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Probing the mechanism of EWSB with a rho-parameter defined in terms of Higgs couplings

J.L. Díaz-Cruz, D.A. López-Falcón

Instituto de Física, BUAP, Apartado Postal J-48, Pue. 72570, Puebla, Mexico

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Abstract

A definition of the rho-parameter based on the Higgs couplings with the gauge bosons, $\rho_{H_i} \equiv g_{H_i WW} / (g_{H_i ZZ} c_W^2)$, is proposed as a new probe into the origin of the mechanism of electroweak symmetry breaking. While $\rho_{h_{SM}} = 1$ holds in the standard model, deviations from one for ρ_{H_i} are predicted in models with extended Higgs sector. We derive a general expression of ρ_{H_i} for a model with arbitrary Higgs multiplets, and discuss its size within the context of specific models with Higgs triplets, including the “Little Higgs” models recently proposed. We find the even for Higgs sectors that incorporate the custodial symmetry to make $\rho = 1$, one could have $\rho_{H_i} \neq 1$, which could be tested at the level of a few percent, with the precision Higgs measurements expected at the next linear collider (NLC).

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1. Introduction

The mechanism of electroweak symmetry breaking (EWSB), which is triggered spontaneously through a Higgs doublet in the minimal standard model (SM), has remained without direct experimental verification so far. Precision measurements of electroweak observables constrain the Higgs mass below about 200 GeV at 95% CL [1–3] within the standard model. Thus, it is expected that a Higgs particle could be discovered at the Run 2 of the Tevatron, provided sufficient luminosity is achieved [4]. But it is intriguing to notice that the EW observables prefer a SM like Higgs with mass below 114.1 GeV [1,3], which is the present lower limit from LEP 2. The data indicate that the Higgs boson should have already been discovered [1], and the fact that it was not, could be taken as a hint of new physics, which could be related with the freedom to choose the Higgs sector [3]. Extensions of the Higgs sector have been proposed for a while [5], and in particular models with Higgs triplets (real or complex) have been considered well motivated, partly because such representations arise in the context of left-right symmetric models [6], or are associated with low-energy mechanisms aimed to generate neutrino masses [7], as an alternative to the usual see-saw mechanism. More recently, Higgs triplets with \mathcal{O} (TeV) masses, have been predicted in connection with the

E-mail addresses: ldiaz@sirio.ifuap.buap.mx (J.L. Díaz-Cruz), dennys@sirio.ifuap.buap.mx (D.A. López-Falcón).

URL: <http://www.ifuap.buap.mx/~dennys/>.

so-called “Little Higgs” models [8], which attempt to explain the required lightness of the Higgs as being associated with a global symmetry.

Models with Higgs triplets can violate the custodial symmetry $SU(2)_c$ of the Higgs–gauge sectors. This symmetry protects the relation between the gauge boson masses and the weak mixing angle, which can be conveniently parameterized through Veltman’s rho-parameter [9], i.e., $\rho = m_W^2/m_Z^2 c_W^2$, which is equal to one at tree-level; loop corrections to this parameter could be very important, as it was exemplified by the prediction of a top quark heavier than originally expected. However, when one considers models with Higgs triplets, with their neutral component acquiring a v.e.v. that contributes to EWSB, then the ρ -parameter could deviate from one even at tree-level. Several Higgs triplets, with ad hoc quantum numbers, are required in order to preserve the custodial symmetry [10].

A simple analysis of the SM Lagrangian reveals that the gauge boson masses and their Higgs couplings originate from the terms:

$$(D^\mu \Phi)^\dagger (D_\mu \Phi) = \Phi^{0*} \Phi^0 [g^2 W^{+\mu} W_\mu^- + g'^2 Z^\mu Z_\mu] + \dots \quad (1)$$

After SSB, one can write the neutral component in terms of the SM Higgs boson (h_0) and the Goldstone boson (G_Z^0), i.e., $\Phi^0 = (v + h^0 + i G_Z^0)/\sqrt{2}$, and the gauge bosons (W^\pm and Z^0) acquire the masses: $m_W^2 = g^2 v^2/4$ and $m_Z^2 = g'^2 v^2/4$, respectively, with $g' = g/c_W$. In this case it happens that the same source of EWSB that contributes to the gauge boson masses, induces the Higgs–gauge couplings, which in turn are given by: $g_{hWW} = g^2 v/2$, $g_{hZZ} = g'^2 v/2$, and therefore one can define the parameters ρ and ρ_h , which satisfy:

$$\rho \equiv \frac{m_W^2}{m_Z^2 c_W^2} = 1 = \frac{g_{hWW}}{g_{hZZ} c_W^2} \equiv \rho_h. \quad (2)$$

We could also express this result by saying that both the neutral Higgs and Goldstone boson have the same couplings to the gauge bosons in the SM. Thus, if the SM is the correct theory of EWSB, a measurement of the Higgs–gauge couplings should give $\rho_h = 1$. However, small deviation from one for ρ_H can be expected to appear because of radiative effects, while the experimental value of ρ_h will deviate from one because of the systematic and statistical errors.

Furthermore, when one considers physics beyond the SM aimed to explain EWSB, it is conceivable that the Goldstone bosons could have a different origin from other neutral scalar of the model, as it could happen in composite scenarios. Alternatively, even if both the Higgs and Goldstone bosons have a common origin, their Higgs–gauge boson couplings could have different values, either because of mixing factors or because of renormalization effects. In all these cases, one would have $\rho \neq \rho_H$. Given the possibility that a scalar particle could be detected in the near future, it will be important to verify whether this particle is indeed a type of Higgs boson, and the parameter ρ_H could play a major role in this regard. This will be illustrated in the next sections with several examples.

The organization of this Letter goes as follows. In Section 2, we shall present a general expression for ρ_H for a Higgs multiplet of arbitrary isospin T and hypercharge Y ; its size is discussed in detail within the context of a minimal extension of the SM that includes one doublet and a real ($Y = 0$) Higgs triplet; one of our main result is the argument that $\rho \simeq 1$ does not implies $\rho_H = 1$. We shall also evaluate a similar parameter, but in terms of the Higgs decay widths, which would be closer to the output from future high-precision experiments for the Higgs boson. We then discuss, in Section 2.2, a model with extended Higgs sectors, which do respects the custodial symmetry, i.e., $\rho = 1$, but the Higgs particles do not necessarily satisfy $\rho_{H_i} = 1$. Then, in Section 4 we shall discuss the above parameter, for the Higgs sector that arises within the context of the “Little Higgs” model. Finally, we shall present our conclusions in Section 5.

2. Higgs multiplets and the rho-parameter

2.1. A general expression for ρ_H

Let us consider a model with an arbitrary Higgs sector, consisting of a number of Higgs multiplets Φ_K of isospin T_K and hypercharge Y_K . From the expression for the kinetic terms, written in terms of the covariant derivative, one obtains the gauge boson masses, which satisfy the following expression for the rho-parameter,

$$\rho = \frac{\sum_K [T_K(T_K + 1) - \frac{1}{4}Y_K^2] v_K^2 c_K}{\sum_K \frac{1}{2}Y_K^2 v_K^2}, \quad (3)$$

where v_K denotes the v.e.v. of the neutral component of the Higgs multiplet, while $c_K = 1/2$ (1) for real (complex) representations. It is well known that Higgs representations for which $T_K(T_K + 1) = \frac{3}{4}Y_K^2$, satisfy $\rho = 1$, regardless of their v.e.v.'s. Examples of this case are: $(T, Y) = (1/2, 1), (3, 4), \dots$. Alternatively, one could choose ad hoc v.e.v.'s for models with several types of Higgs multiplets, such as triplets, to have $\rho = 1$.

On the other hand, when one writes down the gauge boson coupling with the neutral Higgs components Φ_K^0 , which are weak eigenstates, ρ_H satisfies a similar relation, namely:

$$\rho_{\phi_K^0} = \frac{[T_K(T_K + 1) - \frac{1}{4}Y_K^2] v_K^2 c_K}{\frac{1}{2}Y_K^2 v_K^2}. \quad (4)$$

Thus, whatever choice makes $\rho = 1$ for the Higgs multiplet Φ_K , it will also make $\rho_{\phi_K^0} = 1$. However, when one has several multiplets, one needs to consider the Higgs mass eigenstates instead, which are indeed the ones that could be detected and probed at future colliders. Thus, we have to consider the rotations that diagonalizes the real parts of the neutral components, such that the Higgs mass eigenstates H_i are related to the weak eigenstates $\text{Re } \phi_K^0$ as: $\text{Re } \phi_K^0 = U_{Ki} H_i$. Then, the rho-parameter for the Higgs bosons H_i is given by:

$$\rho_{H_i} = \frac{\sum_K [T_K(T_K + 1) - \frac{1}{4}Y_K^2] v_K^2 c_K U_{Ki}}{\sum_K \frac{1}{2}Y_K^2 v_K^2 U_{Ki}}. \quad (5)$$

From this important relation, we can discuss several consequences:

- (1) For models that contains several Higgs multiplets of the same type (say, doublets), for which $T_K(T_K + 1) = \frac{1}{4}Y_K^2$, one gets $\rho_{H_i} = 1$ (as well as $\rho = 1$), because U_{Ki} factorize out in Eq. (5).
- (2) On the other hand, for a model that includes doublets and some other multiplet (say, triplets), for which $\rho \simeq 1$ is satisfied with a hierarchy of v.e.v.'s, i.e., $v_K \ll v_D$, then one has that ρ_{H_i} could be significantly different from one (as will be shown next).
- (3) Finally, if one makes $\rho = 1$ by arranging the v.e.v.'s of several multiplets (as in the model to be discussed in Section 3), then because of the factors U_{Ki} , it turns out that in general $\rho = 1$ does not necessarily imply $\rho_H = 1$, and this could provide an important test of the type of Higgs multiplet that participates in EWSB.

2.2. A model with one doublet and one real triplet

We shall evaluate now the size of ρ_H for an extension of the SM, where the Higgs sector includes one real ($Y = 0$) Higgs triplet, $\mathcal{E} = (\xi^+, \xi^0, \xi^-)$, in addition to the usual SM Higgs doublet Φ . The Higgs potential of the model is written as [11]:

$$V(\Phi, \mathcal{E}) = -\mu_d^2 \Phi^\dagger \Phi + \lambda_1 (\Phi^\dagger \Phi)^2 - \mu_{\text{tr}}^2 \mathcal{E}^\dagger \mathcal{E} + \lambda_2 (\mathcal{E}^\dagger \mathcal{E})^2 + \lambda_3 \Phi^\dagger \Phi \mathcal{E}^\dagger \mathcal{E} - \mu_{\text{dtr}} [\Phi^\dagger (\mathcal{E}_{\text{lin}} \cdot \tau) \Phi], \quad (6)$$

where the last term involves the linear form, namely:

$$\mathcal{E}_{\text{lin}} = \left(\frac{1}{\sqrt{2}}(\xi^+ + \xi^-), \frac{i}{\sqrt{2}}(\xi^+ - \xi^-), \xi^0 \right) \quad (7)$$

and τ is the vector of Pauli's spin matrices.

After constructing the mass matrices, and performing its diagonalization, we arrive to the following mass eigenstates:

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h_d \\ h_{\text{tr}} \end{pmatrix}, \quad (8)$$

where $h_d = \text{Re } \Phi^0$ and $h_{\text{tr}} = \text{Re } \xi^0$; while the mixing angle, α , is defined by:

$$\tan 2\alpha = \frac{4v_D v_{T0}(2\lambda_3 v_{T0} + \sqrt{2}\mu_{\text{dtr}})}{(8\lambda_1 v_{T0} + \sqrt{2}\mu_{\text{dtr}})v_D^2 - 8\lambda_2 v_{T0}^3}, \quad (9)$$

here $v_D = \langle h_d \rangle$ and $v_{T0} = \langle h_{\text{tr}} \rangle$.

The Higgs–gauge Lagrangian is given by:

$$\mathcal{T}_{\text{Cin}} = (D_\Phi^\mu \Phi)^\dagger (D_{\Phi_\mu} \Phi) + (D_\Xi^\mu \Xi)^\dagger (D_{\Xi_\mu} \Xi), \quad (10)$$

where

$$D_\Phi^\mu = \begin{pmatrix} \partial^\mu + i g s_W (A^\mu - Z^\mu t_W) & \frac{i}{\sqrt{2}} g W^{\mu+} \\ \frac{i}{\sqrt{2}} g W^{\mu-} & \partial^\mu - \frac{i}{2} \frac{g}{c_W} Z^\mu \end{pmatrix}, \quad (11)$$

and

$$D_\Xi^\mu = \begin{pmatrix} \partial^\mu + i g (Z^\mu c_W + A^\mu s_W) & i g W^{\mu+} & 0 \\ i g W^{\mu-} & \partial^\mu & i g W^{\mu+} \\ 0 & i g W^{\mu-} & \partial^\mu - i g (Z^\mu c_W + A^\mu s_W) \end{pmatrix}. \quad (12)$$

From this Lagrangian we can identify the masses of the gauge bosons:

$$m_W^2 = \frac{g^2}{4}(v_D^2 + 4v_{T0}^2), \quad m_Z^2 = \frac{m_W^2}{c_W^2} \left(\frac{v_D^2}{v_D^2 + 4v_{T0}^2} \right), \quad (13)$$

and the couplings hWW , hZZ , HWW and HZZ , where $h(H)$ corresponds to the lighter SM-like (heavier) neutral Higgs mass eigenstate:

$$\begin{aligned} g_{hWW} &= g m_W \cos \alpha \left[1 + \tan \alpha \left(\frac{\Delta \rho}{\rho} \right)^{1/2} \right], & g_{hZZ} &= \frac{g m_W}{c_W^2} \cos \alpha, \\ g_{HWW} &= g m_W \sin \alpha \left[\cot \alpha \left(\frac{\Delta \rho}{\rho} \right)^{1/2} - 1 \right], & g_{HZZ} &= \frac{g m_W}{c_W^2} \sin \alpha. \end{aligned} \quad (14)$$

Therefore, in this model we have: $\rho = 1 + 4v_{T0}^2/v_D^2 \equiv 1 + \tan^2 \beta$ and $\rho_h^2 = [1 + \tan \alpha (\Delta \rho / \rho)^{1/2}]^2$ and $\rho_H^2 = [\cot \alpha (\Delta \rho / \rho)^{1/2} - 1]^2$, which are plotted in Fig. 1, as a function of α (we are plotting the square values, just to get positive defined quantities, as future colliders will not know about signs for Higgs couplings). For $\Delta \rho / \rho$, which depends on the parameters of the model, we take the maximum value allowed by data [12], i.e., $\Delta \rho / \rho \simeq 1\%$. We can appreciate that ρ_h can deviate significantly from one for $\alpha \rightarrow \pi/2$, while ρ_H can show large deviating from the SM prediction for $\alpha \rightarrow 0$. Thus clearly $\rho_h \neq \rho \neq \rho_H$.

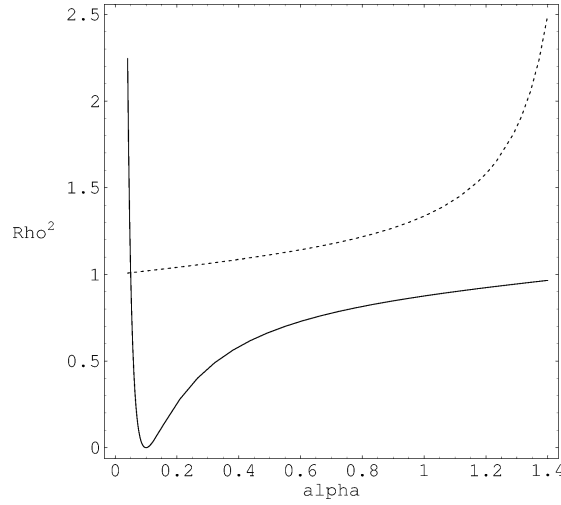


Fig. 1. ρ_h (dotted line) and ρ_H (solid line) as a function of the α mixing-angle.

Given the estimated precision expected for the measurements of the Higgs couplings at NLC, in particular, for the ratios of Higgs–gauge couplings which were analyzed in [13], it happens that ρ_{H_i} could be measured with a precision of order 2%, which will allow to constrain considerably the parameter α in this Higgs triplet model.

On the other hand, one could also use the prediction for the Higgs decays into gauge bosons as a possible test of violations of the custodial symmetry. For the real decays one has:

$$\begin{aligned}
 \Gamma(h \rightarrow WW) &= \cos^2 \alpha \left[1 + \tan \alpha \left(\frac{\Delta \rho}{\rho} \right)^{1/2} \right]^2 \Gamma(h_{\text{SM}} \rightarrow WW), \\
 \Gamma(h \rightarrow ZZ) &= \cos^2 \alpha \Gamma(h_{\text{SM}} \rightarrow ZZ), \\
 \Gamma(H \rightarrow WW) &= \sin^2 \alpha \left[1 + \tan \alpha \left(\frac{\Delta \rho}{\rho} \right)^{1/2} \right]^2 \Gamma(h_{\text{SM}} \rightarrow WW), \\
 \Gamma(H \rightarrow ZZ) &= \sin^2 \alpha \Gamma(h_{\text{SM}} \rightarrow ZZ).
 \end{aligned} \tag{15}$$

Then the ratio $R_{\Gamma_h} = \Gamma(h \rightarrow WW)/2\Gamma(h \rightarrow ZZ)$, is given by:

$$R_{\Gamma_h} = \frac{\Gamma(h_{\text{SM}} \rightarrow WW)}{2\Gamma(h_{\text{SM}} \rightarrow ZZ)} \left[1 + \tan \alpha \left(\frac{\Delta \rho}{\rho} \right)^{1/2} \right]^2, \tag{16}$$

while the ratio $R_{\Gamma_H} = \Gamma(H \rightarrow WW)/2\Gamma(H \rightarrow ZZ)$, is given by:

$$R_{\Gamma_H} = \frac{\Gamma(h_{\text{SM}} \rightarrow WW)}{2\Gamma(h_{\text{SM}} \rightarrow ZZ)} \left[\cot \alpha \left(\frac{\Delta \rho}{\rho} \right)^{1/2} - 1 \right]^2. \tag{17}$$

This ratios are plotted in Fig. 2 as a function of the Higgs mass, for two fixed values of α (0.04 and $\pi/4$), which represent two typical cases of small and large mixing, respectively. In this plot, we have included the decays into one real and one virtual gauge boson, $(h, H) \rightarrow VV^*$, for the appropriate range of Higgs masses.

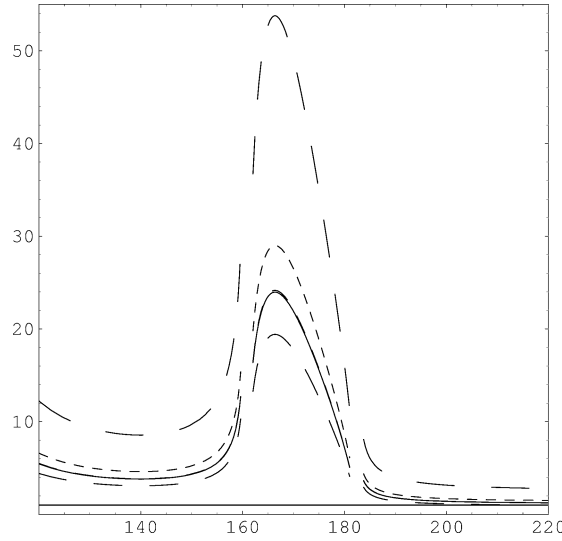


Fig. 2. R_F as a function of m_h [GeV]. The solid line corresponds to the SM, while the short dashed lines corresponding to lighter Higgs (h). The one closer to the SM line (almost overlapping it) corresponds to $\alpha = 0.04$; the next one corresponds to $\alpha = \frac{\pi}{4}$. The long-dashed lines (upper and lower), correspond to the heavy Higgs (with mixing: $\alpha = \frac{\pi}{4}$ and 0.04, respectively). The horizontal straight line indicates the asymptotic (SM) value.

3. An extended model with custodial symmetry

The Higgs sector can be extended to include extra Higgs multiplets in a manner that respects the custodial symmetry. A minimal model with Higgs triplets that gives $\rho = 1$ was discussed in reference [10], and studied in further detail in [14]. This model includes a real ($Y = 0$) triplet, $\mathcal{E} = (\xi^+, \xi^0, \xi^-)$, and a complex ($Y = 2$) Higgs triplet, $\chi = (\chi^{++}, \chi^+, \chi^0)$, in addition to the SM Higgs doublet, $\Phi = (\phi^+, \phi^0)$. The v.e.v. of the neutral components can be chosen such that $\langle \chi^0 \rangle = v_{T2}$, $\langle \xi^0 \rangle = v_{T0}$ and $\langle \phi^0 \rangle = v_D$. Then, when $v_{T2} = v_{T0} = v_T$, the gauge boson masses are given by: $m_W^2 = m_Z^2 c_W^2 = \frac{1}{4} g^2 v^2$, with $v^2 = v_D^2 + 8v_T^2$; in this way one obtains, $\rho = 1$.

The Higgs bosons can be classified according to their transformation properties under the custodial symmetry $SU(2)_c$. The spectrum includes a fiveplet $H_5^{+++,+,0,-,-}$, a threeplet $H_3^{+,0,-}$, and two singlets H_1^0 and $(H')_1^0$. While H_3^0 does not couple to the gauge boson pairs WW and ZZ , the coupling of the remaining neutral states can be written as:

$$g_{H_i^0 WW} = g m_W f_{H_i^0}, \quad g_{H_i^0 ZZ} = \frac{g m_W}{c_W^2} g_{H_i^0}, \quad (18)$$

where the coefficients $f_{H_i^0}$ and $g_{H_i^0}$ are shown in Table 1. From this table we conclude that $\rho_{H_1^0} = \rho_{(H')_1^0} = 1$, while $\rho_{H_5^0} = 1/2$. Thus, using our definition of the rho-parameter, one can clearly distinguish a Higgs state of the type H_5^0 , which transforms non-trivially under the custodial symmetry, from the states H_1^0 and $(H')_1^0$, which are singlets under $SU(2)_c$. However, it should be said that these states are not yet mass eigenstates.

While H_1^0 and $(H')_1^0$ predict $\rho_{H_i} = 1$, their couplings with gauge bosons deviate from the SM prediction. Thus, in order to probe this sector of the model, one could compare the decay widths $\Gamma(H_i^0 \rightarrow ZZ)$, or $\Gamma(H_i^0 \rightarrow WW)$, and using the expected precision on the Higgs measurement, determine that range of parameters that could be excluded.

Table 1

Coefficients $f_{H_i^0}$ and $g_{H_i^0}$ for the Higgs–gauge boson couplings. $t_H \equiv 2\sqrt{v_{T0}^2 + v_{T2}^2}/v_D$

H_i^0	$f_{H_i^0}$	$g_{H_i^0}$
H_1^0	c_H	c_H
$(H')_1^0$	$\frac{2\sqrt{2}}{\sqrt{3}}s_H$	$\frac{2\sqrt{2}}{\sqrt{3}}s_H$
H_3^0	0	0
H_5^0	$\frac{1}{\sqrt{3}}s_H$	$-\frac{2}{\sqrt{3}}s_H$

On the other hand, in terms of mass eigenstates the Higgs–gauge boson couplings induce a ρ_{H_i} parameter, whose expression is given by:

$$\rho_{H_i} = \frac{\frac{1}{2}v_D^2 U_{1i} + v_T^2 U_{2i} + v_T^2 U_{3i}}{\frac{1}{2}v_D^2 U_{1i} + 2v_T^2 U_{2i}}. \quad (19)$$

Thus, as anticipated in Section 2, the choice $v_{T2} = v_{T0} = v_T$, which makes $\rho = 1$, does not imply that $\rho_{H_i} = 1$. In fact, to get $\rho_{H_i} = 1$, for Higgs states that transforms as singlets under the custodial symmetry one would need all the Higgs interactions, including the ones appearing in the Higgs potential to respect the symmetry $SU(2)_c$.

4. Higgs triplets from the Little Higgs models

A new approach was recently proposed to address the naturalness problem of the Higgs sector, dubbed the “little Higgs models”, where the Higgs mass is protected from acquiring quadratic divergences by being promoted as a pseudo-Goldstone boson of a global symmetry [8]. The SM Higgs acquires mass via symmetry breaking at the EW scale (v). While the global symmetry is broken at high-energy scale Λ_s . The important new feature of these models is that the Higgs remains light thanks to the global symmetry, which includes new fields that cancel the quadratic divergences. Furthermore, these extra Higgs fields exist as Goldstone boson multiplets from the global symmetry breaking.

A minimal model, called the “littlest Higgs”, is based on a global symmetry $SU(5)$ which is broken into $SO(5)$ at the scale $\Lambda_s = 4\pi f$, while the locally gauged subgroup is $[SU(2) \times U(1)]^2$, which in turn breaks into the EW gauge symmetry of the SM. This leaves 14 Goldstone bosons, including a real singlet and a real triplet, which become the longitudinal modes of the heavy gauge bosons, as well as a complex doublet and complex triplet, which acquire masses radiatively, of order v and f , respectively. Thus, the “littlest Higgs model”, predicts the existence of several states with $\mathcal{O}(\text{TeV})$ masses, which give place to violations of the custodial symmetry [15].

Following [15] one has that the light (SM-like) gauge bosons masses contribute to the rho-parameter, i.e., $\rho = M_{W_L}^2/M_{Z_L}^2 c_W^2 = 1 + \Delta\rho$, with:

$$\Delta\rho = Ar_f^2 + Br_t^2, \quad (20)$$

where $A = \frac{5}{4}(c'^2 - s'^2)^2$, $B = -4$, $r_f = v/f$, and $r_t = v'/v$; v' denotes the v.e.v. of the Higgs triplet of the model.

On the other hand, for the light Higgs state h , the model predicts the following Higgs–gauge couplings,

$$\begin{aligned} g_{hWW} &= \frac{ig^2v}{2} \left[1 + \left(\frac{1}{2}(c^2 - s^2)^2 - \frac{1}{3} \right) r_f^2 - \frac{1}{2}s_0^2 - 2\sqrt{2}s_0r_t \right], \\ g_{hZZ} &= \frac{ig^2v}{2c_W^2} \left[1 - \left(\frac{5}{2}(c'^2 + s'^2)^2 + \frac{1}{2}(c^2 - s^2)^2 - \frac{1}{3} \right) r_f^2 - \frac{1}{2}s_0^2 + 4\sqrt{2}s_0r_t \right]. \end{aligned} \quad (21)$$

For the purpose of comparison with ρ , we expand ρ_h in terms of r_f and r_t , which gives:

$$\rho_h = 1 + \Delta\rho_h = 1 + A' r_f^2 + B' r_t^2 \quad (22)$$

and now: $A' = (c^2 - s^2)^2 + \frac{5}{2}(c'^2 - s'^2)^2$, and $B' = -6\sqrt{2}s_0$. Therefore, since $A \neq A'$, $B \neq B'$ one clearly has: $\rho \neq \rho_H$. Thus, a measurement of the Higgs couplings at NLC will provide an independent test of the underlying symmetry of the Higgs sector.

For instance, when $\theta = \theta' = \pi/4$, i.e., $A = 0$, and $r_t = 0$, then $\Delta\rho = 0$ exactly, thus, the custodial symmetry is preserved and $\rho_h = 1$ too. Furthermore, even if $r_t = r_f/4 = 1/20$ (maximum value allowed in Ref. [15]), one gets $\Delta\rho \simeq 1\%$ which lays within the experimental limits. In general, for values of parameters $s_0 \simeq 2\sqrt{2}r_t$, $0 \leq r_t < r_f/4$, $1/20 \leq r_f \leq 1/5$, $1/10 \leq \cot\theta = c/s \leq 2$, and $1/10 \leq \tan\theta' = s'/c' \leq 2$, one obtains that $\Delta\rho$ is within the experimental limits. However, even for $\theta = \theta' = \pi/4$, i.e., $A' = 0$ and $r_t = 1/20$, one gets $\rho_H \simeq 0.91$ which could be probed at NLC.

5. Conclusions and discussion

In this Letter we proposed a definition of the rho-parameter based on the Higgs couplings with the gauge bosons, namely, $\rho_{H_i} \equiv g_{H_i WW}/(g_{H_i ZZ} c_W^2)$, as a possible test of the custodial symmetry. We discuss the size of such violation in the context of general models with Higgs triplets, including the “Little Higgs” model recently proposed. We find that even for Higgs models that incorporate the custodial symmetry, to make $\rho = 1$, the Higgs couplings allow $\rho_H \neq 1$. Furthermore, in models where $\rho \simeq 1$ we also obtain that the Higgs bosons could acquire values of ρ_H significantly different from one. We find that ρ_H could be tested at the level of few percent, given the expected Higgs tests that may be achieved at the planned next linear collider (NLC), where we will be entering into the era of precision measurements for the Higgs sector. Violations of the custodial symmetry could also be tested through the ratio of decay widths, $R_\Gamma = \frac{\Gamma(h \rightarrow WW)}{2\Gamma(h \rightarrow ZZ)}$, with similar precision.

In summary, given the possibility that an scalar particle could be detected in the near future, it will be important to verify whether this particle is indeed a type of Higgs boson, and the parameter ρ_H could play a major role in this regard. This parameter measures the transformation properties of the Higgs bosons under the custodial symmetry.

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