Simplified reliability method for spatially variable undrained engineered slopes

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Abstract
A random finite element analysis (RFEA) is employed to study the effect of shear strength spatial variability on the stability of undrained engineered slopes. Based on the RFEA simulation results for 34 (idealized) real engineered slope cases, it is concluded that the coefficient of variation in the spatially variable shear strength and the ratio (vertical scale of fluctuation)/(length of failure curve) have a major effect on the stability reliability. The RFEA simulation results are further used to derive simplified equations for the reliability analysis and the reliability-based design of undrained engineered slopes.

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Keywords: Spatial variability; Slope stability; Reliability; Random finite element; Scale of fluctuation

1. Introduction

Natural soil properties vary considerably in space, even within a single soil layer. In traditional stability analyses of slopes, the factor of safety (FS) is calculated using nominal values for the soil parameters, and the decision is typically made based on this nominal FS. However, due to the spatial variability of soil properties, the nominal FS may not reflect the actual safety status of a slope. To account for such variability, a probabilistic stability analysis is preferable when the safety status is quantified by the probability of failure ($p_f$).

In the literature, a number of researchers have performed probabilistic stability analyses for slopes (e.g., Li and Lumb, 1987; El-Ramly et al., 2002; Baecher and Christian, 2003; Griffiths and Fenton, 2004; Low et al., 2007; Cho, 2010; Wang et al., 2011). In particular, Li and Lumb (1987), El-Ramly et al. (2002), Low et al. (2007), Cho (2010), and Wang et al. (2011) have studied the effect of the spatial variability of soil properties on slope stability using limit equilibrium methods (LEMs). Nonetheless, a more robust method for evaluating the stability of slopes is the random finite element analysis (RFEA) (Griffiths and Fenton, 2004). This method combines the random field theory (Vanmarcke, 1977) with the elasto-plastic finite element analysis to account for the effect of spatial variability. RFEA captures the effect of soil spatial variability well, and it is able to simulate complex failure mechanisms. This approach has been used to quantify the $p_f$ of slopes (e.g., Griffiths and Fenton, 2004; Griffiths et al., 2009; Huang et al., 2010; Hicks and Spencer, 2010). Griffiths et al. (2010) have shown that the $p_f$ estimated using RFEA may differ significantly from that obtained using a random field LEM.

Among the available RFEA literature, most have considered hypothetical slopes. Griffiths and Fenton (2004), Chok et al. (2007), and Griffiths et al. (2009, 2010) investigated a simple
slope with a single soil layer. Huang et al. (2010) considered a slope with two soil layers. Hicks and Spencer (2010) investigated the three-dimensional effect of hypothetical slopes. Most of these studies assumed values for the scale of fluctuation.

The main purpose of this paper is to investigate how the $p_f$ of a spatially variable engineered slope may change with factors such as slope geometry, mean, coefficient of variation (COV), and scales of fluctuation of the spatially variable soil strength. RFEA is adopted in this study to obtain reliable $p_f$ estimates. In order to realistically understand the sensitivity of $p_f$, 34 idealized real engineered slope cases (cut and fill slopes) are analyzed with RFEA, where the random field is used to simulate the vertical and horizontal spatial variabilities of the soil shear strength. To make the analysis more realistic, the mean shear strength and its vertical scale of fluctuation are determined based on actual borehole data. The RFEA results for the 34 idealized real cases are then used to derive a simple relationship between $p_f$ and the nominal FS. This relationship is valuable for the simplified reliability analysis and the reliability-based design for the stability of spatially variable undrained engineered slopes.

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*a Assumed scale of fluctuation.
2. Deterministic slope stability analysis

This section presents the deterministic finite element analysis (FEA) for undrained slopes. RFEA will be presented in a later section. Thirty-four documented cases of undrained engineered slopes are collected from the literature. The basic information for the 34 cases is given in Table 1. There are seven cases of cut slopes and 27 cases of embankments (fill slopes). Slope heights range from 2 to 18.8 m, and slope angles range from 14 to 53°. The undrained shear strength of the soil is determined mostly from unconfined compression (UC) or field vane (FV) shear tests. The nominal factors of safety, calculated in the literature (mainly based on the Swedish circle method or Bishop’s method), are listed in Table 1 as FSd (under the ‘Ref.’ column).

The commercial FEA software, Abaqus (Abaqus/CAE, 2009), is adopted in this study. In FEA, soils are modeled as elasto-perfectly plastic with the Mohr–Coulomb failure criterion using 4-node plane strain quadrilateral reduced integration elements (CPE4R). For all 34 cases, the undrained shear strength (\(s_u\)) is the main input soil parameter. For most cases, the depth-dependent \(s_u\) borehole data are available from the site investigation. These \(s_u\) borehole data are first converted to the mobilized \(s_u\) (Mesri and Huvaj, 2007), denoted by \(s_u\text{mob}\). This depth-dependent \(s_u\text{mob}\) is then modeled as horizontal layers in FEA. For each layer, \(s_u\) is a constant value that is the average value in the \(s_u\text{mob}\) borehole data within that layer. Embankment fills are typically sandy or silty, having an effective friction angle \(\phi’\) and an effective cohesion of \(c’\). However, in FEA, the \(c’\) condition often causes numerical instability. This can be prevented by modeling the embankment layers as equivalent clay layers. As the embankment part typically fails in an active stress state (the vertical effective stress \(\sigma’_v\) = the maximum principal stress), the deviator stress at failure (\(\Delta\sigma_t\)) for an embankment layer is roughly as follows:

\[
\Delta\sigma_t = \sigma’_v - K_a \sigma’_v = [1 - \tan^2(45° - \phi’/2)]\sigma’_v
\]

where \(K_a\) is the active earth pressure coefficient. Therefore, the \(s_u\text{mob}\) for the equivalent clay layer is equal to one half of \(\Delta\sigma_t\), namely

\[
s_u\text{mob} = [1 - \tan^2(45° - \phi’/2)]\sigma’_v / 2
\]

The undrained modulus of elasticity (\(E\)) is estimated from \(s_u\) based on the PI-correlation mentioned in Kulhawy and Mayne (1990). Poisson’s ratio (\(\nu\)) for undrained clays is taken to be 0.49. The friction angle and the dilation angle are assumed to be zero, because the total stress analysis is adopted. Tension cracks are not considered in this study.

The factor of safety (FS) for each case is determined by the strength reduction method (Griffiths and Lane, 1999). The basic idea of the strength reduction method is to divide the shear strength of all the elements by a single strength reduction factor, and the FS is defined to be the strength reduction factor corresponding to “the onset of failure” in FEA. The onset of failure can be defined in various ways. One possibility is the non-convergence of FEA (Griffiths and Lane, 1999). In this study, a displacement-based criterion is taken to define the onset of failure, described as follows. In Griffiths and Lane (1999), the normalized maximum displacement is defined by \(E_{av}D_{max}/(\gamma_{av}H^2)\), where \(D_{max}\) is the maximum absolute nodal displacement in the entire FEA model, \(E_{av}\) is the average undrained modulus of elasticity of the soil layers, \(\gamma_{av}\) is the average unit weight of the soil layers, and \(H\) is the height of the slope. For an elastic slope, \(D_{max}\) is proportional to \(\gamma_{av}\) and \(H^2\), but inversely proportional to \(E_{av}\). The normalization is used to eliminate the effect of \((\gamma_{av}, H, E_{av})\). Namely, \(E_{av}D_{max}/(\gamma_{av}H^2)\) is a constant that does not depend on \((\gamma_{av}, H, E_{av})\), but only depends on the shape of the slope (e.g., the slope angle and the normalized depth of the hard stratum). For the displacement-based criterion adopted in this study, the normalized maximum displacement is calculated for each strength reduction factor, and the onset of failure is assumed to occur at the strength reduction factor where the normalized maximum displacement suddenly increases. The strength reduction factor at the point is, therefore, the FS.

The above displacement-based criterion gives FS results comparable to those obtained in Griffiths and Lane (1999) for an example in that paper. For this particular example, shown in Fig. 1, a two-layer undrained slope is considered, and the FS is determined for different \(s_u2/s_u1\) ratios using the strength reduction method, where \(s_u1\) and \(s_u2\) are the values of

![Fig. 1. Comparison of FS calculated with displacement-based criterion (this study) and Griffiths and Lane’s results: (a) geometry of example slope in Griffiths and Lane (1999); (b) comparison results.](image-url)
undrained shear strength for the upper and lower layers, respectively. $s_u/H$ is fixed at 0.25, while $s_u$ varies. The consistency between the displacement-based strength reduction method and Griffiths and Lane’s results is evident.

Using FEA with the displacement-based strength reduction method, the deterministic safety factors for the 34 cases are shown in Table 1 as $FS_d$ (under the column ‘This study’). These $FS_d$ are obtained based on the nominal (average) values for $s_{u(mob)}$ with all the horizontal thin layers being homogeneous (no random field). A possible explanation for the discrepancy between the $FS_d$ reported in the literature (column ‘Ref.’) and the $FS_d$ determined by FEA (column ‘This study’) is that $s_u$ is converted to $s_{u(mob)}$ in this study. Results from a typical case (case no. 10) are shown in Figs. 2–4. Fig. 2 shows the FEA slope model and the soil parameters for all layers. Fig. 3 shows the normalized maximum displacement versus strength reduction factor curve, where $FS_d$ is determined to be 0.97, as the strength reduction factor $\frac{1}{2} = 0.97$ is the point where the normalized maximum displacement suddenly increases. The following rule of thumb works well. Let the initial normalized maximum displacement be $A$ (see Fig. 3). Draw a horizontal line at $1.45 \times A$ (the horizontal dashed line in the figure). The intersection between the horizontal dashed line and the normalized maximum displacement curve is found to be fairly close to the point where the normalized maximum displacement suddenly increases. This $1.45 \times A$ rule of thumb is used for all 34 cases. Fig. 4 shows the plastic zone at failure for case no. 10, where $L_f$ denotes the total length of the failure curve (composed of few linear segments) in the deterministic FEA.

3. Random finite element analysis

In the random finite element analysis (RFEA), the $s_u$ for each horizontal layer is characterized by a random field with mean $= \mu$ and COV $= \nu$. Since $s_u$ is non-negative, the lognormal random field is adopted. If $s_u$ is lognormally distributed, $\ln(s_u)$ will follow a normal distribution with mean $= \lambda$ and standard deviation $= \xi$

$$\xi = \sqrt{\ln(1 + \nu^2)}$$
$$\lambda = \ln(\mu) - 0.5 \times \xi^2$$

A stationary lognormal random field is used to simulate the spatial variability of $s_u$ within each layer. The mean value of the random field is taken to be the average $s_{u(mob)}$ value within the layer. Except for $s_u$, other soil properties ($E, \nu, \gamma$) are assumed to be homogeneous within each layer.

The anisotropic spatial correlation is considered in the $s_u$ random field—the horizontal scale of fluctuation ($\delta_x$) is larger than the vertical scale of fluctuation ($\delta_z$). Phoon and Kulhawy (1999) have shown that $\delta_x$ may be more than one order of magnitude larger than $\delta_z$. The following anisotropic autocorrelation model is assumed as:

$$\rho(\Delta \chi, \Delta \zeta) = \exp\left(-\frac{2|\Delta \chi/\delta_x - 2|\Delta \zeta/\delta_z}{\delta_x/\delta_z}\right)$$

Fig. 3. Normalized maximum displacement versus strength reduction factor curve for case no. 10.

Fig. 4. Plastic zone developed at failure for case no. 10.
where \( \rho(\Delta x, \Delta z) \) is the autocorrelation function of \( s_u(x,z) \), and \( \Delta x \) and \( \Delta z \) are the horizontal and vertical distances between two points in space. It is clear that as \( \Delta x \) or \( \Delta z \) increases, the magnitude of correlation decreases. The same behavior has been observed for natural soils where soil properties are strongly correlated within a small interval and are weakly correlated for a large interval.

The vertical scale of fluctuation (\( \delta_z \)) for a particular case is evaluated by its \( s_{u(mob)} \) borehole data. The site investigations for the 34 cases are mostly based on UC and FV test results from few boreholes. Based on the \( s_{u(mob)} \) borehole data, \( \delta_z \) can be determined using an approximate method proposed by Vanmarcke (1977), namely

\[
\delta_z \approx 0.8 \bar{d} \tag{5}
\]

where \( \bar{d} \) is the average vertical interval of the intersection points between the \( s_{u(mob)} \) profile and its trend (\( t(z) \)). A typical example of estimating \( \delta_z \) is shown in Fig. 5. \( \delta_z \) is estimated by first calculating the spatial variability \( w(z) = \ln[s_{u(mob)}(z)] - \ln[t(z)] \). For the figure shown, \( \bar{d} = (d_1 + d_2 + \cdots + d_5)/5 \) and \( \delta_z \) is given by Eq. (5). The estimation of the trend \( t(z) \) typically requires judgments. For the \( s_{u(mob)} \) data points in the left plot of Fig. 5, \( t(z) \) is judged to consist of two line segments connected at \( z = 1.8 \) m (\( s_u \) near the ground surface becomes large due to over-consolidation). In this case, the data points above 1.8 m are used to find the shallow trend (e.g., regression using \( t = \alpha + \beta \times z \)), whereas those below 1.8 m are used to find the deep trend (e.g., regression using \( t = \alpha' + \beta' \times z \)). For other cases, the chosen trend function may change if the judgment is made in a different way.

There are more accurate techniques for estimating \( \delta_z \) (e.g., Fenton, 1999; Jaksa et al., 1999, 2004). However, Eq. (5) is adopted in this study because it can be easily implemented by engineers. The resulting \( \delta_z \) values for all 34 cases are listed in Table 1. There are seven cases where only trends \( t(z) \) are known, but the detailed \( s_u \) borehole data are not given. For these cases, \( \delta_z \) is assumed to be 2.5 m, the average value for the \( \delta_z \) of \( s_u \) reported in Phoon and Kulhawy (1999). It will be clear later that the ratio \( \delta_z/L_z \) plays a key factor (recall that \( L_z \) is the total length of the failure curve in the deterministic FEA). This ratio is also listed in Table 1. The \( \delta_z/L_z \) for the 34 cases varies from 0.015 to 0.098. Table 2 lists the statistics of \( \delta_z \), \( L_z \), and \( \delta_z/L_z \) for the 34 cases. It is virtually impossible to determine \( \delta_z \) from the limited number of boreholes for each case. As \( \delta_z \) is more than one order of magnitude larger than \( \delta_s \), three different ratios for \( \delta_z/\delta_s \), namely 10, 20, and 40, are studied.

For each case, the same \( V \), \( \delta_s \), and \( \delta_z \) are taken for all the soil layers (\( V \) is the inherent COV of the \( s_u \) random field). This is obviously a simplification. If we allow various layers to have different \( V \), \( \delta_s \), and \( \delta_z \), the sensitivity study will be intractable. Moreover, random fields for different soil layers are independently simulated. As the thickness of the soil layers (more than
(2 m) is typically larger than $\delta_z$, this assumption of independence between random fields is deemed acceptable.

3.1. Locally averaged random field

In FEA, it is necessary to simulate a single $s_u$ value for each element, rather than simulating the random field $s_u(x,z)$ within each element. A rational choice is to first find the local average of the random field $s_u(x,z)$ over the element and to assign this local average to this element. In this study, the random field is generated using the Fourier series method (Jha and Ching, 2013). This method is able to simulate locally averaged $\ln(s_u)$ for each element, and the locally averaged $s_u$ value for each element is simply the exponential of the locally averaged $\ln(s_u)$. This method is also applicable to non-rectangular elements when implemented with the Gauss quadrature.

In this study, nine ($3 \times 3$) Gauss points are used for each element. The local averaging process adopted in this paper for the lognormal random field is known to produce a mean $s_u$ that is less than the inherent mean of $s_u$ (Griffiths and Fenton, 2004) if $\delta_s$ or $\delta_z$ is significantly less than the element size. However, the FEA element size in this study is typically less than the adopted $\delta_s$ and $\delta_z$ for the 34 cases.

A typical realization of the $s_u$ random field ($V=0.3$, mean value as shown in Fig. 2) for case no. 10 with $\delta_s=2$ m and $\delta_z=40$ m ($\delta_s/\delta_z=20$) is shown in Fig. 6. As expected, $s_u$ in the vertical direction varies more rapidly than in the horizontal direction. After the $s_u$ random field is generated, the displacement-based strength reduction method is taken to determine the FS.

4. Effect of $s_u$ spatial variability on statistics of FS

Due to the randomness of the $s_u$ random field, FS varies from realization to realization. The statistics of FS depend on the inherent COV of the $s_u$ random field ($V$), the scales of fluctuation of the $s_u$ random field ($\delta_s$ and $\delta_z$), and the geometry of the slope at hand. The last factor (geometry) is grossly quantified by $L_f$. For each of the 34 cases, $\delta_s$ is identified from the $s_u$ (mob) borehole data based on the procedure described in Fig. 5, and $L_f$ is determined using a deterministic FEA. A sensitivity study is taken to quantify the effect of the two parameters, $V$ and $\delta_s$; $V$ is taken to be 0.1, 0.3, or 0.5 and $\delta_s$ is taken to be $10 \times \delta_z$, $20 \times \delta_z$, or $40 \times \delta_z$ (namely $\delta_s/\delta_z=10$, 20, or 40). Each of the 34 cases is analyzed under these $3 \times 3 = 9$ different combinations of ($V$, $\delta_s$), and 100 realizations of the $s_u$ random field are simulated for each combination to obtain the mean and the COV of the FS, denoted by $\mu_{FS}$ and $V_{FS}$, respectively. At the end, there are nine sets of ($\mu_{FS}$, $V_{FS}$) for each of the 34 cases. For case no. 10 with $V=0.3$ and $\delta_s/\delta_z=20$, the histogram for the simulated factors of safety is shown in Fig. 7(a)—$\mu_{FS}$ and $V_{FS}$ are 0.848 and 0.087, respectively, whereas the FS determined by the deterministic FEA ($FS_d$) is 0.97. Fig. 7(b) shows the QQ plot for the simulated FS for the lognormal distribution—the lognormal distribution is deemed proper. For most of the 34 cases, it is found that the
simulated factors of safety are roughly lognormally distributed regardless of \((V, \delta_{s}, \delta_{f})\), because the lognormal hypothesis cannot be rejected at a significant level of 5% in the K–S tests.

The mean value of the simulated FS \((\mu_{FS})\) is found to always be less than \(FS_{d}\), and the COV of the simulated FS \((V_{FS})\) is always less than the inherent COV \((V)\) of the \(s_{r}\) random field. For case no. 10 with \(V=0.3\) and \(\delta_{s}/\delta_{f}=20\), it is evident that \(\mu_{FS}=0.85<0.97=FS_{d}\) and that \(V_{FS}=0.087<0.3=V\). The former can be understood as the phenomenon whereby the critical slip curve seeks for the weak zone, whereas the latter can be understood as the phenomenon of the averaging effect along the critical slip curve. Both mechanisms are discussed and quantified in Ching and Phoon (2013) and Ching et al. (in press) for spatially variable soil masses subjected to uniform stress states. However, slopes are not subjected to uniform stress states, so the conclusions in Ching and Phoon (2013) and Ching et al. (in press) cannot be directly applied. Nonetheless, it is of interest to understand how the reduction in \(\mu_{FS}\) and \(V_{FS}\) can be correlated empirically to \(V, \delta_{s}, \delta_{f},\) and \(L_{f}\). In particular, the following correlations are studied:

1. The reduction ratio in the FS mean, namely \((FS_{d}-\mu_{FS})/FS_{d}\), versus the dimensionless factors \((V, \delta_{s}/\delta_{f}, \delta_{f}/L_{f})\).
2. The ratio in the COV \((V_{FS}/V)\) versus the dimensionless factors \((V, \delta_{s}/\delta_{f}, \delta_{f}/L_{f})\).

![Fig. 8. Relationship between \((FS_{d}-\mu_{FS})/FS_{d}\) and \((\delta_{s}/\delta_{f}, \delta_{f}/L_{f})\) for \(V=0.3\).](image)

![Fig. 9. Relationship between \((V_{FS}/V)\) versus \((V, \delta_{s}/\delta_{f}, \delta_{f}/L_{f})\).](image)

### 4.1. \((FS_{d}-\mu_{FS})/FS_{d}\) versus \((V, \delta_{s}/\delta_{f}, \delta_{f}/L_{f})\)

Fig. 8 shows the relationship between the FS reduction ratio \((FS_{d}-\mu_{FS})/FS_{d}\) and \((\delta_{s}/\delta_{f}, \delta_{f}/L_{f})\) for \(V=0.3\). It appears that \((FS_{d}-\mu_{FS})/FS_{d}\) is weakly correlated to \((\delta_{s}/\delta_{f}, \delta_{f}/L_{f})\). This weak correlation is also true for \(V=0.1\) and 0.5, although not shown. However, \((FS_{d}-\mu_{FS})/FS_{d}\) is strongly correlated to \(V\), as seen in Fig. 9. The trend (dashed line) in Fig. 9 is

\[
(FS_{d}-\mu_{FS})/FS_{d} \approx 0.7 \times V^{1.5} = 0.115 \times a_{V}
\]

where

\[
a_{V} = (V/0.3)^{1.5}
\]

The case with \(V=0.3\) is considered as the “baseline case”, and factor \(0.115=0.7 \times 0.3^{1.5}\) is the FS reduction ratio for the baseline case. Coefficient \(a_{V}\) quantifies the effect of \(V\) on the FS reduction ratio; it is used to fine tune the FS reduction ratio with respect to the baseline \(V=0.3\). One can easily verify that \(a_{V}\) becomes unity when \(V=0.3\). The strong correlation between \((FS_{d}-\mu_{FS})/FS_{d}\) and \(V\) makes sense, because \((FS_{d}-\mu_{FS})/FS_{d}\) depends on the tendency to seek for the weak zone, and the weak zone is more pronounced when \(V\) is large. The deviation of \((FS_{d}-\mu_{FS})/FS_{d}\) from the trend line is denoted by \(\varepsilon_{1}=(FS_{d}-\mu_{FS})/FS_{d}-0.7 \times V^{1.5}\). The variability in \(\varepsilon_{1}\) increases with \(V\). The standard deviation of \(\varepsilon_{1}\) is denoted by \(\sigma_{1}\)

\[
\sigma_{1} = 0.06 \times V^{0.85}
\]

As a result

\[
(FS_{d}-\mu_{FS})/FS_{d} = 0.115 \times a_{V} + \sigma_{1} \times Z_{1}
\]

where \(Z_{1}\) is modeled as a standard normal random variable \(N(0, 1)\). In other words

\[
\mu_{FS} = (1-0.115 \times a_{V} - \sigma_{1} \times Z_{1}) \times FS_{d}
\]

\[
= (1-0.115 \times a_{V} - 0.06 \times V^{0.85} \times Z_{1}) \times FS_{d}
\]

### 4.2. \((V_{FS}/V)\) versus \((V, \delta_{s}/\delta_{f}, \delta_{f}/L_{f})\)

The ratio \(V_{FS}/V\) is found to be strongly correlated to \(\delta_{f}/L_{f}\). Fig. 10 shows the relationship between \(V_{FS}/V\) and \((\delta_{s}/\delta_{f}, \delta_{f}/L_{f})\) when \(V\) varies among 0.1, 0.3, and 0.5. It is clear that \(V_{FS}/V\) increases with an increasing \(\delta_{f}/L_{f}\). Again, this makes sense, because \(\delta_{f}/L_{f}\) quantifies how large \(\delta_{f}\) is compared to \(L_{f}\). When \(\delta_{f}/L_{f}\) is large, the averaging effect along the critical slip curve is weak (hence, \(V_{FS}/V\) is large); this is because there are not many fluctuation cycles along the curve. Although not as strong as \(\delta_{f}/L_{f}\), the ratio \(\delta_{s}/\delta_{f}\) also affects \(V_{FS}/V\); \(V_{FS}/V\) slightly increases with \(\delta_{s}/\delta_{f}\). Larger \(\delta_{s}/\delta_{f}\) implies that \(\delta_{s}\) is larger; hence, the horizontal direction will have less variability, which reduces the averaging effect as well. Finally, \(V_{FS}/V\) increases with decreasing \(V\). This phenomenon cannot be easily explained by spatial averaging. In summary, \(\delta_{f}/L_{f}\) has a major effect on \(V_{FS}/V\); \(V\) also has a certain effect on \(V_{FS}/V\), whereas \(\delta_{s}/\delta_{f}\) has the least (but still noticeable) effect on \(V_{FS}/V\). Replacements for the \(\delta_{f}/L_{f}\) factor are attempted, e.g., the \(\delta_{s}/H\) ratio (\(H\) is the slope height) and the \(\delta_{s}/A\) ratio (\(A\) is the area of failed mass, i.e., the area above the failure path), but none of them provides a stronger correlation than \(V_{FS}/V\) versus \(\delta_{f}/L_{f}\).
These coefficients are used to fine tune the $V_{FS}/V$ ratio with respect to the baseline case. One can easily verify that these coefficients all become unity when $\delta_{i}/L_d=0.04$, $\delta_{i}/\delta_z=20$, and $V=0.3$.

The regression analysis shows that the ratio $V_{FS}/V$ can be effectively estimated with the following equation:

$$V_{FS}/V \approx \exp[0.6433 + 1.5027 \times \ln(\delta_z/L_d) + 0.1668 \times \ln(\delta_{i}/\delta_z) - 0.731 \times \ln(V) - 0.1691 \times \ln(V)^2 + 0.1655 \times \ln(\delta_{i}/L_d)^2]$$ (11)

The above equation can be re-written in such a way that the effects of $\delta_{i}/L_d$, $\delta_{i}/\delta_z$, and $V$ are separated. The effects of these three factors are summarized by the following three coefficients, $b_{\delta_{i}/L_d}$, $b_{\delta_{i}/\delta_z}$, and $b_V$:

$$V_{FS}/V \approx (V_{FS}/V)_{\delta_{i}/L_d=0.04, \delta_{i}/\delta_z=20, V=0.3} \times b_{\delta_{i}/L_d} \times b_{\delta_{i}/\delta_z} \times b_V$$ (12)

where $(V_{FS}/V)_{\delta_{i}/L_d=0.04, \delta_{i}/\delta_z=20, V=0.3}=0.2606$ is the $V_{FS}/V$ ratio for the “baseline case” with $\delta_{i}/L_d=0.04$, $\delta_{i}/\delta_z=20$, and $V=0.3$, and

$$b_{\delta_{i}/L_d} = \exp[3.1226 + 1.5027 \times \ln(\delta_{i}/L_d) + 0.1655 \times \ln(\delta_{i}/L_d)^2]$$

$$b_{\delta_{i}/\delta_z} = \exp[-0.4999 + 0.1668 \times \ln(\delta_{i}/\delta_z)]$$

$$b_V = \exp[-0.6349 - 0.731 \times \ln(V) - 0.1691 \times \ln(V)^2]$$ (13)

are the coefficients quantifying the effect of $\delta_{i}/L_d$, $\delta_{i}/\delta_z$, and $V$. These coefficients are used to fine tune the $V_{FS}/V$ ratio with respect to the baseline case. One can easily verify that these coefficients all become unity when $\delta_{i}/L_d=0.04$, $\delta_{i}/\delta_z=20$, and $V=0.3$. Fig. 11 shows the relationship between the actual $V_{FS}/V$ and $V_{FS}/V$ ratio estimated by Eq. (12).

$$V_{FS}/V = 0.2606 \times b_{\delta_{i}/L_d} \times b_{\delta_{i}/\delta_z} \times b_V + \sigma_2 \times Z_2 = 0.2606 \times b_{\delta_{i}/L_d} \times b_{\delta_{i}/\delta_z} \times b_V + 0.0466 \times Z_2$$ (14)

where $Z_2$ is modeled as a standard normal random variable $N(0, 1)$. Fig. 12 shows the $Z_1-Z_2$ plot for the 306 cases—they are roughly independent.

5. Simplified model for failure probability ($\mu_\varepsilon$)

Based on the above results, $\mu_{FS}$ and $V_{FS}$ can be estimated by the following:

$$\mu_{FS} = (1 - 0.115 \times a \varepsilon_v - 0.06 \times V^{0.85} \times Z_1) \times F_\varepsilon$$

$$V_{FS} = (0.2606 \times b_{\delta_{i}/L_d} \times b_{\delta_{i}/\delta_z} \times b_V + 0.0466 \times Z_2) \times V$$ (15)

The $(a, b)$ factors can be calculated based on $\delta_{i}/L_d$, $\delta_{i}/\delta_z$, and $V$ using Eqs. (7) and (13). The two standard normal random variables, $Z_1$ and $Z_2$, reflect the variability in the 34 cases. Namely, Eq. (15) with $Z_1=Z_2=0$ represents the average behavior over the 34 cases; it is not 100% accurate for all cases. There are many realistic factors that are not captured by Eq. (15), such as the detailed $x_p$ trend profile, the detailed slope geometry, the boundary conditions, etc. As a result, $Z_1=Z_2=0$ only represents the average behavior. For $Z_1=Z_2=1$, the calculated $\mu_{FS}$ is lower than the average, and the calculated $V_{FS}$ is higher than the average. Both are conservative. $Z_1=Z_2=1.5$ is a very conservative way to estimate $\mu_{FS}$ and $V_{FS}$. 
5.1. Simplified equation for reliability analysis

The \( p_t \) for a spatially variable undrained slope can be expressed as

\[
p_t = P(\text{FS} < 1) = P[\ln(\text{FS}) < 0]
\]

As \( \text{FS} \) is typically lognormally distributed, \( \ln(\text{FS}) \) is normally distributed, with mean \( \lambda_{\text{FS}} \) and \( \xi_{\text{FS}} \)

\[
\xi_{\text{FS}} = \sqrt{\ln(1 + V_{\text{FS}}^2)}
\]

\[
= \sqrt{\ln[1 + (0.2606 \times b_{\delta_z/L} \times b_{\delta_x/L} \times b_V + 0.0466 \times Z_2)^2 \times V^2]}
\]

(16)

\[
\lambda_{\text{FS}} = \ln(\mu_{\text{FS}}) - \frac{1}{2} \xi_{\text{FS}}^2
\]

\[
= \ln[(1 - 0.115 \times a_V - 0.06 \times V^{0.85} \times Z_1) \times FS_d] - \frac{1}{2} \xi_{FS}^2
\]

(17)

As a result

\[
p_t = P(\lambda_{\text{FS}} + \xi_{\text{FS}} Z < 0) = \Phi(-\lambda_{\text{FS}}/\xi_{\text{FS}})
\]

(18)

where \( Z \) is a standard normal random variable, and \( \Phi \) is the cumulative density function (CDF) for the standard normal random variable. The reliability index \( \beta \) is simply

\[
\beta = \lambda_{FS}/\xi_{FS}
\]

(19)

Consider the baseline case \((V=0.3, \delta_x/\delta_z=20, \text{and } \delta_z/L_d=0.04)\). For this case, all \((a, b)\) factors are unity

\[
\xi_{FS} = \sqrt{\ln(1 + [(0.2606 + 0.0466 \times Z_2) \times 0.3]^2)}
\]

\[
\lambda_{FS} = \ln[(0.885 - 0.06 \times 0.3^{0.85} \times Z_1) \times FS_d] - \frac{1}{2} \xi_{FS}^2
\]

(20)

Let us consider first the average behavior \((Z_1=Z_2=0)\). Hence, \( p_t = \Phi(-\lambda_{FS}/\xi_{FS}) \)

\[
= \Phi\left(-\ln (0.885 \times FS_d) - \frac{1}{2} \times 0.0781^2\right)/0.0781
\]

(21)

(22)
The $p_t$ versus $FS_d$ relationship for $Z_1=Z_2=0$ is shown as the thick line in the upper left plot in Fig. 13. Let us now consider the more conservative case ($Z_1=Z_2=1$)

\[
\xi_{FS} = \sqrt{\ln(1 + [(0.2606 + 0.0466) \times 0.3]^{2})} = 0.092
\]

\[
\lambda_{FS} = \ln \left[ (0.8585 - 0.06 \times 0.3^{0.85}) \times FS_d \right] - \frac{1}{2} \times 0.092^2
\]  
(23)

Hence

\[
p_t \approx \Phi \left[ \ln \left[ (0.8585 - 0.06 \times 0.3^{0.85}) \times FS_d \right] - \frac{1}{2} \times 0.092^2 \right]/0.092
\]  
(24)

The $p_t$ versus $FS_d$ relationship for $Z_1=Z_2=1$ is shown as the dashed line in the upper left plot. A very conservative case with $Z_1=Z_2=1.5$ is shown as the dotted line.

To understand the effect of $\delta/L_L$, $V$, and $\delta/L_\delta$ on the $p_t$ versus $FS_d$ relationship, the other three plots in Fig. 13 show the results when one of the above three factors in the baseline case is altered: the upper right plot reflects a change in $\delta/L_L$ to 0.1, the lower left plot reflects a change in $V$ to 0.5, and the lower right plot reflects a change in $\delta/L_\delta$ to 40. It is evident that $\delta/L_L$ and $V$ have a major effect, but $\delta/L_\delta$ has only a minor effect.

5.2. Simplified equation for reliability-based design

For the reliability-based design, the purpose is to determine the required $FS_d$ to achieve a prescribed target reliability index $\beta_T$. From Eqs. (17)–(20), it is clear that the most general form for $\beta_T$ is

\[
\beta_T = \Phi^{-1}(p_t) = \lambda_{FS}/\xi_{FS} = \frac{\left( \ln \left[ (1 - 0.115 \times a_V - 0.06 \times V^{0.85} \times Z_1) \times FS_d \right] - \frac{1}{2} \ln \left[ 1 + (0.2606 \times b_{h_i}/L_i \times b_{h_i}/b_i \times b_V + 0.0466 \times Z_2)^2 \times V^2 \right] \right)}{\sqrt{\ln \left[ 1 + (0.2606 \times b_{h_i}/L_i \times b_{h_i}/b_i \times b_V + 0.0466 \times Z_2)^2 \times V^2 \right]}}
\]  
(25)

Given the prescribed target reliability index $\beta_T$, the required $FS_d$ can be solved as follows:

\[
FS_d = \frac{\exp \left( \beta_T \sqrt{\ln \left[ 1 + (0.2606 \times b_{h_i}/L_i \times b_{h_i}/b_i \times b_V + 0.0466 \times Z_2)^2 \times V^2 \right]} + \frac{1}{2} \ln \left[ 1 + (0.2606 \times b_{h_i}/L_i \times b_{h_i}/b_i \times b_V + 0.0466 \times Z_2)^2 \times V^2 \right] \right)}{1 - 0.115 \times a_V - 0.06 \times V^{0.85} \times Z_1}
\]  
(26)

6. Calculation example

An example is presented here to show how the simplified reliability analysis and the reliability-based design of an undrained engineered slope can be done. Suppose the engineered slope (say, a cut slope) is analyzed using a deterministic FEA, and suppose the deterministic FS is found to be $FS_d=1.3$ and the resulting plastic zone is $L_f=33$ m. Based on the $s_{w(mob)}$ borehole data, $V$ and $\delta_c$ can be estimated. First, the trend $t(z)$ for the $s_{w(mob)}$ borehole data is estimated. This requires judgment, as mentioned earlier, e.g., for the $s_{w(mob)}$ data points in the left plot of Fig. 5, $t(z)$ is judged to consist of two line segments connecting at $z=1.8$ m, the $(\alpha, \beta)$ constants for each line segment $t=\alpha + \beta \times z$ can be estimated based on the $s_{w(mob)}$ data points. Once the trend $t(z)$ is obtained, the spatial variability $w(z)$ is estimated, the required $FS_d$ that achieves this target. The $FS_d$ can be estimated as

\[
V \approx \sqrt{\exp(\sigma_w^2)} - 1
\]  
(27)

where $\sigma_w^2$ is the sample variance of $w(z)$. Suppose it is determined that $\alpha \approx 1$ m and $V \approx 0.35$. It is then clear that $\delta/L_L \approx 0.03$. The $\delta_c$ information is typically not available, but $\delta/L_\delta$ is assumed to be 20 (it only has a minor effect). Let us consider a conservative $p_t$ estimate based on $Z_1=Z_2=1$. Given the above basic information, the following are the steps for the simplified reliability analysis:

Calculate the $(a, b)$ coefficients using Eqs. (7) and (13)

\[
a_V = (V/0.3)^{1.5} = 1.2601
\]

\[
b_{h_i}/L_i = \exp\left[3.1226 + 1.5027 \times \ln(\delta_\delta/L_i) + 0.1655 \times \ln(\delta/L_i)^2 \right] = 0.8943
\]

\[
b_{h_i}/b_i = \exp\left[-0.4999 + 0.1668 \times \ln(\delta_\delta/\delta_c) \right] = 1.0
\]

\[
b_V = \exp\left[-0.6349 - 0.731 \times \ln(V) - 0.1691 \times \ln(V)^2 \right] = 0.9476
\]  
(28)

Calculate $\lambda_{FS}$ and $\xi_{FS}$ using Eqs. (17) and (18)

\[
zeq
\]  
(29)

Calculate $p_t$ using Eq. (19)

\[
p_t \approx \Phi(-0.0723/0.0934) = 0.2194
\]  
(30)

For the reliability-based design, suppose the target reliability index $\beta_T=2.0$ (target $p_t=0.0228$). The purpose is to determine the required $FS_d$ that achieves this target. The $FS_d$ can be calculated by Eq. (26), namely

\[
FS_d = \exp \left( 2.0 \sqrt{\ln \left[ 1 + (0.2606 \times 0.8943 \times 1.0466 + 0.0466 \times 1)^2 \times 0.35^2 \right]} + \frac{1}{2} \ln \left[ 1 + (0.2606 \times 0.8943 \times 1.0466 + 0.0466 \times 1)^2 \times 0.35^2 \right] \right) / 1 - 0.115 \times 1.2601 - 0.06 \times 0.35^{0.85} \times 1
\]  
(31)

\[
= 1.4577
\]
7. Conclusions

The mean values and the coefficients of variation for the factors of safety (FS) of 34 idealized real engineered slopes have been investigated using a random finite element analysis (RFBA). The spatial variability of the undrained shear strength ($s_u$) has been simulated using a random field in the RFEA. Despite the variety in slope geometries and soil parameters among the 34 cases, some consistent phenomena seem to have been found:

(a) The mean value of FS is always less than the deterministic FS ($FS_d$) (reduction in the mean), and the reduction is more pronounced when the inherent coefficient of variation (COV) of the random field is large. This reduction in the mean is due to the fact that the critical slip curve seeks for the weak zone.

(b) The COV of FS is always less than the inherent COV of the random field (reduction in the variance), and the reduction is more pronounced when the ratio (vertical scale of fluctuation)/(length of failure curve) is small. This reduction in the variance is due to the averaging effect along the critical slip curve. The variance reduction is more pronounced when the inherent COV is large. However, this phenomenon cannot be easily explained.

(c) The above observations, (a) and (b), are consistent with the qualitative and quantitative conclusions made in Ching and Phoon (2013) and Ching et al. (in press), although these two studies are for soils subjected to uniform stress states.

(d) The ratio (horizontal scale of fluctuation)/(vertical scale of fluctuation) has a minor effect on the mean and COV of FS.

Based on the RFEA results, a simplified relationship between the probability of failure ($p_f$) and $FS_d$ is proposed, and simple steps are proposed for the reliability analysis and the reliability-based design for undrained engineered slopes with spatially variable shear strengths.

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