



# Seesaw neutrino mass and new $U(1)$ gauge symmetry

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## ARTICLE INFO

### Article history:

Received 3 November 2008

Received in revised form 2 January 2009

Accepted 8 January 2009

Available online 14 January 2009

Editor: M. Cvetič

## ABSTRACT

The three electroweak doublet neutrinos  $\nu_{e,\mu,\tau}$  of the Standard Model may acquire small seesaw masses, using either three Majorana fermion singlets  $N$  or three Majorana fermion triplets  $(\Sigma^+, \Sigma^0, \Sigma^-)$ . It is well known that the former accommodates the  $U(1)$  gauge symmetry  $B-L$ . It has also been shown some years ago that the latter supports a new  $U(1)_X$  gauge symmetry. Here we study two variations of this  $U(1)_X$ , one for two  $N$  and one  $\Sigma$ , the other for one  $N$  and two  $\Sigma$ . Phenomenological consequences are discussed.

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## 1. Introduction

With the observation of neutrino oscillations, the question of neutrino mass is at the forefront of many theoretical studies in particle physics. A minimal (and essentially trivial) solution is to add three neutral fermion singlets  $N_R$  (commonly referred to as right-handed neutrinos) so that the famous canonical seesaw mechanism, i.e.  $m_\nu \simeq -m_D^2/m_N$ , is realized, where  $m_D$  is the Dirac mass linking  $\nu_L$  to  $N_R$  and  $m_N$  is the heavy Majorana mass of  $N_R$ . On the other hand, this is not the only way to realize the generic seesaw mechanism which is implicit in the unique dimension-five effective operator [1]

$$\mathcal{L}_5 = -\frac{f_{ij}}{2\Lambda} (v_i \phi^0 - l_i \phi^+) (v_j \phi^0 - l_j \phi^+) + \text{H.c.} \quad (1)$$

for obtaining small Majorana masses in the Standard Model (SM) of particle interactions. In fact, there are three tree-level (and three generic one-loop) realizations [2]. The second most often considered mechanism for neutrino mass is that of a scalar triplet  $(\xi^{++}, \xi^+, \xi^0)$ , whereas the third tree-level realization, i.e. that of a fermion triplet  $(\Sigma^+, \Sigma^0, \Sigma^-)$  [3], has not received as much attention. However, it may be essential for gauge-coupling unification [4–7] in the SM, and be probed [8–10] at the Large Hadron Collider (LHC). It is also being discussed in a variety of other contexts [11–15]. A new  $U(1)$  gauge symmetry [16–18] is another remarkable possibility, and in this Letter we study in some detail two versions of this extension, one with two  $N$  and one  $\Sigma$ , the other one  $N$  and two  $\Sigma$ .

**Table 1**

Fermion content of proposed model.

Fermion	$SU(3)_C \times SU(2)_L \times U(1)_Y$	$U(1)_X$
$(u, d)_L$	(3, 2, 1/6)	$n_1$
$u_R$	(3, 1, 2/3)	$n_2$
$d_R$	(3, 1, -1/3)	$n_3$
$(\nu, e)_L$	(1, 2, -1/2)	$n_4$
$e_R$	(1, 1, -1)	$n_5$
$N_R$	(1, 1, 0)	$n_6$
$(\Sigma^+, \Sigma^0, \Sigma^-)_R$	(1, 3, 0)	$n_6$

## 2. New $U(1)$ gauge symmetry

Consider the fermions of the SM plus  $N$  and  $\Sigma$  under a new  $U(1)_X$  gauge symmetry as listed in Table 1. To obtain masses for all the quarks and leptons, four Higgs doublets  $\Phi_i = (\phi^+, \phi^0)_i$  with  $U(1)_X$  charges  $n_1 - n_3$ ,  $n_2 - n_1$ ,  $n_4 - n_5$ , and  $n_6 - n_4$  are required, but some of these may turn out to be the same, depending on the anomaly-free solutions of  $n_i$  to be discussed below. To obtain large Majorana masses for  $N$  and  $\Sigma$ , and to break  $U(1)_X$  spontaneously, the Higgs singlet  $\chi^0$  with  $U(1)_X$  charge  $-2n_6$  or  $2n_6$  will also be required.

Assuming three families of quarks and leptons and the number of  $N$  and  $\Sigma$  to be  $n_N$  and  $n_\Sigma$  with  $n_N + n_\Sigma = 3$ , we consider the conditions for the absence of the axial-vector anomaly [19–21] in the presence of  $U(1)_X$  [16].

$$[SU(3)]^2 U(1)_X: \quad 2n_1 - n_2 - n_3 = 0, \quad (2)$$

$$[SU(2)]^2 U(1)_X: \quad (9/2)n_1 + (3/2)n_4 - 2n_\Sigma n_6 = 0, \quad (3)$$

$$[U(1)_Y]^2 U(1)_X: \quad (1/6)n_1 - (4/3)n_2 - (1/3)n_3 \\ + (1/2)n_4 - n_5 = 0, \quad (4)$$

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**Table 2** $U(1)_X$  properties of Models (A) to (D).

Model	$N_R$	$\Sigma_R$	$n_6$	$n_1 - n_3 = n_2 - n_1$	$n_4 - n_5$	$n_6 - n_4$
(A)	3	0	$-4n_1 + n_2$	$n_2 - n_1$	$n_2 - n_1$	$n_2 - n_1$
(B)	2	1	$(3/4)(3n_1 + n_4)$	$(3/4)(n_1 - n_4)$	$(1/4)(9n_1 - n_4)$	$(1/4)(9n_1 - n_4)$
(C)	1	2	$(3/8)(3n_1 + n_4)$	$(3/4)(n_1 - n_4)$	$(1/4)(9n_1 - n_4)$	$(1/8)(9n_1 - 5n_4)$
(D)	0	3	$(1/4)(3n_1 + n_4)$	$(3/4)(n_1 - n_4)$	$(1/4)(9n_1 - n_4)$	$(3/4)(n_1 - n_4)$

$$U(1)_Y[U(1)_X]^2: n_1^2 - 2n_2^2 + n_3^2 - n_4^2 + n_5^2 = 0, \quad (5)$$

$$[U(1)_X]^3: 3[6n_1^3 - 3n_2^3 - 3n_3^3 + 2n_4^3 - n_5^3] - (3n_\Sigma + n_N)n_6^3 = 0. \quad (6)$$

Furthermore, the absence of the mixed gravitational-gauge anomaly [22–24] requires the sum of  $U(1)_X$  charges to vanish, i.e.

$$U(1)_X: 3[6n_1 - 3n_2 - 3n_3 + 2n_4 - n_5] - (3n_\Sigma + n_N)n_6 = 0. \quad (7)$$

Since the number of  $SU(2)_L$  doublets remains even (it is in fact unchanged), the global  $SU(2)$  chiral gauge anomaly [25] is absent automatically.

Eqs. (2), (4), and (5) do not involve  $n_6$ . Together they allow two solutions:

$$(I) \quad n_4 = -3n_1,$$

$$(II) \quad n_2 = (7n_1 - 3n_4)/4. \quad (8)$$

In the case of solution (I), if  $n_\Sigma \neq 0$ , then Eq. (3) implies  $n_6 = 0$ , from which it can easily be seen that  $U(1)_X$  is proportional to  $U(1)_Y$ , i.e. no new gauge symmetry is obtained. If  $n_\Sigma = 0$ , then  $n_3 = 2n_1 - n_2$  and  $n_5 = -2n_1 - n_2$ , and Eqs. (6) and (7) become

$$3(-4n_1 + n_2)^3 - n_N n_6^3 = 0, \quad (9)$$

$$3(-4n_1 + n_2) - n_N n_6 = 0. \quad (10)$$

For  $n_N = 3$ , we obtain  $n_6 = -4n_1 + n_2$  which has two independent solutions:  $n_1 = 1/6$  and  $n_2 = 2/3$  imply  $U(1)_Y$ , whereas  $n_1 = n_2 = 1/3$  imply  $U(1)_{B-L}$  as is well known. In the case of solution (II),

$$n_3 = (n_1 + 3n_4)/4, \quad n_5 = (-9n_1 + 5n_4)/4, \quad (11)$$

and Eq. (3) yields

$$n_6 = \frac{3}{4n_\Sigma}(3n_1 + n_4). \quad (12)$$

Eqs. (6) and (7) become

$$9(3n_1 + n_4)^3/64 - (3n_\Sigma + n_N)n_6^3 = 0, \quad (13)$$

$$9(3n_1 + n_4)/4 - (3n_\Sigma + n_N)n_6 = 0. \quad (14)$$

The unique solution is thus  $n_N = 0$  and  $n_\Sigma = 3$ . However, if we insist that  $n_N = 3 - n_\Sigma \neq 0$ , then the nonzero  $[U(1)_X]^3$  and  $U(1)_X$  anomalies given by  $(n_\Sigma^3/3 - 2n_\Sigma - 3)n_6^3$  and  $(n_\Sigma - 3)n_6$  may be canceled by the addition of more singlets without affecting the other conditions. For  $n_\Sigma = 2$  ( $n_N = 1$ ), they are  $(-13/6)n_6^3$  and  $-n_6$ , which cannot be canceled by just one chiral fermion. However, a unique solution exists for two right-handed singlets of  $U(1)_X$  charges  $(-5/3)n_6$  and  $(2/3)n_6$ . Similarly, for  $n_\Sigma = 1$  ( $n_N = 2$ ), they are canceled by right-handed singlets of  $U(1)_X$  charges  $(-5/3)n_6$  and  $(-1/3)n_6$ . We list in Table 2 the resulting four models with  $n_\Sigma + n_N = 3$ , where the last three columns correspond to the  $U(1)_X$  charges of possible Higgs doublets  $\Phi_{1,2,3}$  which couple to the quarks, charged leptons, and neutrinos, respectively. Note that these extra singlets  $S_{1R,2R}$  are distinguished from  $N_R$  by their  $U(1)_X$  charges. Whereas  $N_R$  (and  $\Sigma_R$ ) are chosen to be the seesaw anchors for the Majorana neutrino masses through their couplings to the lepton doublets and a Higgs doublet with the appropriate  $U(1)_X$  charge,  $S_{1R,2R}$  are not. However,

**Table 3** $U(1)_X$  content of new particles in Model (B).

Particle	$U(1)_X$
$N_{1R}, N_{2R}, (\Sigma^+, \Sigma^0, \Sigma^-)_R$	$(3/4)(3n_1 + n_4)$
$S_{1R}$	$-(1/4)(3n_1 + n_4)$
$S_{2R}$	$-(5/4)(3n_1 + n_4)$
$(\phi^+, \phi^0)_1$	$(3/4)(n_1 - n_4)$
$(\phi^+, \phi^0)_2$	$(1/4)(9n_1 - n_4)$
$\chi_1$	$-(1/2)(3n_1 + n_4)$
$\chi_2$	$-(3/2)(3n_1 + n_4)$

in the case of Model (C),  $S_{1R}$  just happens to have the required  $U(1)_X$  charge which lets it couple to the lepton doublets through the Higgs doublet which gives rise to quark masses. Note also that we do not consider the exceptional case where one neutrino is massless, hence the number of  $N_R$  plus  $\Sigma_R$  is always set equal to three.

- (A) This is the canonical seesaw model with three singlets. Since the last three columns, corresponding to the  $U(1)_X$  assignments of the Higgs doublets  $\Phi_i$  required for quark, charged-lepton, and neutrino masses, respectively, are the same, only the one standard Higgs doublet is required.
- (D) This is the seesaw model where  $N_R$  is replaced by  $(\Sigma^+, \Sigma^0, \Sigma^-)_R$  per family. Two different Higgs doublets ( $\Phi_1 = \Phi_3$ , and  $\Phi_2$ ) are required.
- (B) Here two  $N_R$  and one  $\Sigma_R$  with the same  $U(1)_X$  assignment are present. One Higgs doublet ( $\Phi_1$ ) couples to quarks, the other ( $\Phi_2 = \Phi_3$ ) to leptons.
- (C) Here one  $N_R$  and two  $\Sigma_R$  are present. Three different Higgs doublets are required, opening up the possibility that neutrino masses are radiative, in the manner proposed first in Ref. [26].

### 3. Model with one triplet

Consider now Model (B) in more detail. In addition to the SM fermions, the other fermions and scalars are listed in Table 3. Quarks acquire masses through  $\Phi_1$  and leptons through  $\Phi_2$ . In addition, the Yukawa terms  $N_R N_R \chi_2$ ,  $\Sigma_R \Sigma_R \chi_2$ ,  $S_{1R} S_{1R} \chi_1^\dagger$ ,  $S_{1R} S_{2R} \chi_2^\dagger$ ,  $N_R S_{1R} \chi_1$ ,  $N_R S_{2R} \chi_1^\dagger$  are allowed. As  $U(1)_X$  is broken spontaneously by the vacuum expectation values  $\langle \chi_{1,2} \rangle$ , all the new fermions acquire large Majorana masses. As for the Higgs potential consisting of  $\Phi_{1,2}$  and  $\chi_{1,2}$ , it has many allowed terms. Two are of particular importance, namely  $\chi_1 \Phi_1^\dagger \Phi_2$  and  $\chi_1^3 \chi_2^\dagger$ , without which there would be two unwanted global  $U(1)$  symmetries.

The  $X$  gauge boson mixes with the  $Z$  boson of the SM because  $\phi_{1,2}^0$  transform under both  $SU(2)_L \times U(1)_Y$  and  $U(1)_X$ . It also contributes directly to quark and lepton neutral-current interactions. Therefore, its mass and coupling are constrained by present experimental data. This is common to Models (B), (C), and (D). Let  $\langle \phi_{1,2}^0 \rangle = v_{1,2}$  and  $\langle \chi_{1,2}^0 \rangle = u_{1,2}$ , then the  $2 \times 2$  mass-squared matrix spanning  $Z$  and  $X$  is given by

$$M_{ZZ}^2 = \frac{1}{2} g_Z^2 (v_1^2 + v_2^2), \quad (15)$$

$$M_{ZX}^2 = M_{XZ}^2 = \frac{3}{8} g_Z g_X (n_1 - n_4) v_1^2 + \frac{1}{8} g_Z g_X (9n_1 - n_4) v_2^2, \quad (16)$$

$$M_{XX}^2 = \frac{1}{2}g_X^2(3n_1 + n_4)^2(u_1^2 + 9u_2^2) + \frac{9}{8}g_X^2(n_1 - n_4)^2v_1^2 + \frac{1}{8}g_X^2(9n_1 - n_4)^2v_2^2. \quad (17)$$

In general, there is  $Z$ - $X$  mixing in their mass matrix, but it must be very small to satisfy present precision electroweak measurements. Of course, increasing  $M_X$  to 10 TeV or so is a possible solution, but there is also a condition for zero  $Z$ - $X$  mass mixing:  $v_2^2/v_1^2 = 3(n_4 - n_1)/(9n_1 - n_4)$ , which requires  $1 < n_4/n_1 < 9$ . For example, if  $v_1^2 = v_2^2 = v^2/2$ , then  $n_4 = 3n_1$ . In that case,

$$M_Z^2 = (1/2)g_Z^2v^2, \quad M_X^2 = 18n_1^2g_X^2(u_1^2 + 9u_2^2) + (9/2)g_X^2v^2. \quad (18)$$

However, there may also be kinetic mixing [27], unless  $U(1)_Y$  and  $U(1)_X$  are orthogonal [28], which is achieved with  $n_4/n_1 = 13/9$ . In that case, it may be avoided up to one loop. For zero mass mixing, this then requires  $v_2^2/v_1^2 = 3/17$ .

#### 4. Low-energy constraints

Precision data at the  $Z$  pole are insensitive to additional direct contributions to fermion pair production from the virtual  $X$  boson. However,  $Z$  pole data can be affected indirectly through  $Z$ - $X$  mixing, generally leading to a shift in the measured  $Z$  mass and a modification of its couplings to SM fermions. The high precision of these data and their good agreement with the SM predictions typically constrain the  $Z$ - $X$  mixing to be well below one percent [29]. For simplicity, we restrict ourselves here to the case with no mixing.

In contrast, precision measurements at energies or momentum transfers much below the electroweak scale can give strong constraints on the interactions of the  $X$  boson, comparable with or stronger than collider limits from the Tevatron (dilepton invariant mass distribution [30] and its forward-backward asymmetry [31]) and LEP 2 [32] (fermion pair production). At low energies, these are interference effects with photon exchange amplitudes which are parametrically suppressed by only two powers of the heavy boson mass, being proportional to  $M_Z^2/M_X^2$ .

In particular, the weak charge  $Q_W$  of heavy nuclei as measured in atomic parity violation (APV) is very sensitive to extra  $U(1)$  gauge bosons. Most accurately known is the weak charge of cesium, where the uncertainties of both the APV measurements [33,34] and the necessary many-body atomic structure calculations [35] are below the 0.5% level. We also include  $Q_W(Tl)$  [36,37] in our analysis. Furthermore, there is the weak charge of the electron which has been extracted by the E-158 Collaboration [38] from polarized Møller scattering at the SLC. For example, at the SM tree level one has  $Q_W^e = 1 - 4\sin^2\theta_W$ , where  $\theta_W$  is the weak mixing angle. This is modified in the presence of the  $X$  boson (and in the absence of  $Z$ - $X$  mixing), viz.,

$$Q_W^e = 1 - 4\sin^2\theta_W - \frac{g_X^2\sin^2\theta_W\cos^2\theta_W M_Z^2}{\pi\alpha M_X^2}(e_L^e - e_R^e),$$

where  $e_L = n_4$  and  $e_R = (5n_4 - 9n_1)/4$ . The weak charges of up and down quarks coherently building up the weak charges of heavy nuclei are modified in a similar way.

There are various measurements of neutrino and anti-neutrino deep inelastic scattering (DIS) cross sections, dominated by the result of the NuTeV Collaboration [39]. The original NuTeV analysis [39] assumed a symmetric strange quark sea for the parton distribution functions. Subsequently, NuTeV determined the strange-quark asymmetry experimentally and found  $S^- \equiv \int_0^1 dx x[S(x) - \bar{S}(x)] = 0.00196 \pm 0.00135 \neq 0$  [40]. As a consequence, we used

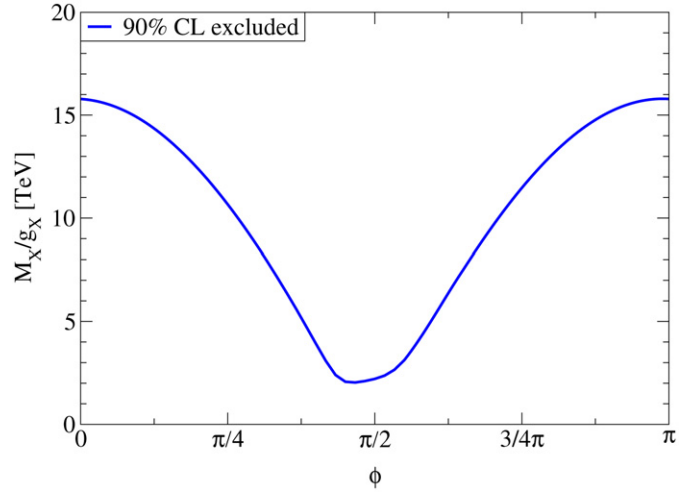


Fig. 1. Lower bound on  $M_X/g_X$  versus  $\phi = \tan^{-1}(n_4/n_1)$ .

Ref. [41] to adjust their value for the left-handed effective coupling,  $g_L^2 = 0.30005 \pm 0.00137$  to  $g_L^2 = 0.3010 \pm 0.0015$ , reducing the initial deviation from the SM of almost 3 standard deviations by about  $1\sigma$ . The right-handed coupling  $g_R^2$  and the older  $\nu$ -DIS results from CDHS [42] and CHARM [43] at CERN and CCFR [44] at FNAL are expected to exhibit shifts due to  $S^- \neq 0$  as well, but these ought to be less significant since their relative experimental uncertainties are larger. For more details, see Ref. [45].

At the one-loop level, the  $X$  boson also contributes to anomalous magnetic moments, but the effect is negligible relative to the experimental uncertainties. Finally, box diagrams containing  $X$  bosons affect tests of CKM unitarity relations, the most precise of which being  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999 \pm 0.0006$  [46]. These effects are rather small and we have not implemented these effects in our analysis.

We plot in Fig. 1 the resulting 95% confidence-level exclusion limit on  $M_X/g_X$  as a function of  $\phi$  where  $\tan\phi = n_4/n_1$  and the normalization  $n_4^2 + n_1^2 = 1$  is assumed. This means that instead of using the couplings  $g_X n_1$  and  $g_X n_4$ , we use  $g_X \cos\phi$  and  $g_X \sin\phi$ .

#### 5. Decays of $X$

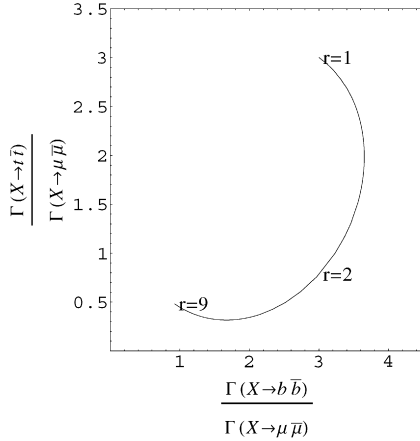
If the  $X$  gauge boson is observed at the LHC, then  $r = n_4/n_1$  may be determined empirically from its decay branching fractions into  $q\bar{q}$ ,  $l\bar{l}$ , and  $\nu\bar{\nu}$ , which will be proportional to  $3(41 - 18r + 9r^2)/8$ ,  $(81 - 90r + 41r^2)/16$ , and  $r^2$ , respectively. The ratios

$$\frac{\Gamma(X \rightarrow t\bar{t})}{\Gamma(X \rightarrow \mu\bar{\mu})} = \frac{3(65 - 42r + 9r^2)}{81 - 90r + 41r^2} \quad \text{and} \quad \frac{\Gamma(X \rightarrow b\bar{b})}{\Gamma(X \rightarrow \mu\bar{\mu})} = \frac{3(17 + 6r + 9r^2)}{81 - 90r + 41r^2} \quad (19)$$

are especially good discriminators [47], as shown in Fig. 2.

#### 6. Model with two triplets

We now examine the structure of Model (C) as shown in Table 4. The fermion content is dictated by the anomaly-free conditions for  $U(1)_X$  to consist of two triplets  $\Sigma_{1R,2R}$  and three singlets  $N_R, S_{1R,2R}$ . Quarks couple to  $\Phi_1$  and charged leptons to  $\Phi_2$ . However,  $(\nu, e)_L$  is connected to  $N_R$  and  $S_R$  through  $\Phi_3$ , and to  $S_{1R}$  through  $\Phi_1$ . To allow all particles to acquire mass, we add the four scalar singlets as shown. We then have the allowed Yukawa terms  $N_R N_R \chi_4$ ,  $\Sigma_R \Sigma_R \chi_4$ ,  $S_{1R} S_{1R} \chi_1$ ,  $N_R S_{2R} \chi_2^\dagger$ ,  $S_{1R} S_{2R} \chi_3^\dagger$ , and the allowed scalar terms  $\chi_1 \chi_2 \chi_4^\dagger$ ,  $\chi_2^\dagger \chi_1^\dagger$ ,  $\chi_3^\dagger \chi_4^\dagger$ ,  $\chi_1^\dagger \chi_2^\dagger \chi_3^\dagger$ ,  $\chi_2^\dagger \chi_4^\dagger$ ,  $\chi_1 \Phi_1^\dagger \Phi_2$ ,

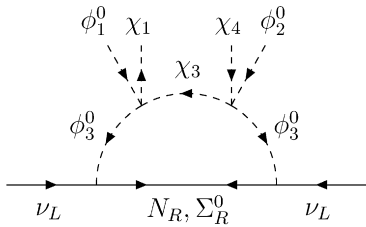


**Fig. 2.** Plot of  $\Gamma(X \rightarrow t\bar{t})/\Gamma(X \rightarrow \mu\bar{\mu})$  versus  $\Gamma(X \rightarrow b\bar{b})/\Gamma(X \rightarrow \mu\bar{\mu})$  as a function of  $r = n_4/n_1$ .

**Table 4**

$U(1)_X$  content of new particles in Model (C).

Particle	$U(1)_X$	$Z_2$
$N_R, (\Sigma^+, \Sigma^0, \Sigma^-)_{1R,2R}$	$(3/8)(3n_1 + n_4)$	-
$S_{1R}$	$(1/4)(3n_1 + n_4)$	+
$S_{2R}$	$-(5/8)(3n_1 + n_4)$	-
$(\phi^+, \phi^0)_1$	$(3/4)(n_1 - n_4)$	+
$(\phi^+, \phi^0)_2$	$(1/4)(9n_1 - n_4)$	+
$(\phi^+, \phi^0)_3$	$(1/8)(9n_1 - 5n_4)$	-
$\chi_1$	$-(1/2)(3n_1 + n_4)$	+
$\chi_2$	$-(1/4)(3n_1 + n_4)$	+
$\chi_3$	$-(3/8)(3n_1 + n_4)$	-
$\chi_4$	$-(3/4)(3n_1 + n_4)$	+



**Fig. 3.** One-loop radiative contribution to neutrino mass.

$\chi_3 \phi_3^\dagger \phi_2$ ,  $\chi_1 \chi_3 \phi_1^\dagger \phi_3$ ,  $\chi_2^2 \phi_1^\dagger \phi_2$ ,  $\chi_3 \chi_4 \phi_3^\dagger \phi_2$ . Thus the resulting Lagrangian has an automatic  $Z_2$  symmetry, which implements exactly the proposal of Ref. [26] for radiative seesaw masses and dark matter, as shown in Fig. 3. The  $3 \times 3$  Majorana neutrino mass matrix receives a tree-level contribution from the coupling of  $S_{1R}$  to a linear combination of  $\nu_i$  through  $\phi_1^0$ , as well as radiative contributions from  $N_R$  and  $\Sigma_R^0$ . This is a natural hierarchical scenario where  $\nu_3 = (\nu_\tau - \nu_\mu)/\sqrt{2}$  for example is heavier than  $\nu_{1,2}$  because the former is the one with a tree-level mass.

The lightest particle of odd  $Z_2$  [48] is now a dark-matter candidate. However, it is unlikely to be a fermion because it will have  $U(1)_X$  gauge interactions with nuclei and a cross section proportional to  $(g_X/m_X)^4$  which is likely to be too big to satisfy the upper limits from direct-search experiments and the requirement of the proper dark-matter relic abundance through its annihilation. If it is a scalar boson, such as the lighter of  $\text{Re}(\phi_3^0)$  and  $\text{Im}(\phi_3^0)$  [26, 49–52] with a mass difference greater than about 1 MeV, then it is an acceptable candidate because the lighter one is prevented from scattering to the heavier one through the  $X$  boson kinematically. On the other hand, the generic quartic scalar term for this splitting, i.e.  $(\lambda_5/2)(\Phi^\dagger \eta)^2 + \text{H.c.}$  where  $\Phi$  is even and  $\eta$  odd under  $Z_2$ ,

is not available here because of the  $U(1)_X$  charges. Nevertheless, splitting does occur in the  $4 \times 4$  mass-squared matrix spanning  $\text{Re}(\phi_3^0)$ ,  $\text{Im}(\phi_3^0)$ ,  $\text{Re}(\chi_3)$ , and  $\text{Im}(\chi_3)$ , which is of the form

$$\mathcal{M}^2 = \begin{pmatrix} m_\phi^2 & 0 & \Delta_2 + \Delta_3 & 0 \\ 0 & m_\phi^2 & 0 & \Delta_2 - \Delta_3 \\ \Delta_2 + \Delta_3 & 0 & m_\chi^2 + \Delta_1 & 0 \\ 0 & \Delta_2 - \Delta_3 & 0 & m_\chi^2 - \Delta_1 \end{pmatrix}. \quad (20)$$

Hence  $m^2[\text{Re}(\chi_3)] - m^2[\text{Im}(\chi_3)] = 2\Delta_1$ , and  $m^2[\text{Re}(\phi_3^0)] - m^2[\text{Im}(\phi_3^0)] = -4\Delta_2\Delta_3/m_\chi^2$ . As for the corresponding relic abundance, there will be contributions from the  $U(1)_X$  gauge interactions and the various allowed Yukawa terms. Note also that the  $Z_2$  symmetry for dark matter here is the conserved remnant [53–58] of  $U(1)_X$ .

## 7. Conclusion

In this Letter, we have discussed some consequences of having one or more Majorana fermion triplets ( $\Sigma^+$ ,  $\Sigma^0$ ,  $\Sigma^-$ ) as seesaw anchors of neutrino masses in the context of a  $U(1)$  extension of the SM. The associated neutral gauge boson  $X$  has prescribed couplings to the usual quarks and leptons in terms of  $g_X$  and  $\phi = \tan^{-1}(n_4/n_1)$ . The exclusion limit on  $M_X/g_X$  from low-energy data has been obtained, showing that  $X$  may be accessible at the LHC if  $g_X$  is of order  $g_Z$ . In the case of one triplet, i.e. Model (B), one Higgs doublet couples to quarks and the other to leptons. In the case of two triplets, i.e. Model (C), there is a third scalar doublet, which allows for the natural implementation of radiative neutrino masses and dark matter.

## Acknowledgements

This work was supported in part by the US Department of Energy under Grant No. DE-FG03-94ER40837, by UNAM as DGAPA-PAPIIT project IN115207, and by CONACyT (México) as project 82291-F. We thank the Institute for Nuclear Theory at the University of Washington for its hospitality and the Department of Energy for partial support during the completion of this work.

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