, the overall welfare change is:

\[
CV_o = PH(\theta CV_{o,\theta} + (1 - \theta)CV_{o,1-\theta})
\]

### 3.2. Vehicle Usage Rationing

For vehicle usage rationing, we consider a “Day without a Car” type of rationing policy under which each vehicle can only be on the road for \(\lambda(0 < \lambda < 1)\) portion out of all days. This is also the current practice of some metropolitan areas, including Beijing, where each vehicle can only drive four out of five workdays. It has to be pointed out that there could be a difference between \(\lambda\) of all working days and all annual miles, depending on the details of vehicle usage restriction policies. For example, the vehicle usage restriction system in Beijing is only effective between 6:00AM and 21:00PM, Monday through Friday. It does not apply during weekends and on national holidays. In this study, we ignore such policy details to keep the modeling framework tractable. Analysis of complicated policy details are left for future studies.

Vehicle usage rationing is different from decreasing the least-wanted (i.e. with the least willingness-to-pay) \(1 - \lambda\) of total travel, which is usually assumed in CS-based analysis. To address this difference, we derive the indirect utility after usage rationing as follows:

\[
V_{usage}(p, Y - C) = \lambda V(p, Y - C) + (1 - \lambda)U(0, Y - C)
\]

\[
= \frac{\lambda}{\beta} \exp(-\beta p) + \frac{1}{1 - \alpha} (Y - C)^{1-\alpha}
\]

where, \(\lambda V(p, Y - C)\) is the contribution of the \(\lambda\) portion of the days when a vehicle is allowed to be driven, while \((1 - \lambda)U(0, Y - C)\) is the contribution of the \(\lambda\) portion of the days when this vehicle usage is rationed.
Applying Roy’s Identity, we can verify that the demand function for individual household derived from this indirect utility function is consistent with our assumption of the usage rationing policy:

$$A_u(p) = \lambda A(p) = \lambda (Y - C)^\alpha \exp(-\beta p)$$  (12)

The new aggregated travel demand under rationing policy is then given by:

$$q_u(p) = PHA_u(p) = \lambda PH(Y - C)^\alpha \exp(-\beta p)$$  (13)

Travel is an induced demand which allows people to fulfill all other activities occurring at different times and locations. Under vehicle usage rationing, people cannot drive during certain days. In the short term, it may be difficult to find replacement activities during the driving days and people would stay at the same level of travel demand in days when they are allowed to drive. One example is commute driving. People are unlikely to commute longer or more often during the days when they are allowed to drive. In other words, such travel demand is not substitutable, especially for the short term. However, in the long term, people may engage in more activities during the driving days to benefit from the traffic reduction due to usage rationing policy. These induced demand may compromise the benefit from initial traffic reduction. As illustrated in Figure 1, point A represents the case where no induced demand is considered, while point B represents the case where induced demand does get considered. In Figure 1, area 1 represents welfare losses due to the amount of driving that is rationed out by the policy, while area 2 represents welfare gains for the remaining trips that benefit from the short-term travel time reduction. When induced demand is considered, more trips enter the network and the welfare changes in this scenario is illustrated in Figure 2.

![Figure 1. Consumer surplus change after usage rationing without induced demand](image)

Given the models representing the two rationing policies, we will analyze their welfare implications and compare them with the pricing policy. The integrated framework allows us to obtain some interesting findings through analytical reasoning under certain conditions. In other cases, numerical examples will be provided to provide insights.
4. Analytical Findings

**Proposition 1.** *When induced demand is taken into account, which is the solution of equation 14, vehicle usage rationing policy will always result in a user welfare loss.*

First, a new equilibrium point \((q_u^*, p_u^*)\) will be obtained from the following equations:

\[
\begin{align*}
q_u &= \lambda PHA(p) = \lambda PH (Y - C)^\alpha \exp(-\beta p), \\
p &= \phi T_0 (1 + \xi (\frac{q_u^F}{F})) (14)
\end{align*}
\]

For the \(P\) portion of population who own a vehicle before the usage rationing policy, their compensating variation is calculated using the following equation:

\[
\lambda V(p_u^{**}, Y - C - CV_{u,\lambda}) + (1 - \lambda) U(0, Y - C - CV_{u,\lambda}) = V(p^*, Y - C) (15)
\]

which is equivalent to:

\[
\frac{\lambda}{\beta} \exp(-\beta p_u^{**}) + \frac{1}{1 - \alpha} (Y - C - CV_{u,\lambda})^{1-\alpha} = \frac{1}{\beta} \exp(-\beta p^*) + \frac{1}{1 - \alpha} (Y - C)^{1-\alpha} (16)
\]

Compensation variation allows the utility of each individual household to stay at the same level. For the remaining population who do not own a vehicle before the rationing policy, their utility is unchanged since their income stays the same. Under homogenous user assumption, we do not consider any forward-looking behavior. If some households are forward-looking, then other households would do the same because of the homogeneity assumption. In equilibrium, nobody can benefit from such behavior. Therefore, total user welfare gain is then \(CV_u = PHCV_{u,\lambda}\).

**Proof.** To prove the proposition, we take first-order derivatives on both sides of 16 with respect to \(\lambda\):

\[
\frac{\partial CV_u}{\partial \lambda} = (Y - C - CV_{u,\lambda})^{\alpha} \exp(-\beta p_u^{**}) \left[\frac{1}{\beta} - \lambda \frac{\partial p_u^{**}}{\partial \lambda}\right] (17)
\]

Also we can derive \(\frac{\partial p_u^{**}}{\partial \lambda}\) from 14:
\[
\frac{\partial p_o^{**}}{\partial \lambda} = \frac{1}{\lambda [\beta + \frac{1}{\psi \phi T_o} (\frac{q_o^{**}}{F})^\alpha]} < \frac{1}{\lambda \beta}
\]

Thus, substituting 18 into 17, we see \( \forall \lambda \in [0, 1) \)

\[
\frac{\partial CV_u}{\partial \lambda} > 0
\]

However, in the extreme case, when the policy rations 0 percentage of vehicle usage \((\lambda = 1)\), the utility level should be unchanged and we have \(CV_u = 0\). Therefore, we obtain:

\[
CV_u(\lambda) < 0, \quad \forall \lambda \in [0, 1) \tag{20}
\]

**Proposition 2.** When road pricing and vehicle ownership rationing are set up in such a way that both policies reduce travel demand by the same amount (or have the same congestion mitigation effects), road pricing will always generate a bigger social welfare gain.

As discussed in our previous study, vehicle usage rationing policy will always cause a social welfare loss when induced demand is considered. Thus, in this section, we will only compare road pricing and vehicle ownership rationing (which is always superior to vehicle usage rationing policy when long-term induced demand is considered).

If we assume the collected toll will be completely used for the benefit of society and no transaction cost is applied, the overall social welfare gain with road pricing will be:

\[
W_p = \tau q_p^{**} \tag{21}
\]

where \(\tau\) represents the price to be charged. Here the toll revenue is used to measure welfare changes because it captures all the benefits brought by pricing under homogeneous user assumption.

Thus, with equation 21, we obtained the difference in social welfare gain for the two policies:

\[
\Delta W = (p_0 - p_o^{**})q_o^{**} - PH\theta CV_{o,\theta}
\]

To evaluate the sign of \(\Delta\), we only need to consider the part within the parenthesis. We define \(h(p_o^{**})\), such that:

\[
h(p_o^{**}) = (p_0 - p_o^{**})(Y - C)^\alpha \exp(-\beta p_o^{**}) - CV_{o,\theta}
\]

We take first derivatives with regard to \(p_o^{**}\) in equation 23 (this is doable because \(CV_{o,\theta}\) is a function of \(p_o^{**}\)),

\[
\frac{dh(p_o^{**})}{dp_o^{**}} = \theta PH[(-Y - C)^\alpha \exp(-\beta p_o^{**})(1 + \beta(p_0 - p_o^{**})) - \frac{\partial CV_{o,\theta}}{\partial p_o^{**}}]
\]

where \(\frac{\partial CV_{o,\theta}}{\partial p_o^{**}}\) is obtained by taking first derivatives in equation 9:

\[
\frac{\partial CV_{o,\theta}}{\partial p_o^{**}} = -(Y - C - CV_{o,\theta})^\alpha \exp(-\beta p_o^{**})
\]

Thus,

\[
\frac{dh(p_o^{**})}{dp_o^{**}} = \exp(-\beta p_o^{**})[(Y - C - CV_{o,\theta})^\alpha - (Y - C)^\alpha(1 + \beta(p_0 - p_o^{**}))] < 0
\]

Since \(h(p_0) = 0\),

\[
h(p) > 0, \forall p \in (0, p_0)
\]
Thus, following equality always holds:
\[ \Delta W > 0 \]  \hspace{1cm} (28)

5. Policy Implications from Numerical Examples

Closed form results for welfare changes are not available for several scenarios (e.g. under vehicle usage restriction policy) in previous analysis. In this section, numerical analysis will be conducted to provide additional insights on welfare implications of both rationing policies.

Ideally, model parameters should be calibrated with vehicle ownership and usage data, which is beyond the scope of this paper with a theoretical focus. In this study, we follow the parameters reported by De Jong (1990):

- Income elasticity of driving \( \alpha = 0.49 \)
- Price elasticity of driving \( \beta = 0.028 \)
- Average annual income \( Y = 35000 \)
- Vehicle Price averaged in years \( C = 2536 \)

We also set the parameters for supply-side function as: \( \varphi = 4 \) and \( \xi = 0.15 \) in equation 6, which is a typical BPR function. We assume the free flow operation cost \( \gamma T_{0} \) to be $2 and capacity \( F \) to be 14PH for convenience (as we are only considering an idealized network, the choice of these two parameters only reflects the network capacity relative to travel demand, but does not have a strict physical meaning).

We first investigate how two policies perform as the rationing ratio, the most important policy parameter, varies. Figure 3 presents the usage welfare gain with regard to the remaining portion after rationing (\( \theta \) for the case of ownership rationing and \( \lambda \) for the case of usage rationing). In other words, 0.8 on the horizontal axis means:

- For ownership rationing policy, only 80% of households who want to purchase vehicles are allowed to do so.
- For usage rationing, vehicle owners can only use their vehicles in 80% of the time (one in every five days).

For vehicle ownership rationing, when \( \theta = 0.40 \), which means to ration out 60% of the vehicles, consumer welfare gain reaches the maximum.

In contrast, with vehicle usage rationing, when \( \lambda = 0.82 \), which means to limit each vehicle roughly to driving 4 out of 5 weekdays, consumer welfare gain reaches the maximum.

With the same rationing ratio, vehicle usage rationing enjoys a greater social welfare gain when rationing ratio is small (less than 0.15 under this set of parameters, right side in Figure 3), while ownership rationing policy offers a greater social welfare gain as rationing ratio increases. This is intuitive because under vehicle ownership rationing, potential drivers who are rationed out of the market are partially compensated by not paying the capital cost of buying a vehicle. However, all drivers under vehicle usage rationing have already paid the initial capital cost. As the rationing ratio increases, the service flow, or utilization, from owning a vehicle keeps on dropping from vehicle usage rationing. When the utility losses become larger than the gain from initial capital cost savings, the vehicle usage rationing policy becomes less attractive than vehicle ownership rationing. In contrast, when the rationing ratio remains small, vehicle usage rationing policy is more attractive because it offers a deeper cut of travel demand (vehicle ownership rationing is long-term, thus also leads to induced demand in our analysis). Another interesting question is how policy performance varies as network congestability differs. We expect both policies to perform better in a network where marginal cost of driving increases significantly as demand increases, compared to a single-bottleneck network where marginal cost remains constant. Here we use vehicle ownership rationing as an example. Figure 4 compares the welfare gain of ownership rationing under two supply models as rationing ratio varies from 0 to 1:

From the figure 4, we see that under the same rationing ratio, the model using BPR function as a supply function always offers a bigger consumer welfare gain compared to that using Vickrey equation. Therefore, the rationing policy can perform better under network conditions associated with higher marginal cost.
Fig. 3. Welfare gain of the two rationing policies using BPR model (only short-term effect for vehicle usage restriction policy is considered since it always leads to welfare losses when induced demand is considered)

Fig. 4. Comparison of welfare gain under ownership rationing using two supply models
Consequently, rationing policies are more likely to succeed in highly congested mega-cities where demand exceeds capacity.

6. Heterogenous Users

To support the analysis of different travel demand management policies in practice, the proposed model must be capable to consider heterogeneity in behavior and be applicable on large networks with multiple origin destination pairs. In this section, we extend the proposed modeling framework to address these issues.

In reality, households may exhibit different preferences for driving due to socio-economic and demographic characteristics or intrinsic preferences. Without losing generality, we assume such heterogeneity in behavior is captured by an error term $e_i$. To facilitate analytical analysis, we further assume $e_i \sim N(0, \sigma^2)$, although other distributions can also be adopted based on empirical evidence.

Following the modeling framework developed in this paper, the demand for driving $A_i$ units of distance for a household is determined by:

$$\ln A_i = \alpha \ln(Y - C) - \beta p + e_i$$  \hspace{1cm} (29)

and the corresponding indirect utility function is:

$$V(p, Y - C) = \frac{1}{\beta} \exp(e_i - \beta p) + \frac{1}{1 - \alpha}(Y - C)^{1 - \alpha}$$  \hspace{1cm} (30)

Following De Jong (1990), we can derive the utility of owning no vehicle by the same logic:

$$\lim_{p \to \infty} V(p, Y) = U(0, Y)$$  \hspace{1cm} (31)

which yields:

$$U(0, Y) = \frac{1}{1 - \alpha} Y^{1 - \alpha}$$  \hspace{1cm} (32)

Thus the probability of owning a car is:

$$P(p) = P(\frac{1}{1 - \alpha}(Y - C)^{1 - \alpha} + \frac{1}{\beta} \exp(e_i - \beta p) \geq \frac{1}{1 - \alpha} Y^{1 - \alpha})$$  \hspace{1cm} (33)

Denote:

$$\zeta = \log[Y^{1 - \alpha} - (Y - C)^{1 - \alpha}] - \log(1 - \alpha) + \log \beta$$  \hspace{1cm} (34)

Then,

$$P(p) = 1 - \Phi(\frac{\zeta + \beta p}{\sigma})$$  \hspace{1cm} (35)

Also, because

$$\ln A_i = \alpha \log(Y - C) - \beta p + e_i$$  \hspace{1cm} (36)

Average travel demand (e.g. measured in VMT) for the driving population is:

$$\bar{A}(p) = \mathbb{E}(A_i | e_i \geq \zeta + \beta p)$$

$$= \frac{e^{\frac{\zeta + \beta p}{\sigma}}[1 - \Phi(\frac{\zeta + \beta p - \sigma^2}{\sigma})]}{1 - \Phi(\frac{\zeta + \beta p}{\sigma})}$$  \hspace{1cm} (37)

Combining 35 and 38, the aggregated demand is:

$$q(p) = H(Y - C)^{1 - \alpha} e^{\frac{\zeta + \beta p - \sigma^2}{\sigma}}[1 - \Phi(\frac{\zeta + \beta p - \sigma^2}{\sigma})]$$  \hspace{1cm} (39)

We adopt the same supply function as in the previous analysis to derive the network equilibrium. We assume $\sigma$ takes a value of 0.35, which is consistent with (De Jong, 1990), although further calibration is required before any application on real policy problems. Figure 5 illustrates the new aggregated travel demand curve and equilibrium state where the demand and supply curves intersect.
7. Application on Beijing Network

After extending the modeling framework to consider heterogeneity in behavior, we will further demonstrate the capacity of the extended model using Beijing sketch network, a fairly large network where various rationing policies have been implemented. In January, 2011, Chinese government initiated a vehicle license plate lottery system to limit the rapid growth of vehicle ownership and usage in Beijing. Each month, about 20,000 license plates will be randomly distributed to people who are willing to buy vehicles. The entry fee to participate is negligible comparing to household income so everyone who are willing to own a vehicle has equal chance to get it. Also, the plates obtained from the lottery is not transferable, which prevents the underground trading of the plates.

7.1. Vehicle Ownership Restriction through Lottery

In a future year, if no policy is implemented, \( HP(p_0) \) of the households will own a vehicle in equilibrium. The quota policy, will restrict the vehicle ownership, issuing \( \theta HP(p_0) \) vehicle plates. Here \( \theta \in (0, 1) \) will be a policy variable the authority needs to decide on.

If the authority requires everyone to participate in the plate lottery, a new equilibrium point \((p^*_o, q^*_o)\) will be achieved. From \(35\), \( HP(p^*_o) \) will enter the lottery to win the \( \theta HP(p_0) \) number of vehicle plates. From \(38\), their average VMT is \( \tilde{A}(p^*_o) \). Thus, The new aggregated demand function will be:

\[
q' = \theta H(Y - C)^a \left[ 1 - \Phi\left(\frac{\zeta + \beta p_0}{\sigma}\right)\right] e^{\frac{\zeta^2 - \beta p^*_o}{\sigma}} \left[ 1 - \Phi\left(\frac{\zeta + \beta p^*_o - \sigma^2}{\sigma}\right)\right]
\]

The new equilibrium is given by the following equation set:

\[
\begin{align*}
q^*_o &= \theta H(Y - C)^a \left[ 1 - \Phi\left(\frac{\zeta + \beta p_0}{\sigma}\right)\right] e^{\frac{\zeta^2 - \beta p^*_o}{\sigma}} \left[ 1 - \Phi\left(\frac{\zeta + \beta p^*_o - \sigma^2}{\sigma}\right)\right] \\
p^*_o &= \phi T_0 (1 + \xi \left(\frac{q^*_o}{F}\right)^\nu).
\end{align*}
\]
Since \( p_o^* < p_o \), \( P(p_o^*) > P(p_o) \). The probability of winning the lottery \( \epsilon \) is:

\[
\epsilon = \frac{\theta[1 - \Phi(\frac{\zeta + \beta p_o}{\sigma})]}{\frac{\zeta + \beta p_o}{\sigma}} < \theta
\]

(42)

For each of the households that initially doesn’t own vehicles but enter the lottery system now and actually win it, their \( e_i \in [\zeta + \beta p_o, \zeta + \beta p_o] \). This type of households exists because under heterogenous user assumption, households who chose not to drive before, but whose utility structure is close to the decision of owning a vehicle may now find driving is more beneficial after the policy is implemented. Such forward-looking behavior is considered in this analysis, although more behavioral study is needed to test this assumption.

Since the probability of winning the lottery is given by 42, the total number of such households is

\[
e H(\Phi(\frac{\zeta + \beta p_o}{\sigma}) - \Phi(\frac{\zeta + \beta p_o}{\sigma}))
\]

By aggregating the \( CV_i \)'s, the CV change from this group of households \( CV_o \) is:

\[
CV_o = e H \int_{\zeta + \beta p_o}^{\zeta + \beta p_o} \{Y - C - [(Y^{1-\alpha} - \frac{1 - \alpha}{\beta} \exp(e_i - \beta p_o)]^{\frac{1}{\alpha}} - \frac{1}{\sqrt{2\pi}\sigma}]e^{\frac{e_i^2}{2\sigma^2}} de_i
\]

(44)

Then consider the households who initially own vehicles, their \( e_i \in [\zeta + \beta p_o, +\infty] \). The total number of such households is \( H(1 - \Phi(\frac{\zeta + \beta p_o}{\sigma})) \)

For the \( 1 - \epsilon \) portion of them who fail the lottery, their individual compensating variation \( CV_i \) is given by the following equation:

\[
\frac{1}{1 - \alpha}(Y - C)^{1 - \alpha} + \frac{1}{\beta} \exp(e_i - \beta p_o) = \frac{1}{1 - \alpha}(Y - CV_i^{1 - \alpha})
\]

(45)

By aggregating the \( CV_i \)'s, the CV change from this group of households (that would have owned a car without the lottery system, but do not win the license plate lottery) \( CV_2 \) is:

\[
CV_2 = (1 - \epsilon) H \int_{\zeta + \beta p_o}^{+\infty} \{Y - [(Y - C)^{1 - \alpha} + \frac{1 - \alpha}{\beta} \exp(-\beta p_0 + e_i)]^{\frac{1}{\alpha}} - \frac{1}{\sqrt{2\pi}\sigma}]e^{\frac{e_i^2}{2\sigma^2}} de_i
\]

(46)

For the \( \epsilon \) portion of them who actually win the lottery, their individual compensating variation \( CV_3 \) is given by the following equation:

\[
\frac{1}{1 - \alpha}(Y - C)^{1 - \alpha} + \frac{1}{\beta} \exp(e_i - \beta p_o) = \frac{1}{1 - \alpha}(Y - C - CV_3^{1 - \alpha} + \frac{1}{\beta} \exp(e_i - \beta p_o))
\]

(47)

By aggregating the \( CV_3 \)'s, the CV change from this group of households (that would have owned a car without the lottery system, and luckily win the license plate lottery after the policy is implemented) \( CV_3 \) is:

\[
CV_3 = e H \int_{\zeta + \beta p_o}^{+\infty} \{(Y - C) - [(Y - C)^{1 - \alpha} - \frac{1 - \alpha}{\beta} \exp(\epsilon p_o - e_i - \beta p_0)]^{\frac{1}{\alpha}} - \frac{1}{\sqrt{2\pi}\sigma}]e^{\frac{e_i^2}{2\sigma^2}} de_i
\]

(48)

The CV change for the whole society is thus:

\[
CV_o = CV_o + CV_2 + CV_3
\]

(49)

\( CV_o \) and \( CV_3 \) are positive because of the mitigation of congestion by the implementation of the quota system. \( CV_2 \) is negative due to the depriving of users from owning a vehicle.
7.2. Numerical Example

This study seeks to demonstrate the proposed theoretical framework for travel demand management policy analysis on large network. We use the Beijing Sketch Network (Figure 6), which contains 153 nodes, 544 links, and 153*153 OD pairs, for illustration.

Fig. 6. Beijing Sketch Network

To be applicable for policy analysis in practice, models need to be carefully calibrated using locally collected data. However, to calibrate a vehicle ownership model in a large city such as Beijing is very complicated and is beyond the scope of this paper. For demonstration purpose, we use the 2010 census data of Beijing and assume the population are evenly distributed within each District. We assume all VMT will be equally allocated to 235 working days (52*5-25(holidays)) by each driver. For other parameters, we will follow assumptions in section 5 and leave further calibration efforts to future research.

One common challenge for integrated models such as the one proposed in this study is how to bridge individual travel demand analysis with aggregate network statistics. According to the modeling framework, demand for driving between each OD pair is decided by the average operating cost. The aggregated driving demand (annual VMT) will then be translated into daily travel demand (trips), using average trip distance derived by certain path usage assumptions (e.g. shortest path). However, after the traffic assignment is conducted, the path flow patterns may be very different from initial assumptions, leading to a total VMT different from aggregate individual choices. This inconsistency requires us to reinvestigate the number of trips using the new path usage pattern and the average trip distance associated with them. To get the path flow pattern from traffic assignment, this study uses the Gradient Project (GP) algorithm (Chen et al., 2002). Although the path-based assignment algorithm is known for its non-uniqueness problem with assignment results and a more integrated framework is preferable to ensure consistency between individual analysis and macroscopic traffic assignment, this iterative process provides a practical solution with reasonable computing efficiency. We leave further investigation in this field for future research.

After any changes in network conditions, the average operating cost between each OD pair will change. This leads to new vehicle usage and ownership decisions for each individual. The aggregated individual
choices will decide new network conditions collectively, which will in turn affect congestion patterns and operating cost. Therefore, people will adjust individual choices and new patterns will then be formed. This iterative process will continue until nobody is willing to make changes. Welfare changes can then be calculated. Figure 7 illustrates the framework for policy analysis on network.

Fig. 7. Framework for Travel Demand Management Policy Analysis

Following the proposed framework, we investigated the welfare implications of the license plate lottery on Beijing Sketch Network. The average volume/capacity ratio before any rationing policy is implemented is 0.80. Figure 8 illustrates welfare changes under different rationing ratio. The x axis represents the remaining portion of drivers compared to the population who are willing to own a car before the lottery policy is implemented. Under current assumptions of model parameter and network settings, the lottery policy will always lead to a welfare loss. However, more model calibration work with local data is required before any application in local policy analysis. Moreover, as illustrated in section 5, vehicle ownership restriction may lead to welfare gains under certain congestion levels. Also, travelers between different origin destination pairs may have very different experiences, ranging from huge welfare improvements to significant welfare losses. Further investigation in this direction may help to address equity concerns of various travel demand management policies. Although only demonstrated through hypothetical scenarios, with more calibration efforts, the modeling framework proposed in this study will become a powerful tool to support various policy analysis on large-scale metropolitan network.

8. Conclusions

As adverse environmental and economic impacts of excessive vehicle usage become more severe, some mega-cities adopt vehicle usage and/or vehicle ownership rationing policies to directly control overall travel
demand. These policies have not been adequately studied in literature. To bridge this gap, this study proposes to analyze rationing policies by extending the joint model of vehicle ownership and mileage models. The numerical analysis in this study suggests that the vehicle usage rationing policies can yield a higher welfare gain when rationing ratio remains small, while vehicle ownership rationing becomes more advantageous when the rationing ratio becomes sufficiently large.

Analysis in this paper shows that with homogeneous users and the assumed demand function, the vehicle usage could yield short term (when the induced demand is minimal and when travelers have not adapted themselves to the new system by exploring various loopholes) social welfare gains, but will unavoidably lead to long-term (when travelers have fully adapted themselves to the new system and explored all opportunities to maximize their personal utility) social welfare losses. This study also compares rationing policies with the more popular road pricing policy. When both policies achieve the same congestion mitigation effects, road pricing will always generate a bigger social welfare gain if no transaction cost is assumed.

Equity issues are a big concern for travel demand management policies. People usually argue against pricing policy by pointing out that it favors people with high income. Models that are applicable for real world policy analysis must be able to consider heterogeneity among travelers and large networks with multiple OD pairs. This study extended the theoretical framework to address these issues and its potentials are demonstrated through applications on the Beijing sketch network. Various issues about modeling consistency are discussed and an iterative approach is applied to provide practical solutions. However, to draw convincing conclusions about policy implications, models need to be carefully calibrated with locally collected data, which represents important directions for future research. If well calibrated and carefully applied, models built in this study could provide a powerful theoretical framework for analyzing a set of policies that is attracting growing interest.

References


Appendix A. Summary of Notation

$\alpha$: Income elasticity of driving;
$\beta$: Price elasticity of driving;
$\eta$: household socio-economic and demographic characteristics;
$\phi$: Value of time;
$1 - \theta$: Vehicle ownership rationing ratio;
$1 - \lambda$: Vehicle usage rationing ratio;
$\xi, \varphi$: Parameters in BPR function;
$A(p)$: Annual VMT by a household;
$A_o(p)$: Annual household VMT after vehicle usage rationing;
$C$: Annualized capital cost of owning a car;
$CS_u$: Aggregated consumer surplus gain after vehicle usage rationing without induced demand;
$CV_{o,0}$: Compensating variation for individual household who keeps their vehicles after vehicle ownership rationing;
$CV_{o,1-\theta}$: Compensating variation for individual household who no longer owns vehicles after rationing;
$CV_o$: Aggregated compensating variation after vehicle ownership rationing;
$CV_{u,\lambda}$: Compensating variation for individual driving household after vehicle usage rationing with induced demand;
$CV_u$: Aggregated compensating variation after vehicle usage rationing with induced demand;
$F$: Road capacity;
$H$: Number of households;
$p$: operation cost per mile;
$p_0$: Maximal operation cost beyond which nobody is willing to drive;
$P$: Percentage of households who own vehicles;
$q$: Aggregated travel demand;
$q_0$: Total driving amount at the critical price $p_0$;
$q_u$: Aggregated travel demand under vehicle usage rationing;
$(q^*, p^*)$: Network equilibrium point;
$(q_o^*, p_o^*)$: Network equilibrium point after vehicle ownership rationing;
$(q_u^*, p_u^*)$: Equilibrium point under vehicle usage rationing with induced demand;
$t(q)$: Travel time;
$T_0$: Free-flow travel time;
$U(A, X)$: Direct utility function;
$V(p, Y - C)$: Indirect utility function;
$V_{usage}(p, Y - C)$: Indirect utility function after vehicle usage rationing;
$Y$: Annual household income;