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Towards frame-like gauge invariant formulation for massive mixed symmetry bosonic fields

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Abstract

In this paper, as a first step towards frame-like gauge invariant formulation for massive mixed symmetry bosonic fields, we consider mixed tensors, corresponding to Young tableau with two rows with $k \geqslant 2$ boxes in the first row and only one box in the second row. We construct complete Lagrangian and gauge transformations describing massive particles in (anti) de Sitter space–time with arbitrary dimension $d \geqslant 4$ and investigate all possible massless and partially massless limits.

0. Introduction

As is well known, in $d = 4$ dimensions for the description of arbitrary spin particles it is enough to consider completely symmetric (spin-)tensor fields only. At the same time, in dimensions greater than four in many cases like supergravity theories, superstrings and higher spin theories, one has to deal with mixed symmetry (spin-)tensor fields [1–4]. There are different approaches to investigation of such fields both light-cone [5,6], as well as explicitly Lorentz covariant ones (e.g. [7–13]). For the investigation of possible interacting theories for higher spin particles as well as of gauge symmetry algebras behind them it is very convenient to use so-called frame-like formulation [14–16] (see also [17–19]) which is a natural generalization of well-known frame formulation of gravity in terms of veilbein $e_\mu{}^a$ and Lorentz connection $\omega_\mu{}^{ab}$.

Till now, most of the papers on frame-like formulation for mixed symmetry fields deal with massless case [20–25] (see however [26]). The aim of this work is to start an extension of frame-like formulation to the case of massive mixed symmetry fields. Namely, we will begin

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construction of gauge invariant formulation for such mixed symmetry massive fields in general $(A)dS_d$ space–times with non-zero cosmological constant and arbitrary space–time dimension $d \geqslant 4$. There are two general approaches to gauge invariant description of massive fields. One of them uses powerful BRST approach [11,12,27–30]. Another one, which we will follow in this work, [19,31–34] (see also [9,26,35–37]) is a generalization to higher spin fields of well-known mechanism of spontaneous gauge symmetry breaking. In this, one starts with appropriate set of massless fields with all their gauge symmetries and obtain gauge invariant description of massive field as a smooth deformation.

One of the nice feature of gauge invariant formulation for massive fields is that it allows us effectively use all known properties of massless fields serving as building blocks. There are two different frame-like formulations for massless mixed symmetry bosonic fields. For simplicity, let us restrict ourselves with mixed symmetry tensors corresponding to Young tableau with two rows. In what follows we will denote $Y(k,l)$ a tensor $\Phi^{a_1\ldots a_k,b_1\ldots b_l}$ which is symmetric both on first k as well as last l indices, completely traceless on all indices and satisfies a constraint $\Phi^{(a_1\ldots a_k,b_1)b_2\ldots b_l} = 0$, where round brackets mean symmetrization. In the first approach [21–24] for the description of $Y(k,l)$ tensor ($k \neq l$) one use a one-form $e_\mu{}^{Y(k-1,l)}$ as a main physical field. In this, only one of two gauge symmetries is realized explicitly and such approach is very well adapted for the $(A)dS$ spaces. Another formulation [25] uses two-form $e_{\mu\nu}{}^{Y(k-1,l-1)}$ as a main physical field in this, both gauge symmetries are realized explicitly. Such formalism works in flat Minkowski space while deformations into $(A)dS$ space requires introduction of additional fields [38]. In this paper we will use the second formalism. As we have already seen in all cases considered previously and we will see again in this paper, gauge invariant description of massive fields always allows smooth deformation into $(A)dS$ space without introduction of any additional fields besides those that are necessary in flat Minkowski space so that restriction mentioned above will not be essential for us.

Mixed symmetry tensor fields have more gauge symmetries compared with well-known case of completely symmetric tensors and, as a result, gauge invariant formulation for them requires more additional fields making construction much more involved. In this paper, as a first step towards gauge invariant frame-like formulation of mixed symmetry bosonic fields, we consider $Y(k,1)$ tensors for arbitrary $k \geqslant 2$. This case turns out to be special and anyway requires separate consideration. Indeed, in general case $Y(k,l)$, $l > 1$, auxiliary field analogous to Lorentz connection has to be a two-form $\omega_{\mu\nu}{}^{Y(k-1,l-1,1)}$, while for the $Y(k,1)$ case one has to introduce one-form $\omega_\mu{}^{Y(k-1,1,1)}$ instead. Thus this case turns out to be a natural generalization of the simplest model for $Y(2,1)$ tensor constructed by us before [26]. The structure of the paper is simple. In Section 1 we reproduce our results for simplest $Y(2,1)$ tensor. Then, in Section 2 we consider more complicated case – $Y(3,1)$ which shows practically all general features. At last, in Section 3 we construct massive theory for general $Y(k,1)$ tensor field. In all cases we construct complete Lagrangian and gauge transformations describing massive particles in $(A)dS_d$ spaces with arbitrary cosmological constant and arbitrary space–time dimension $d \geqslant 4$ and investigate all possible massless and partially massless limits [17,33,39–41].

1. Tensor $Y(2,1)$

In this case frame-like formulation requires two tensors [20]: two-form $\Phi_{\mu\nu}{}^a$ as a main physical field and one-form $\Omega_\mu{}^{abc}$, antisymmetric on abc, as analogue of Lorentz connection. To describe correct number of physical degrees of freedom, massless Lagrangian has to be invariant under the following gauge transformations:

$$\delta\Phi_{\mu\nu}{}^a = \partial_{[\mu}\xi_{\nu]}{}^a + \eta_{\mu\nu}{}^a, \qquad \delta\Omega_\mu{}^{abc} = \partial_\mu\eta^{abc}, \tag{1}$$

where η^{abc} is completely antisymmetric. Note that ξ-transformations are reducible, i.e.

$$\xi_\mu{}^a = \partial_\mu\chi^a \quad \Rightarrow \quad \delta\Phi_{\mu\nu}{}^a = 0.$$

One of the advantages of frame-like formulation is the possibility to construct an object ("torsion") out of first derivatives of main physical field $\Phi_{\mu\nu}{}^a$ which is invariant under ξ-transformations

$$T_{\mu\nu\alpha}{}^a = \partial_\mu\Phi_{\nu\alpha}{}^a + \partial_\alpha\Phi_{\mu\nu}{}^a + \partial_\nu\Phi_{\alpha\mu}{}^a = \partial_{[\mu}\Phi_{\nu\alpha]}.$$

To find a correct form of massless Lagrangian one can use the following simple trick. Let us consider an expression

$$\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc}T_{\nu\alpha\beta}{}^d, \qquad \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\} = \delta_a^{[\mu}\delta_b^{\mu}\delta_c^{\alpha}\delta_d^{\beta]}$$

and make a substitution $T_{\mu\nu\alpha}{}^a \to \Omega_{[\mu,\nu\alpha]}{}^a$. We obtain

$$\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc}T_{\nu\alpha\beta}{}^d \quad \Rightarrow \quad \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc}\Omega_{\nu,\alpha\beta}{}^d \quad \Rightarrow \quad \left\{{}^{\mu\nu}_{ab}\right\}\Omega_\mu{}^{acd}\Omega_\nu{}^{bcd}.$$

Thus we will look for massless Lagrangian in the form

$$\mathcal{L}_0 = a_1\left\{{}^{\mu\nu}_{ab}\right\}\Omega_\mu{}^{acd}\Omega_\nu{}^{bcd} + a_2\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc}T_{\nu\alpha\beta}{}^d.$$

It is (by construction) invariant under the ξ-transformations, while invariance under η-transformations requires $a_1 = -9a_2$. We choose $a_1 = -3$, $a_2 = \frac{1}{3}$ and obtain finally

$$\mathcal{L}_0 = -3\left\{{}^{\mu\nu}_{ab}\right\}\Omega_\mu{}^{acd}\Omega_\nu{}^{bcd} + \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc}\partial_\nu\Phi_{\alpha\beta}{}^d. \tag{2}$$

All things are very simple in a flat Minkowski space, but if one tries to consider a deformation of this theory into $(A)dS$ space then it turns out to be impossible [38]. Thus we turn to the massive particle and consider the most general case – massive particle in $(A)dS$ space with arbitrary cosmological constant. First of all, we have to determine which additional fields we need to construct gauge invariant formulation of such massive particle. In general, for each gauge transformation of main physical field we need appropriate Goldstone field but in most cases this Goldstone field turns out to be gauge field by itself so we need Goldstone fields of second order and so on. But for the mixed symmetry bosonic fields one has to take into account reducibility of gauge transformations. Let us illustrate this procedure on our present (simplest) case [9]. Our main physical field $Y(2,1)$ has two gauge transformations with parameters which are symmetric $Y(2,0)$ and antisymmetric $Y(1,1)$ tensors respectively. Thus we need two primary Goldstone fields corresponding to $Y(2,0)$ and $Y(1,1)$. Both have their own gauge transformations with vector parameter $Y(1,0)$, but due to reducibility of gauge transformations of the main field, we have to introduce one secondary Goldstone field $Y(1,0)$ only. This field has its own gauge transformation with parameter $Y(0,0)$, but due to reducibility of gauge transformations of antisymmetric second rank tensor $Y(1,1)$, the procedure stops here. It is natural to use frame-like formulation for all fields, so we introduce four pairs of tensors: $(\Omega_\mu{}^{abc}, \Phi_{\mu\nu}{}^a)$, $(\omega_\mu{}^{ab}, h_\mu{}^a)$, $(\Omega^{abc}, \Phi_{\mu\nu})$ and (ω^{ab}, h_μ).

We start with the sum of kinetic terms for all fields

$$\mathcal{L}_0 = -3\{^{\mu\nu}_{ab}\}\Omega_\mu{}^{acd}\Omega_\nu{}^{bcd} + \{^{\mu\nu\alpha\beta}_{abcd}\}\Omega_\mu{}^{abc}D_\nu\Phi_{\alpha\beta}{}^d + \{^{\mu\nu}_{ab}\}\omega_\mu{}^{ac}\omega_\nu{}^{bc} - \{^{\mu\nu\alpha}_{abc}\}\omega_\mu{}^{ab}D_\nu h_\alpha{}^c - \Omega_{abc}{}^2 + \{^{\mu\nu\alpha}_{abc}\}\Omega^{abc}D_\mu\Phi_{\nu\alpha} + \omega_{ab}{}^2 - 2\{^{\mu\nu}_{ab}\}\omega^{ab}D_\mu h_\nu \tag{3}$$

as well as appropriate set of initial gauge transformations

$$\delta_0\Phi_{\mu\nu}{}^a = D_{[\mu}\xi_{\nu]}{}^a + \eta_{\mu\nu}{}^a, \qquad \delta_0\Omega_\mu{}^{abc} = D_\mu\eta^{abc},$$
$$\delta_0 h_\mu{}^a = D_\mu\zeta^a + \chi_\mu{}^a, \qquad \delta_0\omega_\mu{}^{ab} = D_\mu\chi^{ab},$$
$$\delta_0\Phi_{\mu\nu} = D_{[\mu}\xi_{\nu]}, \qquad \delta_0 h_\mu = D_\mu\zeta, \tag{4}$$

where all partial derivatives are replaced by $(A)dS$ covariant ones. Here and in what follows, we will use the following convention on covariant derivatives:

$$[D_\mu, D_\nu]\xi^a = -\kappa\left(e_\mu{}^a\xi_\nu - e_\nu{}^a\xi_\mu\right), \quad \kappa = \frac{2\Lambda}{(d-1)(d-2)}. \tag{5}$$

Note, that due to non-commutativity of covariant derivatives such Lagrangian is not invariant under the initial gauge transformations

$$\delta_0\mathcal{L}_0 = \kappa\{^{\mu\nu}_{ab}\}\left[3(d-3)\left(2\Omega_\mu{}^{abc}\xi_\nu{}^c + \eta^{abc}\Phi_{\mu\nu}{}^c\right) - (d-2)\left(\omega_\mu{}^{ab}\zeta_\nu - \chi^{ab}h_{\mu\nu}\right)\right],$$

so we have to take this non-invariance into account later on. Now to proceed with the construction of gauge invariant formulation for massive particle, we have to add to the Lagrangian all possible cross terms of order m (i.e. with the coefficients having dimension of mass). Moreover, as our previous experience shows, we need to introduce cross terms for the nearest neighbours only, i.e. main field with primary Goldstone fields, primary fields with secondary ones and so on. For the case at hands all possible such terms could be written as follows:

$$\mathcal{L}_1 = \{^{\mu\nu\alpha}_{abc}\}\left[a_1\omega_\mu{}^{ab}\Phi_{\nu\alpha}{}^c + a_2\Omega_\mu{}^{abc}\Phi_{\nu\alpha}\right] + \{^{\mu\nu}_{ab}\}\left[a_3\Omega_\mu{}^{abc}h_\nu{}^c + a_4\Omega^{abc}\Phi_{\mu\nu}{}^c\right] + \{^{\mu\nu}_{ab}\}\left[a_5\omega_\mu{}^{ab}h_\nu + a_6\omega^{ab}\Phi_{\mu\nu}\right] + a_7\{^{\mu}_{a}\}\omega^{ab}h_\nu{}^b. \tag{6}$$

Non-invariance of these terms under the initial gauge transformations could be compensated by the following corrections to gauge transformations:

$$\delta_1\Phi_{\mu\nu}{}^a = \frac{\beta_1}{12(d-3)}e_{[\mu}{}^a\zeta_{\nu]} - \frac{3\alpha_1}{(d-3)}e_{[\mu}{}^a\xi_{\nu]}, \qquad \delta_1\Omega_\mu{}^{abc} = \frac{\beta_1}{6(d-3)}e_\mu{}^{[a}\chi^{bc]},$$
$$\delta_1 h_\mu{}^a = \beta_1\xi_\mu{}^a + \frac{4\rho_0}{d-2}e_\mu{}^a\zeta, \qquad \delta_1\omega_\mu{}^{ab} = -\frac{\beta_1}{2}\eta_\mu{}^{ab},$$
$$\delta_1\Phi_{\mu\nu} = \alpha_1\xi_{[\mu,\nu]}, \qquad \delta_1\Omega^{abc} = -3\alpha_1\eta^{abc},$$
$$\delta_1 h_\mu = \rho_0\zeta_\mu + \beta_0\xi_\mu, \qquad \delta_1\omega^{ab} = -2\rho_0\chi^{ab}, \tag{7}$$

provided

$$a_1 = a_3 = \frac{\beta_1}{2}, \qquad a_2 = a_4 = -3\alpha_1, \qquad a_5 = a_7 = 4\rho_0, \qquad a_6 = \beta_0.$$

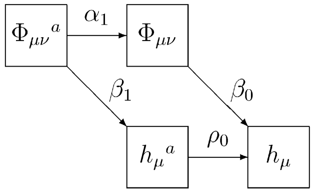

Fig. 1. General massive theory for $Y(2,1)$ tensor.

Thus we have $\delta_0\mathcal{L}_1 + \delta_1\mathcal{L}_0 = 0$ and this leaves us with variations of order m^2 (taking into account non-invariance of kinetic terms due to non-commutativity of covariant derivatives) $\delta_0\mathcal{L}_0 + \delta_1\mathcal{L}_1$. In general, to compensate this non-invariance one has to introduce mass-like terms into the Lagrangian as well as appropriate corrections for gauge transformations. But in this case there are no possible mass-like terms (the only possible term $\{{}^{\mu\nu}_{ab}\}h_\mu{}^a h_\nu{}^b$ is forbidden by ζ-invariance). Note that if we go from frame-like to metric-like formulation by solving algebraic equations for auxiliary fields $\Omega_\mu{}^{abc}$, $\omega_\mu{}^{ab}$ and Ω^{abc} then we obtain mass-like terms exactly as in [9]. Nevertheless, it turns out to be possible to achieve complete invariance without any explicit mass-like terms just by adjusting the values for our four main parameters α_1, β_1, β_0 and ρ_0. We obtain

$$\rho_0 = \sqrt{\frac{3(d-2)}{4(d-3)}}\alpha_1, \qquad \beta_0 = -\sqrt{\frac{3(d-2)}{4(d-3)}}\beta_1, \qquad \beta_1^2 - 36\alpha_1^2 = -12\kappa(d-3).$$

Now we are ready to analyze results obtained. First of all, recall that there is no strict definition of what is mass in $(A)dS$ space (see e.g. discussion in [42]). Working with gauge invariant description of massive particles it is natural to define massless limit as a limit where all Goldstone fields decouple from the main gauge field. Such a limit, if it exists at all, leads to the particle having exactly the same number of physical degrees of freedom as massless particle in flat Minkowski space. To make analyze more transparent, let us give here a Fig. 1 showing the roles played by our four parameters. One can easily see that massless limit is a limit where both $\alpha_1 \to 0$ and $\beta_1 \to 0$ simultaneously. But from the last relation above it is immediately follows that such a limit is possible in flat Minkowski space ($\kappa = 0$) only. For non-zero values of cosmological constant one can obtain partially massless limits instead.[1] Indeed, in AdS space ($\kappa < 0$) one can put $\alpha_1 = 0$ (and this gives $\rho_0 = 0$). Then our system decomposes into two disconnected subsystems. One of them with the fields $\Phi_{\mu\nu}{}^a$ and $h_\mu{}^a$ describe partially massless theory [38] with the Lagrangian

$$\mathcal{L} = \mathcal{L}_0(\Phi_{\mu\nu}{}^a) + \mathcal{L}_0(h_\mu{}^a) + \frac{\beta_1}{2}\{{}^{\mu\nu\alpha}_{abc}\}\omega_\mu{}^{ab}\Phi_{\nu\alpha}{}^c + \frac{\beta_1}{2}\{{}^{\mu\nu}_{ab}\}\Omega_\mu{}^{abc}h_\nu{}^c, \tag{8}$$

which is invariant under the following gauge transformations:

$$\delta\Phi_{\mu\nu}{}^a = D_{[\mu}\xi_{\nu]}{}^a + \eta_{\mu\nu}{}^a + \frac{\beta_1}{12(d-3)}e_{[\mu}{}^a\zeta_{\nu]},$$

$$\delta\Omega_\mu{}^{abc} = D_\mu\eta^{abc} + \frac{\beta_1}{6(d-3)}e_\mu{}^{[a}\chi^{bc]},$$

[1] Here and in what follows we will call particle to be partially massless if its number of physical degrees of freedom lies between that of massless and massive one. Such particles correspond to irreducible representations of (anti) de Sitter group that have no analogue among irreducible representations of Poincaré group.

$$\delta h_\mu{}^a = D_\mu \zeta^a + \chi_\mu{}^a + \beta_1 \xi_\mu{}^a, \qquad \delta \omega_\mu{}^{ab} = D_\mu \chi^{ab} - \frac{\beta_1}{2} \eta_\mu{}^{ab}, \tag{9}$$

where $\beta_1{}^2 = -12\kappa(d-3)$. In this, two other fields $\Phi_{\mu\nu}$ and h_μ provide gauge invariant description of massive antisymmetric second rank tensor field. In turn, in dS space ($\kappa > 0$) one can put $\beta_1 = 0$ (and this gives $\beta_0 = 0$). In this case our system also decompose into two disconnected subsystems. One of them with the fields $\Phi_{\mu\nu}{}^a$ and $\Phi_{\mu\nu}$ gives another example of partially massless theory with the Lagrangian

$$\mathcal{L} = \mathcal{L}_0\big(\Phi_{\mu\nu}{}^a\big) + \mathcal{L}_0(\Phi_{\mu\nu}) - 3\alpha_1 \left\{{}^{\mu\nu\alpha}_{abc}\right\} \Omega_\mu{}^{abc} \Phi_{\nu\alpha} - 3\alpha_1 \left\{{}^{\mu\nu}_{ab}\right\} \Omega^{abc} \Phi_{\mu\nu}{}^c, \tag{10}$$

which is invariant under the following gauge transformations:

$$\begin{aligned} &\delta \Phi_{\mu\nu}{}^a = D_{[\mu} \xi_{\nu]}{}^a + \eta_{\mu\nu}{}^a - \frac{3\alpha_1}{(d-3)} e_{[\mu}{}^a \xi_{\nu]}, \qquad \delta \Omega_\mu{}^{abc} = D_\mu \eta^{abc}, \\ &\delta \Phi_{\mu\nu} = D_{[\mu} \xi_{\nu]} + \alpha_1 \xi_{[\mu,\nu]}, \qquad \delta \Omega^{abc} = -3\alpha_1 \eta^{abc}, \end{aligned} \tag{11}$$

where $3\alpha_1{}^2 = \kappa(d-3)$. In this, two other fields $h_\mu{}^a$ and h_μ provide gauge invariant description of partially massless spin 2 particle [17,19,26].

2. Tensor $Y(3,1)$

As we have already mentioned in the Introduction, frame-like formulation for massless $Y(k,1)$ tensors turns out to be special because it requires that auxiliary field be one form and not two form field. The Lagrangian for such massless $Y(k,1)$ tensors could be, in principle, extracted from the general formula (3.21) of [25], though the only explicit example given there deals with $Y(2,1)$ case. Anyway, it can be easily constructed as a natural generalization of the simplest example given above. Namely, we introduce two-form $\Phi_{\mu\nu}{}^{ab}$ which is symmetric and traceless on ab as a main physical field as well as auxiliary one-form $\Omega_\mu{}^{abc,d}$ which is completely antisymmetric on abc, traceless and satisfies a constraint $\Omega_\mu{}^{[abc,d]} = 0$. To provide correct number of physical degrees of freedom massless Lagrangian has to be invariant under the following gauge transformations:

$$\delta \Phi_{\mu\nu}{}^{ab} = \partial_{[\mu} \xi_{\nu]}{}^{ab} + \eta_{\mu\nu}{}^{(a,b)}, \qquad \delta \Omega_\mu{}^{abc,d} = \partial_\mu \eta^{abc,d}. \tag{12}$$

Here $\xi_\mu{}^{ab}$ is symmetric and traceless on ab, while $\eta^{abc,d}$ has the same properties on local indices as $\Omega_\mu{}^{abc,d}$. Note, that these gauge transformations are also reducible

$$\xi_\mu{}^{ab} = \partial_\mu \chi^{ab} \quad \Rightarrow \quad \delta \Phi_{\mu\nu}{}^{ab} = 0.$$

To construct appropriate massless Lagrangian we will use the same trick as before. We introduce a "torsion" tensor $T_{\mu\nu\alpha}{}^{ab} = \partial_{[\mu} \Phi_{\nu\alpha]}{}^{ab}$ which is invariant under ξ-transformations, consider an expression $\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\} \Omega_\mu{}^{abc,e} T_{\nu\alpha\beta}{}^{de}$ and make a substitution $T_{\mu\nu\alpha}{}^{ab} \Rightarrow \Omega_{[\mu,\nu\alpha]}{}^{(a,b)}$. We obtain

$$\begin{aligned} \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\} \Omega_\mu{}^{abc,e} T_{\nu\alpha\beta}{}^{de} \quad &\Rightarrow \quad \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\} \Omega_\mu{}^{abc,e} \big(\Omega_{\nu,\alpha\beta}{}^{d,e} + \Omega_{\nu,\alpha\beta}{}^{e,d}\big) \\ &\Rightarrow \quad \left\{{}^{\mu\nu}_{ab}\right\} \big[3\Omega_\mu{}^{acd,e} \Omega_\nu{}^{bcd,e} + \Omega_\mu{}^{cde,a} \Omega_\nu{}^{cde,b}\big]. \end{aligned}$$

Thus we will look for the massless Lagrangian in the form

$$\mathcal{L}_0 = a_1 \left\{{}^{\mu\nu}_{ab}\right\} \big[3\Omega_\mu{}^{acd,e} \Omega_\nu{}^{bcd,e} + \Omega_\mu{}^{cde,a} \Omega_\nu{}^{cde,b}\big] + a_2 \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\} \Omega_\mu{}^{abc,e} T_{\nu\alpha\beta}{}^{de}.$$

This Lagrangian is (by construction) invariant under ξ-transformations, while invariance under η-transformations requires $a_1 = -3a_2$. We choose $a_1 = 1$, $a_2 = -\frac{1}{3}$ and finally obtain

$$\mathcal{L}_0\big(\Phi_{\mu\nu}{}^{ab}\big) = \left\{{}^{\mu\nu}_{ab}\right\}\big[3\Omega_\mu{}^{acd,e}\Omega_\nu{}^{bcd,e} + \Omega_\mu{}^{cde,a}\Omega_\nu{}^{cde,b}\big] - \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc,e}\partial_\nu\Phi_{\alpha\beta}{}^{de}. \quad (13)$$

As in the previous case, it is not possible to deform this massless Lagrangian into $(A)dS$ space without introduction of additional fields. Thus we turn to the general case – massive particle in $(A)dS$ space with arbitrary cosmological constant. First of all, we have to determine the set of additional fields which is necessary for gauge invariant description of such massive particle. Our main gauge field $Y(3,1)$ has two gauge transformations (combined into one $\xi_\mu{}^{ab}$ transformation in frame-like formalism) with parameters corresponding to $Y(2,1)$ and $Y(3,0)$. Recall that these transformations are reducible with the reducibility parameter $Y(2,0)$. Thus we have to introduce two primary Goldstone fields – $Y(2,1)$ and $Y(3,0)$. The first one also has two gauge transformations with parameters $Y(1,1)$ and $Y(2,0)$ with the reducibility $Y(1,0)$, while the second field has one gauge transformation $Y(2,0)$ only. Taking into account reducibility of main field gauge transformations it is enough to introduce two secondary fields $Y(1,1)$ and $Y(2,0)$ only. Both have gauge transformations with parameters $Y(1,0)$, but due to reducibility of gauge transformations for $Y(2,1)$ field it is enough to introduce one additional field $Y(1,0)$. It has its own gauge transformation $Y(0,0)$, but due to reducibility of gauge transformations for $Y(1,1)$ field, the procedure stops here. Thus we need six fields – $Y(l,1)$, $Y(l,0)$, $1 \leqslant l \leqslant 3$.

Again we will use frame-like formalism for the description of all fields and introduce six pairs: $(\Omega_\mu{}^{abc,d}, \Phi_{\mu\nu}{}^{ab})$, $(\omega_\mu{}^{a,bc}, h_\mu{}^{ab})$, $(\Omega_\mu{}^{abc}, \Phi_{\mu\nu}{}^a)$, $(\omega_\mu{}^{ab}, h_\mu{}^a)$, $(\Omega^{abc}, \Phi_{\mu\nu})$ and (ω^{ab}, h_μ). Note, that here and in what follows we use the same conventions for the frame-like formulation of $Y(k,0)$ fields as in [19]. We start with the sum of kinetic terms for all six fields

$$\begin{aligned}
\mathcal{L}_0 &= \left\{{}^{\mu\nu}_{ab}\right\}\big[3\Omega_\mu{}^{acd,e}\Omega_\nu{}^{bcd,e} + \Omega_\mu{}^{cde,a}\Omega_\nu{}^{cde,b}\big] - \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc,e}D_\nu\Phi_{\alpha\beta}{}^{de} \\
&- \left\{{}^{\mu\nu}_{ab}\right\}\left[\frac{1}{2}\omega_\mu{}^{a,cd}\omega_\nu{}^{b,cd} + \omega_\mu{}^{c,ad}\omega_\nu{}^{c,bd}\right] + 2\left\{{}^{\mu\nu\alpha}_{abc}\right\}\omega_\mu{}^{a,bd}D_\nu h_\alpha{}^{cd} \\
&- 3\left\{{}^{\mu\nu}_{ab}\right\}\Omega_\mu{}^{acd}\Omega_\nu{}^{bcd} + \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc}D_\nu\Phi_{\alpha\beta}{}^d \\
&+ \left\{{}^{\mu\nu}_{ab}\right\}\omega_\mu{}^{ac}\omega_\nu{}^{bc} - \left\{{}^{\mu\nu\alpha}_{abc}\right\}\omega_\mu{}^{ab}D_\nu h_\alpha{}^c \\
&- \Omega_{abc}{}^2 + \left\{{}^{\mu\nu\alpha}_{abc}\right\}\Omega^{abc}\partial_\mu\Phi_{\nu\alpha} + \frac{1}{2}\omega_{ab}{}^2 - \left\{{}^{\mu\nu}_{ab}\right\}\omega^{ab}\partial_\mu h_\nu
\end{aligned} \quad (14)$$

as well as with appropriate set of initial gauge transformations

$$\begin{aligned}
&\delta_0\Phi_{\mu\nu}{}^{ab} = D_{[\mu}\xi_{\nu]}{}^{ab} + \eta_{\mu\nu}{}^{(a,b)}, \qquad \delta_0\Omega_\mu{}^{abc,d} = D_\mu\eta^{abc,d}, \\
&\delta_0\Phi_{\mu\nu}{}^a = D_{[\mu}\xi_{\nu]}{}^a + \eta_{\mu\nu}{}^a, \qquad \delta_0\Omega_\mu{}^{abc} = D_\mu\eta^{abc}, \\
&\delta_0 h_\mu{}^{ab} = D_\mu\zeta^{ab} + \chi_\mu{}^{ab}, \qquad \delta_0\omega_\mu{}^{a,bc} = D_\mu\chi^{a,bc}, \\
&\delta_0 h_\mu{}^a = D_\mu\zeta^a + \chi_\mu{}^a, \qquad \delta_0\omega_\mu{}^{ab} = D_\mu\chi^{ab}, \\
&\delta_0\Phi_{\mu\nu} = D_{[\mu}\xi_{\nu]}, \qquad \delta_0 h_\mu = D_\mu\zeta,
\end{aligned} \quad (15)$$

where all derivatives are now $(A)dS$ covariant ones. As usual, due to non-commutativity of covariant derivatives this Lagrangian is not invariant under the initial gauge transformations

$$\delta_0\mathcal{L}_0 = -3\kappa(d-2)\left\{{}^{\mu\nu}_{ab}\right\}\left(2\Omega_\mu{}^{abc,d}\xi_\nu{}^{cd} - \eta^{abc,d}\Phi_{\mu\nu}{}^{cd}\right)$$
$$+ 3\kappa(d-1)\left(\omega_\mu{}^{\mu,ab}\zeta^{ab} - \chi^{\mu,ab}h_\mu{}^{ab}\right)$$
$$+ \kappa\left\{{}^{\mu\nu}_{ab}\right\}\left[3(d-3)\left(2\Omega_\mu{}^{abc}\xi_\nu{}^c + \eta^{abc}\Phi_{\mu\nu}{}^c\right) - (d-2)\left(\omega_\mu{}^{ab}\zeta_\nu - \chi^{ab}h_{\mu\nu}\right)\right],$$

but we will take this non-invariance into account later on.

To construct gauge invariant description of massive particles we proceed by adding cross terms of order m (i.e. terms with the coefficients with dimension of mass) to the Lagrangian. As we have already noted, one has to introduce such cross terms for the nearest neighbours only, i.e. main gauge field with primary ones, primary with secondary and so on. To simplify the presentation we consider these terms step by step.

$\Phi_{\mu\nu}{}^{ab} \Leftrightarrow \Phi_{\mu\nu}{}^a, h_\mu{}^{ab}$. In this case additional terms to the Lagrangian could be written in the following form:

$$\mathcal{L}_1 = \left\{{}^{\mu\nu\alpha}_{abc}\right\}\left[a_1\Omega_\mu{}^{abc,d}\Phi_{\nu\alpha}{}^d + a_2\Phi_{\mu\nu}{}^{ad}\Omega_\alpha{}^{bcd} + a_3\Phi_{\mu\nu}{}^{ad}\omega_\alpha{}^{b,cd}\right]$$
$$+ \left\{{}^{\mu\nu}_{ab}\right\}a_4\Omega_\mu{}^{abc,d}h_\nu{}^{cd}. \tag{16}$$

As usual, their non-invariance under the initial gauge transformations could be compensated by appropriate corrections to gauge transformations

$$\delta_1\Phi_{\mu\nu}{}^{ab} = -\frac{4\alpha_2}{d-2}\left[e_{[\mu}{}^{(a}\xi_{\nu]}{}^{b)} + \frac{2}{d}g^{ab}\xi_{[\mu,\nu]}\right] + \frac{\beta_2}{6(d-2)}e_{[\mu}{}^{(a}\zeta_{\nu]}{}^{b)},$$
$$\delta_1\Omega_\mu{}^{abc,d} = -\frac{\alpha_2}{d}\left[3e_\mu{}^d\eta^{abc} + e_\mu{}^{[a}\eta^{bc]d} - \frac{4}{(d-2)}g^{d[a}\eta^{bc]}{}_\mu\right]$$
$$+ \frac{\beta_2}{3(d-3)}\left[e_\mu{}^{[a}\chi^{b,c]d} - \frac{1}{d-2}g^{d[a}\chi^{b,c]}{}_\mu\right],$$
$$\delta_1\Phi_{\mu\nu}{}^a = \alpha_2\xi_{[\mu,\nu]}{}^a, \qquad \delta_1\Omega_\mu{}^{abc} = -4\alpha_2\eta^{abc}{}_\mu,$$
$$\delta h_\mu{}^{ab} = \beta_2\xi_\mu{}^{ab}, \qquad \delta_1\omega_\mu{}^{a,bc} = -\frac{\beta_2}{2}\eta_\mu{}^{a(b,c)}, \tag{17}$$

provided $a_1 = 4\alpha_2$, $a_2 = 3\alpha_2$, $a_3 = a_4 = -\beta_2$.

$\Phi_{\mu\nu}{}^a, h_\mu{}^{ab} \Leftrightarrow \Phi_{\mu\nu}, h_\mu{}^a$. Now additional terms to the Lagrangian have the form

$$\Delta\mathcal{L}_1 = \left\{{}^{\mu\nu\alpha}_{abc}\right\}\left[a_5\omega_\mu{}^{ab}\Phi_{\nu\alpha}{}^c + a_6\Omega_\mu{}^{abc}\Phi_{\nu\alpha}\right] + \left\{{}^{\mu\nu}_{ab}\right\}\left[a_7\Omega_\mu{}^{abc}h_\nu{}^c + a_8\Omega^{abc}\Phi_{\mu\nu}{}^c\right]$$
$$+ \left\{{}^{\mu\nu}_{ab}\right\}\left[a_9\omega_\mu{}^{a,bc}h_\nu{}^c + a_{10}h_\mu{}^{ac}\omega_\nu{}^{bc}\right]. \tag{18}$$

To compensate their non-invariance under the initial gauge transformations we introduce the following corrections to gauge transformations:

$$\delta_1\Phi_{\mu\nu}{}^a = \frac{\beta_1}{12(d-3)}e_{[\mu}{}^a\zeta_{\nu]} - \frac{3\alpha_1}{d-3}e_{[\mu}{}^a\xi_{\nu]}, \qquad \delta_1\Omega_\mu{}^{abc} = \frac{\beta_3}{6(d-3)}e_\mu{}^{[a}\chi^{bc]},$$
$$\delta_1 h_\mu{}^{ab} = \frac{\rho_1}{d-1}\left[e_\mu^{(a}\zeta^{b)} - \frac{2}{d}g^{ab}\zeta_\mu\right],$$
$$\delta_1\omega_\mu{}^{a,bc} = \frac{\rho_1}{d}\left[\chi^{a(b}e_\mu{}^{c)} + \frac{1}{d-1}\left(2g^{bc}\chi_\mu{}^a - g^{a(b}\chi_\mu{}^{c)}\right)\right],$$

$$\delta_1 h_\mu{}^a = \beta_1 \xi_\mu{}^a + \rho_1 \zeta_\mu{}^a, \qquad \delta_1 \omega_\mu{}^{ab} = -\frac{\beta_1}{2}\eta_\mu{}^{ab} + \rho_1 \chi^{[a,b]}{}_\mu,$$
$$\delta_1 \Phi_{\mu\nu} = \alpha_1 \xi_{[\mu,\nu]}, \qquad \delta_1 \Omega^{abc} = -3\alpha_1 \eta^{abc}, \tag{19}$$

where $a_5 = a_7 = \beta_1/2$, $a_6 = a_8 = -3\alpha_1$, $a_9 = a_{10} = -2\rho_1$.

$\Phi_{\mu\nu}, h_\mu{}^a \Leftrightarrow \Phi_{\mu\nu}, h_\mu$. Finally, we add to the Lagrangian terms (we already familiar with)

$$\Delta\mathcal{L}_1 = \left\{{}^{\mu\nu}_{ab}\right\}\left[a_{11}\omega_\mu{}^{ab} h_\nu + a_{12}\omega^{ab}\Phi_{\mu\nu}\right] + a_{13}\omega^{ab} h_{ab} \tag{20}$$

and corresponding corrections to gauge transformations

$$\delta h_\mu{}^a = \frac{2\rho_0}{d-2} e_\mu{}^a \zeta, \qquad \delta h_\mu = \rho_0 \zeta_\mu + \beta_0 \xi_\mu, \qquad \delta \omega^{ab} = -2\rho_0 \chi^{ab}, \tag{21}$$

where $a_{11} = a_{13} = 2\rho_0$, $a_{12} = \beta_0/2$.

Collecting all pieces together, we obtain complete set of cross terms

$$\begin{aligned}
\mathcal{L}_1 = {} & \left\{{}^{\mu\nu\alpha}_{abc}\right\}\left[4\alpha_2 \Omega_\mu{}^{abc,d}\Phi_{\nu\alpha}{}^d + 3\alpha_2 \Phi_{\mu\nu}{}^{ad}\Omega_\alpha{}^{bcd}\right. \\
& \left. - \beta_2 \Phi_{\mu\nu}{}^{ad}\omega_\alpha{}^{b,cd}\right] - \beta_2 \left\{{}^{\mu\nu}_{ab}\right\}\Omega_\mu{}^{abc,d} h_\nu{}^{cd} + \left\{{}^{\mu\nu\alpha}_{abc}\right\}\left[\frac{\beta_1}{2}\omega_\mu{}^{ab}\Phi_{\nu\alpha}{}^c - 3\alpha_1 \Omega_\mu{}^{abc}\Phi_{\nu\alpha}\right] \\
& + \left\{{}^{\mu\nu}_{ab}\right\}\left[\frac{\beta_1}{2}\Omega_\mu{}^{abc} h_\nu{}^c - 3\alpha_1 \Omega^{abc}\Phi_{\mu\nu}{}^c\right] + \left\{{}^{\mu\nu}_{ab}\right\}\left[-2\rho_1 \omega_\mu{}^{a,bc} h_\nu{}^c - 2\rho_1 h_\mu{}^{ac}\omega_\nu{}^{bc}\right. \\
& \left. + 2\rho_0 \omega_\mu{}^{ab} h_\nu + \frac{\beta_0}{2}\omega^{ab}\Phi_{\mu\nu}\right] + 2\rho_0 \omega^{ab} h_{ab}.
\end{aligned} \tag{22}$$

Now, as we have achieved cancellation of all variations of order m, i.e. $\delta_0\mathcal{L}_1 + \delta_1\mathcal{L}_0 = 0$, we have to take care on variations of order m^2 (including contribution from kinetic terms due to non-commutativity of covariant derivatives) $\delta_0\mathcal{L}_0 + \delta_1\mathcal{L}_1$. As in the previous case, there are no any explicit mass-like terms allowed here, but complete invariance of the Lagrangian could be achieved just by adjusting the values of remaining free parameters $\alpha_{1,2}$, $\beta_{0,1,2}$ and $\rho_{0,1}$

$$\beta_1 = -2\sqrt{\frac{d-1}{d-2}}\beta_2, \qquad \beta_0 = -\sqrt{\frac{6(d-1)}{d-3}}\beta_2,$$
$$\rho_1 = 2\sqrt{\frac{d-1}{d-2}}\alpha_2, \qquad \rho_0 = \sqrt{\frac{3(d-2)}{2(d-3)}}\alpha_1,$$
$$24\alpha_2^2 - \beta_2^2 = 6(d-2)\kappa, \qquad 12(d+1)\alpha_2^2 - 3d\alpha_1^2 = d(d+1)\kappa.$$

The role that each of the parameters plays could be easily seen from Fig. 2. Now we are ready to analyze the results obtained. First of all, note that the massless limit (i.e. decoupling of main gauge fields from all others) requires $\alpha_2 = \beta_2 = 0$. As the first of last two relations clearly shows this is possible in flat Minkowski space ($\kappa = 0$) only. In this, for non-zero values of cosmological constant there exists a number of partially massless limits.

In *dS* space ($\kappa > 0$) one can put $\beta_2 = 0$ (and this simultaneously gives $\beta_1 = \beta_0 = 0$). In this complete system decompose into two disconnected subsystems, as shown on Fig. 3. One of them, with the fields $\Phi_{\mu\nu}{}^{ab}$, $\Phi_{\mu\nu}{}^a$ and $\Phi_{\mu\nu}$ gives new example of partially massless theory with the Lagrangian

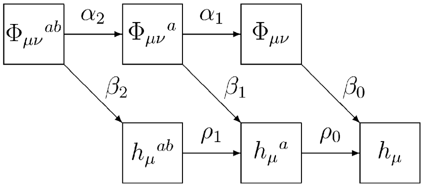

Fig. 2. General massive theory for $Y(3,1)$ tensor.

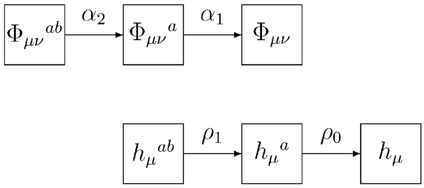

Fig. 3. Partially massless limit in *dS* space.

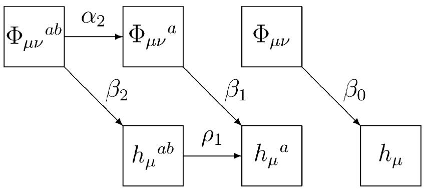

Fig. 4. Non-unitary partially massless theory.

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_0\big(\Phi_{\mu\nu}{}^{ab}\big) + \mathcal{L}_0\big(\Phi_{\mu\nu}{}^{a}\big) + \mathcal{L}_0(\Phi_{\mu\nu}) \\
&\quad + \alpha_2 \left\{{}^{\mu\nu\alpha}_{abc}\right\}\big[4\Omega_\mu{}^{abc,d}\Phi_{\nu\alpha}{}^{d} + 3\Phi_{\mu\nu}{}^{ad}\Omega_\alpha{}^{bcd}\big] \\
&\quad - 3\alpha_1\big[\left\{{}^{\mu\nu\alpha}_{abc}\right\}\Omega_\mu{}^{abc}\Phi_{\nu\alpha} + \left\{{}^{\mu\nu}_{ab}\right\}\Omega^{abc}\Phi_{\mu\nu}{}^{c}\big],
\end{aligned} \tag{23}$$

where $4\alpha_2^2 = (d-2)\kappa$, $3d\alpha_1^2 = 2(d+1)(d-3)\kappa$, which is invariant under the following gauge transformations (for simplicity we reproduce here gauge transformations for physical fields only):

$$\begin{aligned}
\delta\Phi_{\mu\nu}{}^{ab} &= D_{[\mu}\xi_{\nu]}{}^{ab} + \eta_{\mu\nu}{}^{(a,b)} - \frac{4\alpha_2}{d-2} e_{[\mu}{}^{(a}\xi_{\nu]}{}^{b)}, \\
\delta\Phi_{\mu\nu}{}^{a} &= D_{[\mu}\xi_{\nu]}{}^{a} + \eta_{\mu\nu}{}^{a} + \alpha_2\xi_{[\mu,\nu]}{}^{a} - \frac{3\alpha_1}{d-3} e_{[\mu}{}^{a}\xi_{\nu]}, \\
\delta\Phi_{\mu\nu} &= D_{[\mu}\xi_{\nu]} + \alpha_1\xi_{[\mu,\nu]}.
\end{aligned} \tag{24}$$

At the same time, three other fields $h_\mu{}^{ab}$, $h_\mu{}^{a}$ and h_μ give gauge invariant description of partially massless spin 3 particle [17,19].

One more example of partially massless theory appears if one put $\alpha_1 = 0$ (and hence $\rho_0 = 0$). In this, complete system also decompose into two disconnected subsystems as shown on Fig. 4. Note, however that in this case we obtain $12\alpha_2^2 = d\kappa$, $\beta_2^2 = -4(d-3)\kappa$, so that such theory (as is often to be the case) “lives” inside unitary forbidden region.

From the other hand, in *AdS* space ($\kappa < 0$) one can put $\alpha_2 = 0$ (and hence $\rho_1 = 0$). In this, decomposition into two subsystems looks as shown on Fig. 5. Thus we obtain one more example

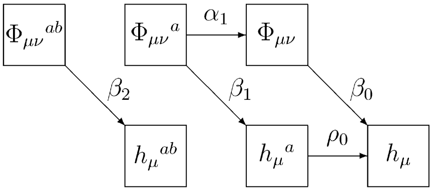

Fig. 5. Partially massless limit in *AdS* space.

of partially massless theory with two fields $\Phi_{\mu\nu}{}^{ab}$ and $h_\mu{}^{ab}$. The Lagrangian

$$\mathcal{L} = \mathcal{L}_0(\Phi_{\mu\nu}{}^{ab}) + \mathcal{L}_0(h_\mu{}^{ab}) - \beta_2 \left\{{}^{\mu\nu\alpha}_{abc}\right\} \Phi_{\mu\nu}{}^{ad} \omega_\alpha{}^{b,cd} - \beta_2 \left\{{}^{\mu\nu}_{ab}\right\} \Omega_\mu{}^{abc,d} h_\nu{}^{cd}, \tag{25}$$

where $\beta_2^2 = -6(d-2)\kappa$, is invariant under the following gauge transformations:

$$\delta\Phi_{\mu\nu}{}^{ab} = D_{[\mu}\xi_{\nu]}{}^{ab} + \eta_{\mu\nu}{}^{(a,b)} + \frac{\beta_2}{6(d-2)} e_{[\mu}{}^{(a}\zeta_{\nu]}{}^{b)},$$
$$\delta h_\mu{}^{ab} = D_\mu\zeta^{ab} + \chi_\mu{}^{ab} + \beta_2\xi_\mu{}^{ab}. \tag{26}$$

In this, four remaining fields $\Phi_{\mu\nu}{}^a$, $h_\mu{}^a$, $\Phi_{\mu\nu}$ and h_μ just gives the same gauge invariant massive theory as in the previous section.

3. Tensor $Y(k,1)$

For the description of massless particles we introduce main physical field – two-form $\Phi_{\mu\nu}{}^{a_1\ldots a_{k-1}} = \Phi_{\mu\nu}{}^{(k-1)}$ (here and in what follows we will use the same condensed notations for tensor objects as in [19]) which is completely symmetric and traceless on local indices and auxiliary one-form $\Omega_\mu{}^{abc,(k-2)}$ which is completely antisymmetric on abc, traceless on all local indices and satisfies a constraint $\Omega_\mu{}^{[abc,d](k-3)} = 0$. To have correct number of physical degrees of freedom massless Lagrangian has to be invariant under the following gauge transformations:

$$\delta\Phi_{\mu\nu}{}^{(k-1)} = \partial_{[\mu}\xi_{\nu]}{}^{(k-1)} + \eta_{\mu\nu}{}^{(1,k-2)}, \qquad \delta\Omega_\mu{}^{abc,(k-2)} = \partial_\mu\eta^{abc,(k-2)}, \tag{27}$$

where properties of parameters ξ and η correspond to that of $\Phi_{\mu\nu}$ and Ω_μ. To find appropriate Lagrangian, we introduce a tensor $T_{\mu\nu\alpha}{}^{(k-1)} = \partial_{[\mu}\Phi_{\nu\alpha]}{}^{(k-1)}$, which is invariant under ξ-transformations, consider an expression $\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc,(k-2)} T_{\nu\alpha\beta}{}^{d(k-2)}$ and make a substitution $T_{\mu\nu\alpha}{}^{(k-1)} \to \Omega_{[\mu,\nu\alpha]}{}^{(1,k-2)}$. We obtain

$$\begin{aligned}
&\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc,(k-2)} T_{\nu\alpha\beta}{}^{d(k-2)} \\
&\quad\Rightarrow\quad \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc,(k-2)} \Omega_{\nu,\alpha\beta}{}^{(d,k-2)} \\
&\quad\Rightarrow\quad \left\{{}^{\mu\nu}_{ab}\right\}\left[3\Omega_\mu{}^{acd(k-2)}\Omega_\nu{}^{bcd,(k-2)} + (k-2)\Omega_\mu{}^{cde,a(k-3)}\Omega_\nu{}^{cde,b(k-3)}\right].
\end{aligned}$$

Thus we will look for massless Lagrangian in the form

$$\begin{aligned}
\mathcal{L}_0 = {} & a_1\left\{{}^{\mu\nu}_{ab}\right\}\left[3\Omega_\mu{}^{acd,(k-2)}\Omega_\nu{}^{bcd,(k-2)} + (k-2)\Omega_\mu{}^{cde,a(k-3)}\Omega_\nu{}^{cde,b(k-3)}\right] \\
& + a_2\left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc,(k-2)} T_{\nu\alpha\beta}{}^{d(k-2)}.
\end{aligned}$$

It is by construction invariant under the ξ-transformations, while invariance under the η-transformations requires $a_1 = -3a_2$. We choose $a_1 = (-1)^{k-1}$, $a_2 = -(-1)^{k-1}/3$ and obtain finally

$$(-1)^{k-1}\mathcal{L}_0 = \left\{{}^{\mu\nu}_{ab}\right\}\left[3\Omega_\mu{}^{acd,(k-2)}\Omega_\nu{}^{bcd,(k-2)} + (k-2)\Omega_\mu{}^{cde,a(k-3)}\Omega_\nu{}^{cde,b(k-3)}\right] - \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc,(k-2)}\partial_\nu\Phi_{\alpha\beta}{}^{d(k-2)}. \tag{28}$$

As in the previous cases, it is not possible to deform this massless theory into $(A)dS$ space without introduction of additional fields, so we will turn to the general case – massive particle in $(A)dS$ space with arbitrary cosmological constant. Our first task – to determine the set of additional fields which are necessary for gauge invariant description of such massive particle. Our main gauge field $Y(k,1)$ has two gauge transformations with parameters $Y(k-1,1)$ and $Y(k,0)$ with the reducibility $Y(k-1,0)$, thus we need two primary Goldstone fields $Y(k-1,1)$ and $Y(k,0)$. The first one has two own gauge transformations with parameters $Y(k-2,1)$ and $Y(k-1,0)$ with reducibility $Y(k-2,0)$, while the second one has one gauge transformation with parameter $Y(k-1,0)$ only. So we need two secondary fields $Y(k-2,1)$ and $Y(k-1,0)$ and so on. As in the previous cases, this procedure stops at vector field $Y(1,0)$, thus we totally have to introduce fields $Y(l,1)$ and $Y(l,0)$ with $1 \leqslant l \leqslant k$.

Let us start with the sum of kinetic terms for all these fields

$$\begin{aligned}
\mathcal{L}_0 &= \sum_{l=2}^{k}\mathcal{L}_0\big(\Phi_{\mu\nu}{}^{(l-1)}\big) - \Omega_{abc}{}^2 + \left\{{}^{\mu\nu\alpha}_{abc}\right\}\Omega^{abc}D_\mu\Phi_{\nu\alpha} \\
&\quad + \sum_{l=2}^{k}\mathcal{L}_0\big(h_\mu{}^{(l-1)}\big) + \frac{1}{2}\omega_{ab}{}^2 - \left\{{}^{\mu\nu}_{ab}\right\}\omega^{ab}D_\mu h_\nu, \\
(-1)^l\mathcal{L}_0\big(\Phi_{\mu\nu}{}^{(l)}\big) &= \left\{{}^{\mu\nu}_{ab}\right\}\left[3\Omega_\mu{}^{acd,(l-1)}\Omega_\nu{}^{bcd,(l-1)} + (l-1)\Omega_\mu{}^{cde,a(l-2)}\Omega_\nu{}^{cde,b(l-2)}\right] \\
&\quad - \left\{{}^{\mu\nu\alpha\beta}_{abcd}\right\}\Omega_\mu{}^{abc,(l-1)}D_\nu\Phi_{\alpha\beta}{}^{d(l-1)}, \\
(-1)^l\mathcal{L}_0\big(h_\mu{}^{(l)}\big) &= -\left\{{}^{\mu\nu}_{ab}\right\}\left[\omega_\mu{}^{c,a(l-1)}\omega_\nu{}^{c,b(l-1)} + \frac{1}{l}\omega_\mu{}^{a,(l)}\omega_\nu{}^{b,(l)}\right] \\
&\quad + 2\left\{{}^{\mu\nu\alpha}_{abc}\right\}\omega_\mu{}^{a,b(l-1)}D_\nu h_\alpha{}^{c(l-1)},
\end{aligned} \tag{29}$$

as well as appropriate set of initial gauge transformations

$$\begin{aligned}
&\delta\Phi_{\mu\nu}{}^{(l)} = D_{[\mu}\xi_{\nu]}{}^{(l)} + \eta_{\mu\nu}{}^{(1,l-1)}, \qquad \delta\Omega_\mu{}^{abc,(l-1)} = D_\mu\eta^{abc,(l-1)}, \qquad \delta\Phi_{\mu\nu} = D_{[\mu}\xi_{\nu]}, \\
&\delta h_\mu{}^{(l)} = D_\mu\zeta^{(l)} + \chi_\mu{}^{(l)}, \qquad \delta\omega_\mu{}^{a,(l)} = D_\mu\chi^{a,(l)}, \qquad \delta h_\mu = D_\mu\zeta,
\end{aligned} \tag{30}$$

where all derivatives are now $(A)dS$ covariant ones. Due to non-commutativity of covariant derivatives this Lagrangian is not invariant under the initial gauge transformations

$$\begin{aligned}
\delta_0\mathcal{L}_0 &= \sum_{l=2}^{k}(-1)^l\kappa\left\{{}^{\mu\nu}_{ab}\right\}\Big[3(d+l-4)\big(-2\Omega_\mu{}^{abc,(l-1)}\xi_\nu{}^{c(l-1)} + \eta^{abc,(l-1)}\Phi_{\mu\nu}{}^{c(l-1)}\big) \\
&\quad + 2(d+l-3)\left(\omega_\mu{}^{a,b(l-1)}\zeta_\nu{}^{(l-1)} - \frac{l+1}{l}\chi^{\mu,(l)}h_\mu{}^{(l)}\right)\Big],
\end{aligned}$$

but we will take this non-invariance into account later on.

To proceed with the construction of gauge invariant description of massive particle, we have to add to the Lagrangian cross terms of order m (i.e. with the coefficients having dimension of mass). As we have already noted above, one has to introduce such cross terms for the nearest neighbours only (i.e. main field with primary ones, primary with secondary and so on). For the case at hands, this means introduction cross terms between pairs $Y(l+1,1)$, $Y(l+2,0)$ and $Y(l,1)$, $Y(l+1,0)$. Moreover, due to symmetry and tracelessness properties of the fields, there exists such terms for three possible cases $Y(l+1,1) \Leftrightarrow Y(l,1)$, $Y(l+1,1) \Leftrightarrow Y(l+1,0)$, $Y(l+2,0) \Leftrightarrow Y(l+1,0)$ only. We consider these three possibilities in turn.

$\Omega_\mu{}^{abc,(l-1)}, \Phi_{\mu\nu}{}^{(l)} \Leftrightarrow \Omega_\mu{}^{abc,(l-2)}, \Phi_{\mu\nu}{}^{(l-1)}$. Here additional terms to the Lagrangian could be written as follows:

$$\mathcal{L}_1 = (-1)^l \left\{{}^{\mu\nu\alpha}_{abc}\right\}\left[a_{1l}\Omega_\mu{}^{abc,(l-1)}\Phi_{\nu\alpha}{}^{(l-1)} + a_{2l}\Omega_\mu{}^{abd,(l-2)}\Phi_{\nu\alpha}{}^{cd(l-2)}\right]. \tag{31}$$

Their non-invariance under the initial gauge transformations could be compensated by the following corrections to gauge transformations:

$$\begin{aligned}
\delta_1\Phi_{\mu\nu}{}^{(l)} &= -\frac{(l+2)\alpha_l}{(l-1)(d+l-4)}\left[e_{[\mu}{}^{(1}\xi_{\nu]}{}^{l-1)} + \frac{2}{d+2l-4}\xi_{[\mu,\nu]}{}^{(l-2}g^{12)}\right],\\
\delta_1\Omega_\mu{}^{abc,(l-1)} &= -\frac{\alpha_l}{(l-1)d}\left[3\eta^{abc,(l-2}e_\mu^{1)} + e_\mu{}^{[a}\eta^{bc](1,l-2)} - \mathrm{Tr}\right],\\
\delta_1\Phi_{\mu\nu}{}^{(l-1)} &= \alpha_l\xi_{[\mu,\nu]}{}^{(l-1)}, \qquad \delta_1\Omega_\mu{}^{abc,(l-2)} = -\frac{(l+2)\alpha_l}{l-1}\eta^{abc,(l-2)}{}_\mu,
\end{aligned} \tag{32}$$

provided

$$a_{1l} = \frac{(l+2)}{(l-1)}\alpha_l, \qquad a_{2l} = 3\alpha_l.$$

$\Omega_\mu{}^{abc,(l-1)}, \Phi_{\mu\nu}{}^{(l)} \Leftrightarrow \omega_\mu{}^{a,(l)}, h_\mu{}^{(l)}$. This time additional terms to the Lagrangian have the form

$$\mathcal{L}_1 = (-1)^l\left[a_{3l}\left\{{}^{\mu\nu}_{ab}\right\}\Omega_\mu{}^{abc,(l-1)}h_\nu{}^{c(l-1)} + a_{4l}\left\{{}^{\mu\nu\alpha}_{abc}\right\}\omega_\mu{}^{a,b(l-1)}\Phi_{\nu\alpha}{}^{c(l-1)}\right] \tag{33}$$

and their non-invariance under the initial gauge transformations could be compensated by

$$\begin{aligned}
\delta_1\Phi_{\mu\nu}{}^{(l)} &= \frac{\beta_l}{6(d+l-4)}e_{[\mu}{}^{(1}\zeta_{\nu]}{}^{l-1)},\\
\delta_1\Omega_\mu{}^{abc,(l-1)} &= \frac{\beta_l}{3(d-3)}\left[e_\mu{}^{[a}\chi^{b,c](l-1)} - \mathrm{Tr}\right],\\
\delta_1 h_\mu{}^{(l)} &= \beta_l\xi_\mu{}^{(l)}, \qquad \delta_1\omega_\mu{}^{a,(l)} = -\frac{\beta_l}{2}\eta_\mu{}^{a(1,l-1)},
\end{aligned} \tag{34}$$

provided $a_{3l} = a_{4l} = -\beta_l$.

$\omega_\mu{}^{a,(l+1)}, h_\mu{}^{(l+1)} \Leftrightarrow \omega_\mu{}^{a,(l)}, h_\mu{}^{(l)}$. This case (that has already been considered in [19]) requires additional terms to the Lagrangian in the form

$$\mathcal{L}_1 = (-1)^l\left\{{}^{\mu\nu}_{ab}\right\}\left[a_{5l}\omega_\mu{}^{a,b(l)}h_\nu{}^{(l)} + a_{6l}\omega_\mu{}^{a,(l)}h_\nu{}^{b(l)}\right] \tag{35}$$

as well as the following corrections to gauge transformations

$$\delta_1 h_\mu{}^{(l+1)} = \frac{(l+1)\rho_l}{l(d+l-2)}\left[e_\mu{}^{(1}\xi^{l)} - \mathrm{Tr}\right], \qquad \delta_1\omega_\mu{}^{a,(l+1)} = \frac{(l+1)\rho_l}{l(d+l-1)}\left[\eta^{a,(l}e_\mu{}^{1)} - \mathrm{Tr}\right],$$

$$\delta_1 h_\mu{}^{(l)} = \rho_l \xi_\mu{}^{(l)}, \qquad \delta_1 \omega_\mu{}^{a,(l)} = \frac{\rho_l}{l}\big[\eta_\mu{}^{a(l)} + (l+1)\eta^{a,(l)}{}_\mu - \mathrm{Tr}\big], \tag{36}$$

where $a_{5l} = a_{6l} = \frac{2(l+1)}{l}\rho_l$.

Collecting all pieces together, we obtain finally

$$\begin{aligned}
\mathcal{L}_1 = \sum_{l=2}^{k-1}(-1)^l \bigg[&\{^{\mu\nu\alpha}_{abc}\}\alpha_l \bigg[\frac{l+2}{l-1}\Omega_\mu{}^{abc,(l-1)}\Phi_{\nu\alpha}{}^{(l-1)} + 3\Omega_\mu{}^{abd,(l-2)}\Phi_{\nu\alpha}{}^{cd(l-2)}\bigg] \\
&- \beta_l\big[\{^{\mu\nu}_{ab}\}\Omega_\mu{}^{abc,(l-1)}h_\nu{}^{c(l-1)} + \{^{\mu\nu\alpha}_{abc}\}\omega_\mu{}^{a,b(l-1)}\Phi_{\nu\alpha}{}^{c(l-1)}\big] \\
&+ \frac{2(l+1)}{l}\rho_l\{^{\mu\nu}_{ab}\}\big[\omega_\mu{}^{a,b(l)}h_\nu{}^{(l)} + \omega_\mu{}^{a,(l)}h_\nu{}^{b(l)}\big]\bigg] \\
&+ \{^{\mu\nu}_{ab}\}\big[-3\alpha_1\Omega^{abc}\Phi_{\mu\nu}{}^c + 2\rho_0\omega_\mu{}^{ab}h_\nu + \beta_0\omega^{ab}\Phi_{\mu\nu}\big] \\
&- 3\alpha_1\{^{\mu\nu\alpha}_{abc}\}\Omega_\mu{}^{abc}\Phi_{\nu\alpha} + 2\rho_0\omega^{ab}h_{ab}.
\end{aligned} \tag{37}$$

As for the corrections to gauge transformations, we once again restrict ourselves with the transformations for physical fields only

$$\begin{aligned}
\delta_1\Phi_{\mu\nu}{}^{(l)} &= \alpha_{l+1}\xi_{[\mu,\nu]}{}^{(l)} - \frac{(l+2)\alpha_l}{(l-1)(d+l-4)}\big[e_{[\mu}{}^{(1}\xi_{\nu]}{}^{l-1)} - \mathrm{Tr}\big] \\
&\quad + \frac{\beta_l}{6(d+l-4)}e_{[\mu}{}^{(1}\zeta_{\nu]}{}^{l-1)}, \\
\delta_1\Phi_{\mu\nu}{}^a &= \alpha_2\xi_{[\mu,\nu]}{}^a - \frac{3\alpha_1}{d-3}e_{[\mu}{}^a\xi_{\nu]} + \frac{\beta_1}{6(d-3)}e_{[\mu}{}^a\zeta_{\nu]}, \qquad \delta_1\Phi_{\mu\nu} = \alpha_1\xi_{[\mu,\nu]}, \\
\delta_1 h_\mu{}^{(l)} &= \beta_l\xi_\mu{}^{(l)} + \rho_l\zeta_\mu{}^{(l)} + \frac{l\rho_{l-1}}{(l-1)(d+l-3)}\big[e_\mu{}^{(1}\zeta^{l-1)} - \mathrm{Tr}\big], \\
\delta_1 h_\mu{}^a &= \beta_1\xi_\mu{}^a + \rho_1\zeta_\mu{}^a + \frac{\rho_0}{d-2}e_\mu{}^a\zeta, \qquad \delta_1 h_\mu = \beta_0\xi_\mu + \rho_0\zeta_\mu.
\end{aligned} \tag{38}$$

Now, having achieved cancellation of all variations of order m $\delta_0\mathcal{L}_1 + \delta_1\mathcal{L}_0 = 0$, we have to take care on variations of order m^2 (including contribution of kinetic terms due to non-commutativity of covariant derivatives) $\delta_0\mathcal{L}_0 + \delta_1\mathcal{L}_1$. As in the previous cases, complete invariance of the Lagrangian could be achieved without introduction of any explicit mass-like terms into the Lagrangian (and appropriate corrections to gauge transformations). Indeed, rather long calculations give four relations

$$\begin{aligned}
&\alpha_l\beta_{l-1} = -\beta_l\rho_{l-1}, \\
&(l+1)(d+l-3)\beta_l\rho_l = -(l+3)(d+l-2)\alpha_{l+1}\beta_{l+1}, \\
&-\frac{6(l+3)(d+l-4)(d+2l)}{l(d+l-3)(d+2l-2)}\alpha_{l+1}^2 + \frac{6(l+2)}{l-1}\alpha_l^2 - \beta_l^2 = 6\kappa(d+l-4), \\
&\frac{d+l-3}{3(d+l-4)}\beta_l^2 + \frac{2(l+1)(d+l-3)(d+2l)}{l(d+l-2)(d+2l-2)}\rho_l^2 - \frac{2l}{l-1}\rho_{l-1}^2 = -2\kappa(d+l-3).
\end{aligned}$$

To solve these relations we proceed as follows. From the first one we get

$$\rho_l = -\frac{\beta_l}{\beta_{l+1}}\alpha_{l+1}.$$

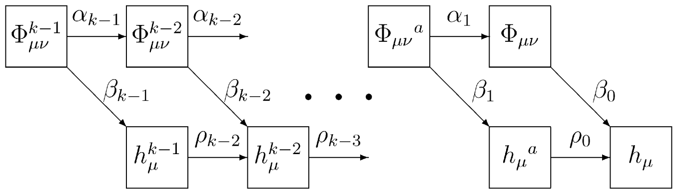

Fig. 6. General massive $Y(k,1)$ theory.

Putting this relation into the second one, we obtain recurrent relation on parameters β

$$\beta_l^2 = \frac{(l+3)(d+l-2)}{(l+1)(d+l-3)}\beta_{l+1}^2.$$

This allows us to express all parameters β_l in terms of β_{k-1}

$$\beta_l^2 = \frac{k(k+1)(d+k-4)}{(l+1)(l+2)(d+l-3)}\beta_{k-1}^2.$$

In this, one can show that fourth equation is equivalent to third one. When all parameters β are known, the third equation becomes recurrent relation on parameters α and this allows us (taking into account that $\alpha_k = 0$) to express all α_l in terms of α_{k-1}. Let us introduce a notation $M^2 = \frac{k(k+1)}{k-2}\alpha_{k-1}^2$, then the expression for α_l could be written as follows:

$$\alpha_l^2 = \frac{(l-1)(d+k+l-3)}{(l+1)(l+2)(d+2l-2)}\big[M^2 - (k-l-1)(d+k+l-4)\kappa\big].$$

Thus we are managed to express all parameters in terms of two main ones β_{k-1} and M (or α_{k-1}), in this the following relation must hold:

$$6M^2 - k\beta_{k-1}^2 = 6k(d+k-5)\kappa.$$

Now we are ready to analyze the results obtained. In complete theory we have three sets of parameters α, β and ρ and the roles they play could be easily seen from Fig. 6.

First of all note, that massless limit (that requires $M \to 0$ and $\beta_{k-1} \to 0$ simultaneously) is indeed possible in flat Minkowski space only, while for non-zero values of cosmological constant we can obtain a number of partially massless theories. In *AdS* space ($\kappa < 0$) one can put $\alpha_{k-1} = 0$ (and this gives $\rho_{k-2} = 0$), in this two fields $\Phi_{\mu\nu}{}^{(k-1)}$ and $h_\mu{}^{(k-1)}$ decouple and describe partially massless theory with the Lagrangian (Fig. 7)

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_0\big(\Phi_{\mu\nu}{}^{(k-1)}\big) + \mathcal{L}_0\big(h_\mu{}^{(k-1)}\big)\\ &\quad + (-1)^k\beta_{k-1}\big[\left\{{}^{\mu\nu}_{ab}\right\}\Omega_\mu{}^{abc,(k-2)}h_\nu{}^{c(k-2)} + \left\{{}^{\mu\nu\alpha}_{abc}\right\}\omega_\mu{}^{a,b(k-2)}\Phi_{\nu\alpha}{}^{c(k-2)}\big],\end{aligned} \tag{39}$$

which is invariant under the following gauge transformations:

$$\begin{aligned}\delta\Phi_{\mu\nu}{}^{(k-1)} &= D_{[\mu}\xi_{\nu]}{}^{(k-1)} + \eta_{\mu\nu}{}^{(1,k-2)} + \frac{\beta_{k-1}}{6(d+k-5)}e_{[\mu}{}^{(1}\zeta_{\nu]}{}^{k-2)},\\ \delta h_\mu{}^{(k-1)} &= D_\mu\zeta^{(k-1)} + \chi_\mu{}^{(k-1)} + \beta_{k-1}\xi_\mu{}^{(k-1)}.\end{aligned} \tag{40}$$

Such particle corresponds to irreducible representation of anti-de Sitter group in complete agreement with general discussion in [38]. Indeed, a pair of representations $Y(k,1)$ and $Y(k,0)$ of the

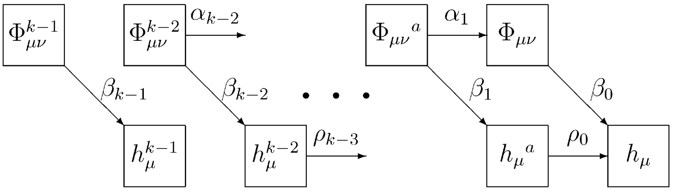

Fig. 7. Partially massless limit in *AdS* space.

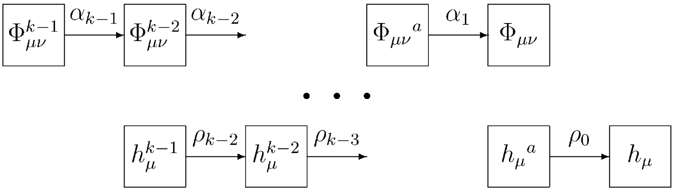

Fig. 8. Partially massless limit in *dS* space.

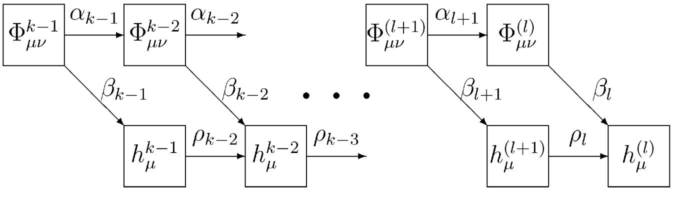

Fig. 9. Example of non-unitary partially massless theory.

Lorentz group perfectly combine into one $Y(k,1)$ representation of anti-de Sitter group (compare with Eq. (43) of [38]). As we have already mentioned, massless $Y(k,1)$ particle in flat Minkowski space has two gauge transformations with parameters corresponding to $Y(k-1,1)$ and $Y(k,0)$. If one goes to metric-like formulation by solving algebraic equations for auxiliary fields, then one can see that in the absence of h field only gauge symmetry with the parameter $Y(k-1,1)$ survives. At the same time, all other fields besides $\Phi_{\mu\nu}{}^{(k-1)}$ and $h_\mu{}^{(k-1)}$ just give gauge invariant description of massive $\Phi_{\mu\nu}{}^{(k-2)}$ tensor.

On the other hand, in *dS* space ($\kappa > 0$) one can put $\beta_{k-1} = 0$ (and this results in $\beta_l = 0$ for all l). In this case complete system decompose into two disconnected subsystems (Fig. 8). One subsystem with the fields $\Phi_{\mu\nu}{}^{(l)}$, $0 \leqslant l \leqslant k-1$ gives new example of partially massless theory. Again if one goes to metric-like formulation one can see that in this case only gauge symmetry with the parameter $Y(k,0)$ survives. At the same time partially massless theory described by the second subsystem $h_\mu{}^{(k)}$, $0 \leqslant l \leqslant k-1$ is already known [17,19]. Besides, a number of (non-unitary) partially massless theories appears then one put one of the $\alpha_l = 0$ (and hence $\rho_{l-1} = 0$). In this, complete system also decompose into two disconnected subsystems. One of them gives partially massless theory with the fields $\Phi_{\mu\nu}{}^{(n)}$, $h_\mu{}^{(n)}$, $l \leqslant n \leqslant k-1$ (Fig. 9), while the rest of fields just give massive theory for the $\Phi_{\mu\nu}{}^{(l-1)}$ tensor.

4. Conclusion

In this paper, using the simplest mixed symmetry tensors $Y(k,1)$ as an example, we have shown that frame-like gauge invariant formulation of massive higher spin particles [19] could be extended to the case of mixed symmetry fields. All that one needs for that is to determine proper collection of massless fields to start with taking into account all gauge symmetries and their reducibility. It is clear that for general mixed symmetry tensors such construction will require a lot of fields so that calculations become very lengthy and involved. Thus we need some more powerful methods, for example, some kind of oscillator formalism adapted to frame-like formulation with its separation of world and local indices. Let us also note here an interesting method of dimensional digression proposed recently [43].

References

[1] T. Curtright, Generalized gauge fields, Phys. Lett. B 165 (1985) 304.
[2] C.S. Aulakh, I.G. Koh, S. Ouvry, Higher spin fields with mixed symmetry, Phys. Lett. B 173 (1986) 284.
[3] J.M. Labastida, T.R. Morris, Massless mixed symmetry bosonic free fields, Phys. Lett. B 180 (1986) 101.
[4] J.M. Labastida, Massless particles in arbitrary representations of the Lorentz group, Nucl. Phys. B 322 (1989) 185.
[5] R.R. Metsaev, Massless arbitrary spin fields in $AdS_{(5)}$, Phys. Lett. B 531 (2002) 152, hep-th/0201226.
[6] R.R. Metsaev, Mixed symmetry massive fields in $AdS_{(5)}$, Class. Quantum Grav. 22 (2005) 2777, hep-th/0412311.
[7] C. Burdik, A. Pashnev, M. Tsulaia, On the mixed symmetry irreducible representations of the Poincaré group in the BRST approach, Mod. Phys. Lett. A 16 (2001) 731, hep-th/0101201.
[8] X. Bekaert, N. Boulanger, Tensor gauge fields in arbitrary representations of $GL(D,R)$: Duality and Poincaré lemma, Commun. Math. Phys. 245 (2004) 27, hep-th/0208058.
[9] Yu.M. Zinoviev, On massive mixed symmetry tensor fields in Minkowski space and $(A)dS$, hep-th/0211233.
[10] X. Bekaert, N. Boulanger, Tensor gauge fields in arbitrary representations of $GL(D,R)$: II. Quadratic actions, Commun. Math. Phys. 271 (2007) 723, hep-th/0606198.
[11] I.L. Buchbinder, V.A. Krykhtin, H. Takata, Gauge invariant Lagrangian construction for massive bosonic mixed symmetry higher spin fields, Phys. Lett. B 656 (2007) 253, arXiv: 0707.2181.
[12] P.Yu. Moshin, A.A. Reshetnyak, BRST approach to Lagrangian formulation for mixed-symmetry fermionic higher-spin fields, JHEP 0710 (2007) 040, arXiv: 0707.0386.
[13] A. Campoleoni, D. Francia, J. Mourad, A. Sagnotti, Unconstrained higher spins of mixed symmetry. I. Bose fields, arXiv: 0810.4350.
[14] M.A. Vasiliev, 'Gauge' form of description of massless fields with arbitrary spin, Sov. J. Nucl. Phys. 32 (1980) 439.
[15] V.E. Lopatin, M.A. Vasiliev, Free massless bosonic fields of arbitrary spin in d-dimensional de Sitter space, Mod. Phys. Lett. A 3 (1988) 257.
[16] M.A. Vasiliev, Free massless fermionic fields of arbitrary spin in d-dimensional de Sitter space, Nucl. Phys. B 301 (1988) 26.
[17] E.D. Skvortsov, M.A. Vasiliev, Geometric formulation for partially massless fields, Nucl. Phys. B 756 (2006) 117, hep-th/0601095.
[18] D.P. Sorokin, M.A. Vasiliev, Reducible higher-spin multiplets in flat and AdS spaces and their geometric frame-like formulation, arXiv: 0807.0206.
[19] Yu.M. Zinoviev, Frame-like gauge invariant formulation for massive high spin particles, arXiv: 0808.1778.
[20] Yu.M. Zinoviev, First order formalism for mixed symmetry tensor fields, hep-th/0304067.
[21] K.B. Alkalaev, O.V. Shaynkman, M.A. Vasiliev, On the frame-like formulation of mixed-symmetry massless fields in $(A)dS_{(d)}$, Nucl. Phys. B 692 (2004) 363, hep-th/0311164.
[22] K.B. Alkalaev, Two-column higher spin massless fields in $AdS_{(d)}$, Theor. Math. Phys. 140 (2004) 1253, hep-th/0311212.
[23] K.B. Alkalaev, O.V. Shaynkman, M.A. Vasiliev, Lagrangian formulation for free mixed-symmetry bosonic gauge fields in $(A)dS_{(d)}$, JHEP 0508 (2005) 069, hep-th/0501108.
[24] K.B. Alkalaev, O.V. Shaynkman, M.A. Vasiliev, Frame-like formulation for free mixed-symmetry bosonic massless higher-spin fields in $AdS_{(d)}$, hep-th/0601225.
[25] E.D. Skvortsov, Frame-like actions for massless mixed-symmetry fields in Minkowski space, arXiv: 0807.0903.

[26] Yu.M. Zinoviev, First order formalism for massive mixed symmetry tensor fields in Minkowski and $(A)dS$ spaces, hep-th/0306292.
[27] I.L. Buchbinder, V.A. Krykhtin, Gauge invariant Lagrangian construction for massive bosonic higher spin fields in D dimensions, Nucl. Phys. B 727 (2005) 537, hep-th/0505092.
[28] I.L. Buchbinder, V.A. Krykhtin, L.L. Ryskina, H. Takata, Gauge invariant Lagrangian construction for massive higher spin fermionic fields, Phys. Lett. B 641 (2006) 386, hep-th/0603212.
[29] I.L. Buchbinder, V.A. Krykhtin, P.M. Lavrov, Gauge invariant Lagrangian formulation of higher spin massive bosonic field theory in AdS space, Nucl. Phys. B 762 (2007) 344, hep-th/0608005.
[30] I.L. Buchbinder, V.A. Krykhtin, A.A. Reshetnyak, BRST approach to Lagrangian construction for fermionic higher spin fields in $(A)dS$ space, Nucl. Phys. B 787 (2007) 211, hep-th/0703049.
[31] Yu.M. Zinoviev, Gauge invariant description of massive high spin particles, Preprint 83-91, IHEP, Protvino, 1983.
[32] S.M. Klishevich, Yu.M. Zinoviev, On electromagnetic interaction of massive spin-2 particle, Phys. At. Nucl. 61 (1998) 1527, hep-th/9708150.
[33] Yu.M. Zinoviev, On massive high spin particles in $(A)dS$, hep-th/0108192.
[34] R.R. Metsaev, Gauge invariant formulation of massive totally symmetric fermionic fields in $(A)dS$ space, Phys. Lett. B 643 (2006) 205–212, hep-th/0609029.
[35] P. de Medeiros, Massive gauge-invariant field theories on spaces of constant curvature, Class. Quantum Grav. 21 (2004) 2571, hep-th/0311254.
[36] M. Bianchi, P.J. Heslop, F. Riccioni, More on La Grande Bouffe, JHEP 0508 (2005) 088, hep-th/0504156.
[37] K. Hallowell, A. Waldron, Constant curvature algebras and higher spin action generating functions, Nucl. Phys. B 724 (2005) 453, hep-th/0505255.
[38] L. Brink, R.R. Metsaev, M.A. Vasiliev, How massless are massless fields in AdS_d, Nucl. Phys. B 586 (2000) 183, hep-th/0005136.
[39] S. Deser, A. Waldron, Gauge invariance and phases of massive higher spins in $(A)dS$, Phys. Rev. Lett. 87 (2001) 031601, hep-th/0102166.
[40] S. Deser, A. Waldron, Partial masslessness of higher spins in $(A)dS$, Nucl. Phys. B 607 (2001) 577, hep-th/0103198.
[41] S. Deser, A. Waldron, Null propagation of partially massless higher spins in $(A)dS$ and cosmological constant speculations, Phys. Lett. B 513 (2001) 137, hep-th/0105181.
[42] T. Garidi, What is mass in desitterian physics? hep-th/0309104.
[43] A.Yu. Artsukevich, M.A. Vasiliev, On dimensional degression in $AdS_{(d)}$, arXiv: 0810.2065.